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STRESSES IN TALL BUILDINGS

BY

CYRUS A. MELICK

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STRESSES IN TALL BUILDINGS

AN INVESTIGATION OF THE STRESSES IN TALL STEEL BUILDINGS OF THE CAGE CONSTRUCTION TYPE WITH PORTAL BRACING

BY

CYRUS ALAN MELICK, D.C.E.

COLUMBUS, OHIO
1912

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INTRODUCTION.

This investigation has been undertaken with a three-fold purpose in view, as follows:

First, to develop a practicable mathematical solution of the stresses in tall buildings with portal bracing for all types of loadings, and, by its application, to determine the principal errors which exist in present methods of design.

Second, to endeavor by experiment on an existing structure to find by actual measurement the distribution and amount of stresses and distortions due to wind pressures, the only loading which can be practically varied on such a structure.

Third, to give in a final chapter a full and definite resumé of all findings and suggestions for designers as derived therefrom, so as to make the results of the investigation of value to the practicing engineer or architect without wasting his time in perusal of the body of the investigation, a proceeding which would, it is believed, be a discouraging one for a man who has been too busy in practical work to follow up closely a mathematical discussion.

This investigation consists, then, of a series of mathematical demonstrations, followed by an extensive series of applications and comparisons, and, finally, what is believed to be some very valuable conclusions offered in the last chapter, with references to the chapters from which the conclusions are drawn in case more detailed information is desired. Results have been shown graphically as far as possible, in addition to giving the actual tabular values. It is believed the investigation will be equally of value to the practical as well as to the theoretical man.

The mathematical treatment is based on an entirely elastic structure with what is believed to be an entire freedom from erroneous or doubtful assumptions.

This study has been made possible through the generosity of the late Professor Stillman W. Robinson, who established at the Ohio State University the Stillman W. Robinson Fellowship in Engineering. The writer has, during the years 1909–11, endeavored to carry out the purpose Professor Robinson had in view in establishing the fellowship and now takes great pleasure in presenting to the Ohio State University and, through this institution, to the engineering profession generally, the results of a careful and diligent study of the stresses in tall buildings.

Acknowledgments are made and the author's sincere thanks extended as follows:

To the Ohio State University for financing the investigation.

To the Robinson Fellowship Committee for interest shown and advice given.

To Professor C. T. Morris, professor of structural engineering, for the interest shown, and advice and suggestions given throughout the work.

To Mr. J. Warren Smith, Section Director of the U. S. Weather Bureau.

To Mr. Frank L. Packard, architect, and his engineer, Mr. E. F. Babbitt, for furnishing plans and data.

To Mr. A. L. Fisher, superintendent of the structure used for the experiments.

Many others have, by suggestions and minor aid, been of assistance, and thanks are hereby extended to them collectively.

CYRUS A. MELICK.

COLUMBUS, OHIO,
June, 1911.

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NOTATION.

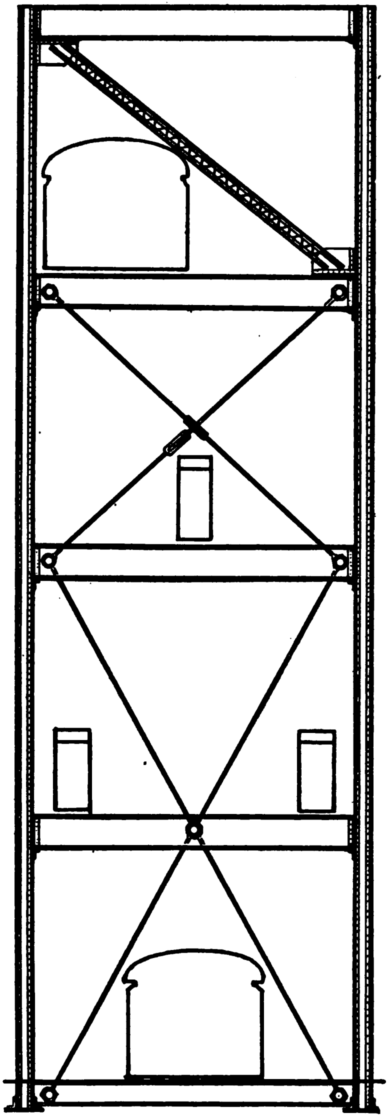
The following symbols are used throughout this investigation with the designations here given except in cases where there can be no confusion and in such cases a note is made of the exception.

Subscripts used in connection with the symbols below or any other symbols given later on always designate the floor in which the quantity occurs unless otherwise noted.

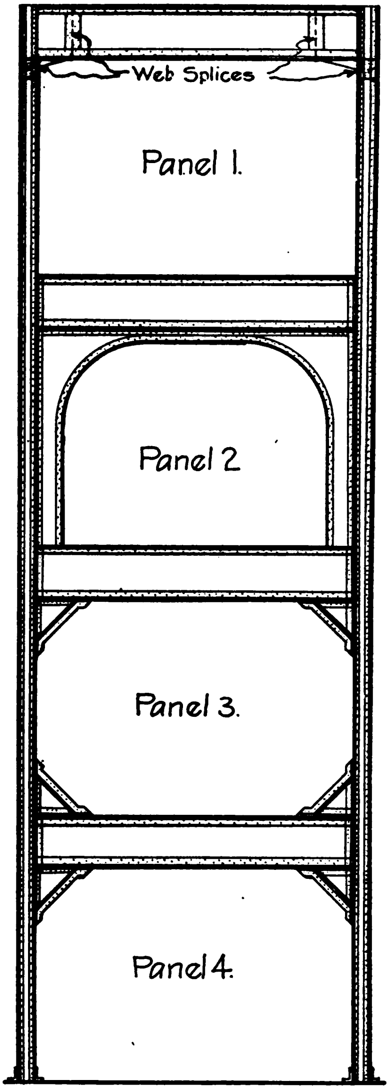
Exponents used in connection with the symbols below or any other symbols given later on always designate the member to which the symbol applies or the expression or equation in which it occurs unless otherwise noted. Thus I_3^A represents the moment of inertia of the third floor section of column A.

All the following quantities are in inch and pound units.

I = moment of inertia.
A = area of cross section.
E = modulus of elasticity.
b = distance center to center of columns.
c = story height measured between centers of gravity of girder flanges.
d = depth of floor girder, center to center of gravity of flanges.
V = axial compressive stress in a column.
H = axial compressive stress in a floor girder.
S = horizontal shear across a column section.
M = bending moment at foot of a column section ($x = 0$).
s = axial unit stress.
s' = extreme fibre unit stress from bending only.
f = resultant extreme fibre unit stress.
r = minimum radius of gyration.
v = distance from neutral axis of column to the extreme fibre.
W = total load on any floor girder (considered as uniform).
w = floor load per lineal inch of girder.
P = concentrated load applied directly or indirectly to a column.
F = concentrated wind pressure for one story—considered as applied to the building at mid depth of the floor girders.



Diagonal. Bracing.
Fig 1a



Portal Bracing.
Fig 1b.

of this class are statically determinate and the ease with which stresses may be computed and the design facilitated make this class a desirable structure. However, the diagonal bracing required imposes the condition that a comparatively thick partition be placed in the plane of the bracing for cover and protection of the steel. In the ordinary office building this condition divides the floor surface into sets of box-like suites, usually of the same shape and size. No large opening, no freedom in the selection of a position for an opening or corridor—in short, no effective architectural medium—can be used for joining adjacent suites through such a partition. Usually the doors must be small and either squarely in the center of a panel or on one side, according to the type of diagonal bracing used. Corridors must either extend along the wall, thus cutting off a great deal of window light from the offices, or down the center of a panel, creating a row of offices on each side of the hall. These limitations would often result in very shallow offices and sometimes in no direct communication between suites. By referring to Fig. 1, *a*, it will be seen that, from an architectural standpoint, single rigid diagonal bracing is preferable to the intersection systems of tension bars.

In practice the diagonal members frequently can not be designed so as to have the gravity axes of columns, girders and diagonal members meet in a common point and there result some bending stresses in the columns and girders. These are usually small and frequently neglected in the calculations. The floor girders in this class of buildings are usually shallow and, though the connections are riveted, they can not be said to be rigidly connected to the columns because the flanges are but very meagrely connected. In all calculations it is sufficiently accurate to consider them as connected to the columns by pins. The bending moments in the columns and girders, due to whatever rigidity these connections may have is thus neglected. As will appear from a later part of this investigation these stresses are certainly comparatively small. Buildings of Class *A* will receive but small consideration in the following pages.

Class B Buildings.—There are several styles of bracing used in this class of buildings; these are best comprehended by referring to Fig. 1, *b*.

"Panel 1" illustrates plain plate girder portal bracing in which the girders are made as deep as the design will permit and, as will be noticed, are *rigidly* fastened to the columns. *This can only be accomplished by properly connecting the flange angles as well as the web to the column.* This requirement is very rarely fulfilled in practice, because the large bending moments at connections require so many rivets to connect the flange angles that an extension of the web of the column is necessary to make the connections properly and this type of connection is difficult to make in the field. Panels 2, 3, and 4 show the common but very unsatisfactory method of making these connections.

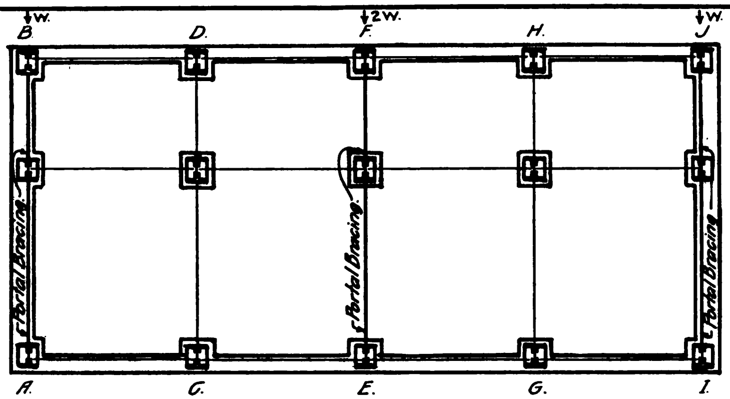
"Panel 2" illustrates a type of bracing which has been employed to some extent. It consists of a solid web plate, out of which the arches' opening is deducted, stiffened by angles bent to the curve of the opening. The bracing is field riveted by means of connection angles to the flanges of the columns and girders. This type of bracing is not so simple of analysis as plate girder portal bracing, it does not transfer stress directly to the neutral axes of columns and girders as a proper design should do, and the resistance of the girder connection itself is usually neglected in the calculations. This is not an economical form of bracing and has the added fault that some of the rivets in the connections are stressed in tension on the rivet heads.

"Panel 3" illustrates full knee braced plate girder portal bracing and this type, when the girders are properly connected, is undoubtedly the best and most economical of all types of portal bracing. The knee bracing may be merely one or two angles or, preferably, one or two angles and a solid triangular web as shown. In general no appreciable advantage is gained by making top and bottom knees of different depths. The knees should not be deep, as, in such case, the bottom knees will materially interfere with the floor space.

"Panel 4" is the same as "Panel 3" except for the omission of the bottom knees. Very heavy knees must here be used if any appreciable reduction in the bending stress at the foot of the column is to be effected.

The above statements concerning knee bracing will be fully verified in this investigation.

The arrangement of portal bracing, in plan, may be best discussed by reference to Fig. 2. Evidently sections *AB* and *IJ* are stiffened very materially by the rear and front walls respectively, while section *EF* has nothing to stiffen it but light partitions and even these are usually very much cut up by openings for doors, light, etc., and are subject to removal at any time. Therefore, if wind bracing is to be provided at section *EF* it should be stronger than that which is provided at sections *AB* and *IJ* irrespective of the fact that it gets twice as much wind load. If no bracing is placed in sections *CD* and *GH* then the wind coming to these sections must reach sections *AB*, *EF*, and *IJ* by being transferred through the floors alone, at each floor level. The girders on sections *CD* and *GH*, in such a case, need only be designed for floor loads and connected to the columns by ordinary methods and thus may well be considered as having hinged connections. It does not seem likely that the columns on these sections can receive any appreciable wind stress, either axial or bending. The other girders must be designed to act as wind bracing, in addition to supporting the floor, and the connecting columns must be proportioned for both axial and bending stresses from wind. The proper selection of sections for the economical disposition of wind bracing must remain a matter of judgment.



Plan... Showing Location of Bracing.
Fig. 2.

CHAPTER II.

DESIGN OF PORTAL BRACED BUILDINGS — RIGID GIRDERS.

This chapter is devoted to the design of plain, plate girder portal braced structures. A simple modification of the methods here given, required when knee braces are used, is given in Chapter XII.

General Method.

The following assumptions are involved in this discussion:

(*a*) That the floor-girders are absolutely rigid, *i. e.*, that they neither bend nor change in length when subjected to stress. Assuming the girders to be properly connected to the columns, this assumption implies that the columns are straight for the full depth of the girders and that all columns at floor levels have the same slopes and lateral deflections.

(*b*) The columns are assumed to bend under the action of lateral forces, but not to be affected in length by axial stresses. This implies that floor girders will always be horizontal and the columns vertical at floor levels

(*c*) That the columns at their anchorages will always be maintained in a fixed vertical direction.

(*d*) That the elastic limit of the material will never be exceeded.

(*e*) That the column material shall be uniform throughout the structure.

(*f*) The increase of bending moment leverage due to the deflection of the columns is neglected.

(*g*) That the distribution of axial unit stresses among the several columns will be the same as if the building were considered as a solid beam, *i. e.*, that these unit stresses will vary directly as the distance from a neutral axis.

It is evident at a glance how impossible are assumptions *a* and *b*, and what a serious disagreement exists between these assumptions and assumption *g*. Nevertheless, the method based on these assumptions will, in later chapters, be shown to be sufficiently reliable for taking care of wind pressures (with a slight exception) and the simplicity of the method has been the factor in commending it to the engineering profession.

SLOPES, BENDING STRESSES AND DEFLECTIONS OF COLUMNS.

Referring to Fig. 3, *b*, which is a general figure for any column section in the building:

At any point $P \equiv (x, y)$ on the elastic line of the column section there exists a bending moment $= M - Sx - Vy$ which, when the small term Vy is neglected, becomes $M - Sx$.

It is shown by the differential calculus that, sufficiently accurate for the purpose, the radius of curvature of the elastic line at any point is expressed as the reciprocal of d^2y/dx^2 at that point. From this and the experimentally verified principle that stress and the accompanying deformations are proportional within the elastic limit, it is further shown in mechanics that $EI(d^2y/dx^2) =$ the bending moment at P. I will not vary in the distance c and E is constant.

The differential equation of the elastic line of the column section is then written

$$E\frac{d^2y}{dx^2} = \frac{M}{I} - \frac{S}{I}x.$$

It will be noticed that the origin of coordinates is at the foot of the column section whose length is c. Integrating the above expression it is found that

$$E\frac{dy}{dx} = \frac{M}{I}X - \frac{S}{2I}x^2 + K$$

where K is a constant of integration. Integrating this expression it is found that

$$Ey = \frac{M}{2I}x^2 - \frac{S}{6I}x^3 + Kx + K'$$

where K' is also a constant of integration. But $dy/dx = 0$ when $x = 0$, hence $K = 0$ and therefore

$$E\frac{dy}{dx} = \frac{M}{I}x - \frac{S}{2I}x^2.$$

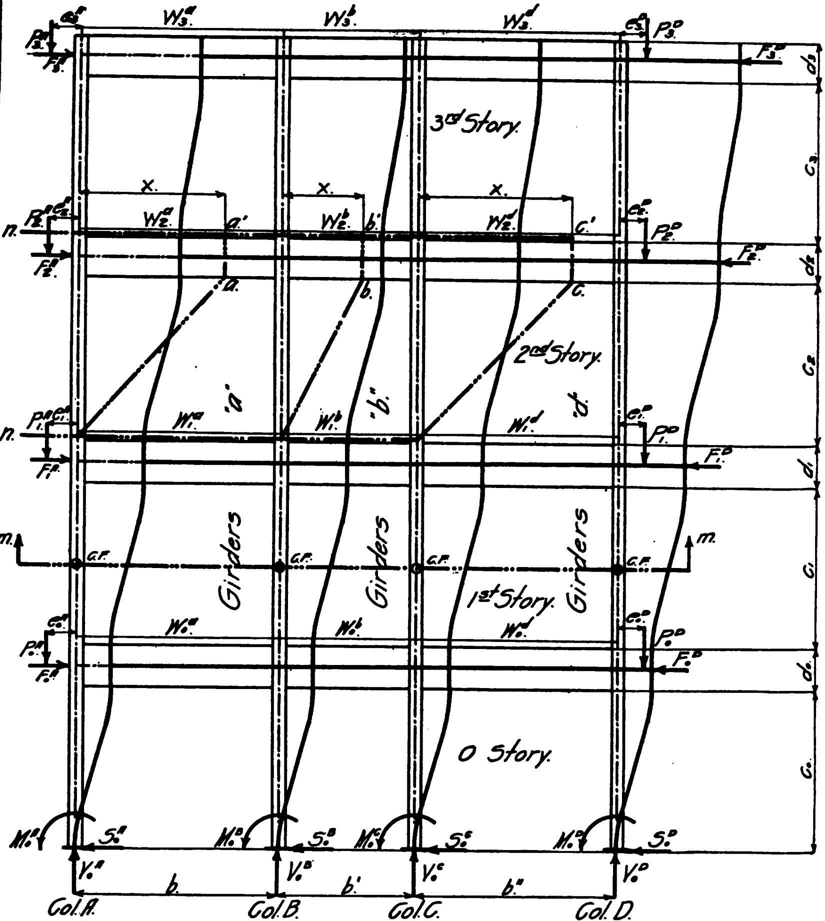


FIG. 3a.

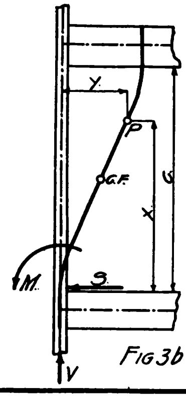


FIG 3b

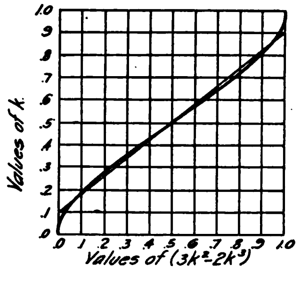


FIG 3c.

Also $y = 0$ when $x = 0$, hence $K' = 0$ and therefore

$$Ey = \frac{M}{2I}x^2 - \frac{S}{6I}x^3.$$

But $dy/dx = 0$ when $x = c$, hence $M = S\,(c/2)$ and therefore CF, *the point of contra-flexure, always occurs at mid height regardless of type or amount of loading.*

Substituting the value of M, just found, in the equation of the elastic line the equation becomes

$$Ey = \frac{S}{12I}(3cx^2 - 2x^3)$$

or, letting $x = kc$ where k is a decimal fraction,

$$Ey = \frac{Sc^3}{12I}(3k^2 - 2k^3).$$

By assigning to k values varying from 0 to 1, the shape of the elastic line is determined and this curve is shown in Fig. 3*c*. The shape of the curve is the same for all column sections in the building, the *amount* of the deflection being proportional to Sc^3/I. The light straight line shown in Fig. 3*c* suggests a quick approximate method for constructing these curves, the major part of the curve being nearly straight.

When

$$x = c, \quad Ey = \delta = \frac{Sc^3}{12I};$$

but, since the girders are rigid

$$\delta^A = \delta^B = \delta^C = \delta^D$$

(Fig. 3, *a*). Hence

$$\frac{S^A c^3}{12I^A} = \frac{S^B c^3}{12I^B} = \frac{S^C c^3}{12I^C} = \frac{S^D c^3}{12I^D}$$

or

$$\frac{S^A}{I^A} = \frac{S^B}{I^B} = \frac{S^C}{I^C} = \frac{S^D}{I^D}$$

or

$$S^B = S^A\frac{I^B}{I^A}; \quad S^C = S^A\frac{I^C}{I^A}; \quad S^D = S^A\frac{I^D}{I^A}.$$

Now $S^A + S^B + S^C + S^D = \Sigma F$, the total wind shear on a section of the building cutting columns A, B, C, and D. Therefore, substituting the above values of S^B, S^C, and S^D and solving,

$$S^A = \frac{I^A}{I^A + I^B + I^C + I^D}\Sigma F$$

and therefore

$$S^B = \frac{I^B}{I^A + I^B + I^C + I^D}\Sigma F; \quad S^C = \frac{I^C}{I^A + I^B + I^C + I^D}\Sigma F$$

and

$$S^D = \frac{I^D}{I^A + I^B + I^C + I^D}\Sigma F.$$

It must now be clear that the bending moment in a column, by this theory is entirely independent of both floor and eccentric column loads, depending entirely on lateral or wind forces, and that this moment is maximum and equal at the two extremities of each column section and at these points amounts to $S(c/2)$. Also, since ΣF and $I^A + I^B + I^C + I^D$ are constant for any given story the shears and maximum moments in the several columns are directly proportional to the moments of inertia of the several respective columns.

AXIAL STRESSES IN COLUMNS.

It has often been suggested that a building under the action of wind pressure acts like a solid beam. Referring to Fig. 4, *a*, let NA be the neutral axis of the structure shown, in accordance with the beam theory and let s be the unit axial compressive stress at units distance to the right of NA—due to flexure alone.

Now, passing a horizontal section "*mm*" through the points of contra-flexure of the columns, suppose there exists at P (Fig. 4*a*) a clockwise external bending moment X due to forces of any nature acting on the upper part of the structure cut by "*mm*." If any or all of the load producing this bending moment X is vertical, the resultant unit axial stresses in the columns will be represented by Fig. 4, *b*, as the quantities AA', BB', CC', DD', and EE'. The unit stresses shown in heavy lines and marked s^A, s^B, s^C, s^D, and s^E are the stresses due to the bending moment X alone, while the unit stresses $AA'' = BB'' = CC'' = DD'' = EE''$ are due to the effect the vertical load would have if concentrated at the neutral axis. If Y represents the amount of this vertical load and the unit stress AA'' be represented by P, then

$$P(A^A + A^B + A^C + A^D + A^E) = Y$$

or

$$P = \frac{Y}{A^A + A^B + A^C + A^D + A^E}.$$

It must be evident from this equation that the "beam assumption" will not hold for vertical loads, for

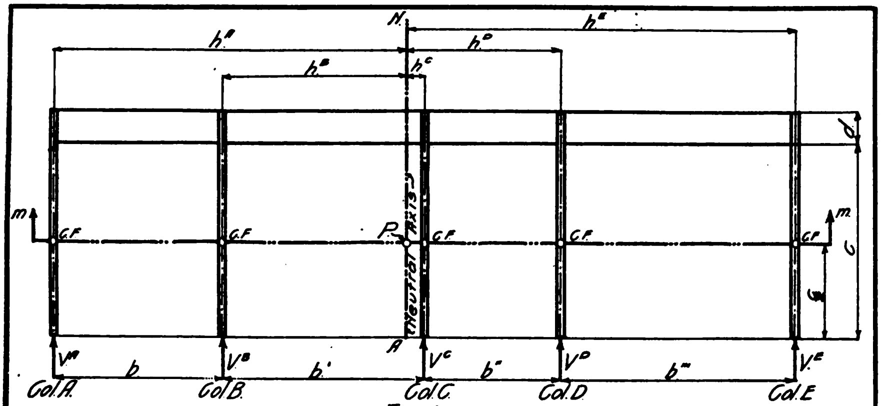


FIG. 4a

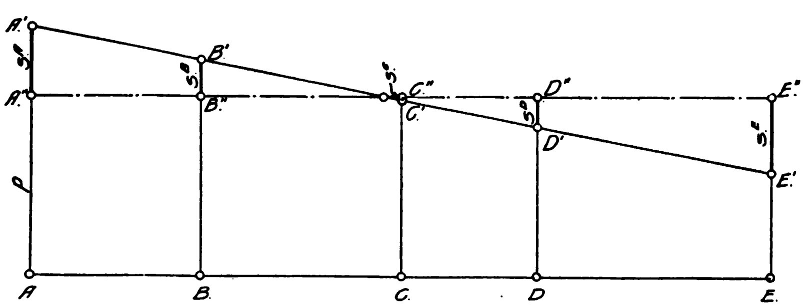


FIG. 4b

Suppose a heavy concentrated load placed vertically over P (Fig. 4, a), Columns A, B, C, D, and E would, by this theory, settle equal amounts under the load and receive the same unit stress. Considering the *very small size* of this settlement an engineer would not be justified in estimating any of this load to be carried through the girders to columns A, D, or E, but should estimate it all as going to columns B and C. However, since buildings are usually designed for constant uniform loads covering the entire floor and since the areas A, of the columns, are estimated roughly on the basis of a constant value of ρ for any floor, the above method would not differ widely, as a rule, from more practical methods. It is not the purpose of this chapter to give the suggestions and methods resulting from this investigation (see Chapter XVI), but rather to develop and apply a consistent theory based on rigidity of floor girders.

Referring now to the unit stresses due to the bending moment X alone, these stresses may be written

$$s^A = -h^A s; \; s^B = -h^B s; \; s^C = h^C s; \; s^D = h^D s; \; s^E = h^E s,$$

since the unit stress from flexure on a beam is zero at the neutral axis and varies directly as the distance from the neutral axis. The sum of all of the total column stresses due to X must equal zero. Hence

$$s^A A^A + s^B A^B + s^C A^C + s^D A^D + s^E A^E = 0$$

or, substituting the above values of s^A, s^B, s^C, s^D and s^E and remembering that

$$h^A = b + b' - h^C; \; h^B = b' - h^C; \; h^D = b'' + h^C$$

and

$$h^E = b'' + b''' + h^C$$

there results

$$s[-A^A(b + b') - A^B b' + h^C(A^A + A^B + A^C + A^D + A^E) + A^D b'' + A^E(b'' + b''')] = 0$$

or

$$h^C = \frac{A^A(b + b') + A^B b' - A^D b'' - A^E(b'' + b''')}{A^A + A^B + A^C + A^D + A^E}.$$

Thus, after estimating the column areas the position of the neutral axis is readily determined; then h^A, h^B, h^D, and h^E, are evaluated as above. Now in order to balance the bending moment X the following relation must exist, viz.,—

$$V^A h^A - V^B h^B + V^C h^C + V^D h^D + V^E h^E = X$$

where

$$V^A = s^A A^A;\quad V^B = s^B A^B;\quad V^C = s^C A^C;\quad V^D = s^D A^D;\quad V^E = s^E A^E.$$

Making the substitution there results

$$s[A^A h^{A^2} + A^B h^{B^2} + A^C h^{C^2} + A^D h^{D^2} + A^E h^{E^2}] = X,$$

from which s is readily found. Now s^A, s^B, s^C, s^D, and s^E may be found from relations previously given and then the column stresses due to X, viz., V^A, V^B, V^C, V^D, and V^E. A short summary of the method of designing columns is now given as follows:

First, assume the depths of girder and story heights; all the columns can then be designed independently of the girders.

Second, estimate roughly the width of column web plate such that the minimum top sections will not be too wasteful of material and the bottom sections will not be uneconomical due to a too small radius of gyration.

Third, start at the top of the building so that column and fireproofing weights may be added as the design progresses. Compute and tabulate values of X and Y for each floor.

Fourth, estimate by trial the sections for the top floor, computing the position of the neutral axis and the axial and flexural unit column stresses, rough approximations being taken as a start and the design revised until satisfactory. Then proceed to the next floor below.

It is well to note here that there are two methods in use in arranging column sections. They differ only in the location of column splices. In the early history of tall buildings column splices were considered points of weakness and there originated the method of "staggered splices," *i. e.*, only every alternate column spliced at a floor. In later practice column splices and confidence in them improved; when properly made they may be considered stronger than the main section of the column. On this basis the columns may as well be all spliced at the same floor, thus facilitating erection. Especially is this method valuable in a reinforced concrete building, allowing all columns and floor slabs for the same floor to be poured at one time. It should be remembered that columns are usually designed in two story lengths for economy in shipping, erection, and number of splices, two stories usually making about a car length (30 feet). When splices are "staggered" the method of approximating and recalculating is a little more difficult. The example given later illustrates this case.

STRESSES AND DESIGN OF FLOOR GIRDERS.

Referring to Fig. 3, *a*, stresses are obtained as follows:

Girder A_2—Top Flange.—Pass a section *naa'n* as shown; then, taking moments around point "*a*" the top flange compressive stress equals

$$\frac{1}{d_2}\left\{S_2^A c_2 + S_3^A d_2 + (V_2^A - V_3^A)x + F_2^A\frac{d_2}{2} - P_2^A(e_2^A + x) - \frac{w_2x^2}{2} - M_2^A + M_3^A\right\}$$

$$= S_2^A\frac{c_2}{2d_2} + S_3^A\frac{(c_3+2d_2)}{2d_2} + \frac{F_2^A}{2} - P_2^A\frac{(e_2^A+x)}{d_2} + (V_2^A - V_3^A)\frac{x}{d_2} - \frac{w_2x^2}{2d_2}.$$

The maximum compression in the top flange is usually given by this expression when $x = 0$ and the maximum tension when $x = b$, changing the signs throughout.

Girder A_2—Bottom Flange.—Using the same section as above and taking moments around point a' the bottom flange tensile stress equals

$$S_2^A\frac{(c_2+2d_2)}{2d_2} + S_3^A\frac{c_3}{2d_2} - \frac{1}{2}F_2^A - P_2^A\frac{(e_2^A + x)}{d_2} + (V_2^A - V_3^A)\frac{x}{d_2} - \frac{w_2x^2}{2d_2}.$$

The maximum tension in the bottom flange is given when $x = 0$ and the maximum compression when $x = b$, changing signs throughout.

Girder b_2—Top Flange.—Pass a section *nbb'n* as shown and take moments about *b*. The top flange compressive stress equals

$$(S_2^A + S_2^B)\frac{c_2}{2d_2} + (S_3^A + S_3^B)\frac{(c_3 + 2d_2)}{2d_2} + \tfrac{1}{2}F_2^A - P_2^A\frac{(e_2^A + b + x)}{d_2}$$

$$+ (V_2^A - V_3^A)\frac{b}{d_2} + (V_2^A - V_3^A + V_2^B - V_3^B)\frac{x}{d_2} - \frac{w_2b^2}{2d_2} - \frac{w_2\left(bx + \frac{x^2}{2}\right)}{d_2},$$

the maximum compression occurring for $x = 0$ and maximum tension for $x = b'$, changing all signs.

Girder b_2—Bottom Flange.—Using the same section as above and taking moments around point b' the bottom flange tensile stress equals

$$(S_2^A+S_2^A)\frac{(c_2+2d_2)}{2d_2}+(S_3^A+S_3^B)\frac{c_3}{2d_2}-\tfrac{1}{2}F_2^A-P_2^A\frac{(e_2^A+b+x)}{d_2}$$
$$+(V_2^A-V_3^A)\frac{b}{d_2}+(V_2^A-V_3^A+V_2^B-V_3^B)\frac{x}{d_2}-\frac{w_2b^2}{2d_2}-\frac{w_2\left((bx+\frac{x^2}{2}\right)}{d_2},$$

the maximum tension occurring for $x = 0$ and maximum compression for $x = b'$, changing all signs.

Girder c_2—Top Flange.—Pass a section *ncc'n* as shown and take moments about *c*. The top flange compressive stress equals

$$(S_2^A+S_2^B+S_2^C)\frac{c_2}{2d_2}+(S_3^A+S_3^B+S_3^C)\frac{(c_3+2d_2)}{2d_2}$$
$$+\tfrac{1}{2}F_2^A-P_2^A\frac{(e_2^A+b+b'+x)}{d_2}+(V_2^A-V_3^A)\frac{b}{d_2}$$
$$+(V_2^A-V_3^A+V_2^B-V_3^B)\frac{b'}{d_2}+(V_2^A-V_3^A+V_2^B-V_3^B+V_2^C-V_3^C)\frac{x}{d_2}$$
$$-\frac{w_2(b+b')^2}{2d_2}-\frac{w_2\left[(b+b')x+\frac{x^2}{2}\right]}{d_2},$$

the maximum compression occurring for $x = 0$ and maximum tension for $x = b''$, changing all signs.

Girder c_2—Bottom Flange.—Using the same section as above and taking moments around point c' the bottom flange tensile stress equals

$$(S_2^A+S_2^B+S_2^C)\frac{(c_2+2d_2)}{2d_2}+(S_3^A+S_3^B+S_3^C)\frac{c_3}{2d_2}$$
$$-\tfrac{1}{2}F_2^A-P_2^A\frac{(e_2^A+b+b'+x)}{d_2}+(V_2^A-V_3^A)\frac{b}{d_2}$$
$$+(V_2^A-V_3^A+V_2^B-V_3^B)\frac{b'}{d_2}+(V_2^A-V_3^A+V_2^B-V_3^B+V_2^C)-V_3^C)\frac{x}{d_2}$$
$$-\frac{w_2(b+b')^2}{2d_2}-\frac{w_2\left[(b+b')x+\frac{x^2}{2}\right]}{d_2},$$

the maximum tension occurring for $x = 0$ and maximum compression for $x = b''$, changing all signs.

The above girder equations have been written for the second story. For any other story equations may be written by merely changing the subscripts in these to correspond with the story under consideration. It must also be clear now how the equations change for changes in the number of panels constituting the width of the building.

THE DESIGN OF A PLAIN PLATE GIRDER PORTAL BRACED BUILDING — GIRDERS ASSUMED RIGID.

The method just given will now be applied to a five story structure with only two columns in a cross section. (See Fig. 5, *a*.) A structure of this simple type is taken as fully illustrating the method of design and as a basis for the investigation of stresses in future chapters on the assumption of a *perfectly elastic structure.* Fig. 5, *a*, shows the dimensions of the structure and the location of the staggered column splices. Fig. 5, *b*, shows the location of floor beams and area contributing load to the columns and girders. All floors are arranged alike and *every* transverse column section of the structure is considered as portal braced; it is believed that this is most economical and gives a more uniform design. Figs. 5*c* and 5*d* show the location and eccentricity of wall beams, the connection of these beams to the columns and the position of the front and rear walls relative to the columns. In this design the floor load is not considered as uniform but as concentrated at the beam connections.

Unit Loads—Live Load.—Roof, 30 ℔ per. sq. ft.; 1st, 2d, and 3d floors, 50 ℔ per sq. ft.; 0 floor, 100 ℔ per sq. ft.

Dead Load.—Roof and 1st, 2d, and 3d floors, 90℔ per sq. ft.; 0 floor, 110 ℔ per sq. ft.; walls, face, 94 ℔ per sq. ft. of surface (75 per cent of solid wall), rear, 125 ℔ per sq. ft. of surface (no windows); parapet on face, extending 5 feet above centre of roof girder @ 125 ℔ per sq. ft. of surface; column fireproofing, 100 ℔ per lineal foot of column.

Wind Load.—30 ℔ per sq. ft. of exposed surface. Consider that the wind may act on face or rear and that the exposed surface of face and rear are equal, *i. e.*, as if the rear wall also had a parapet.

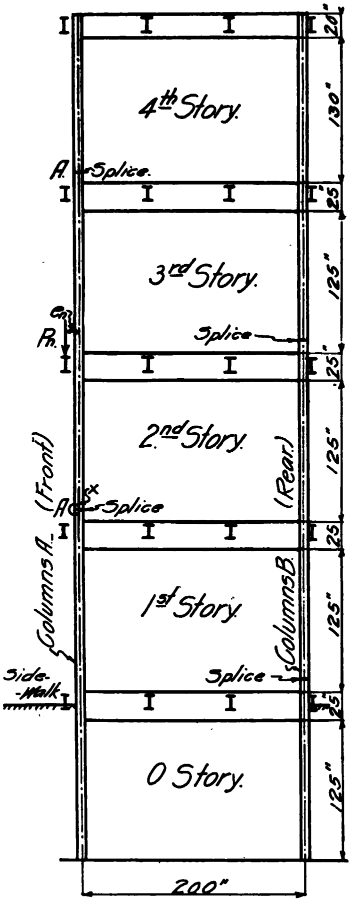


FIG 5a.

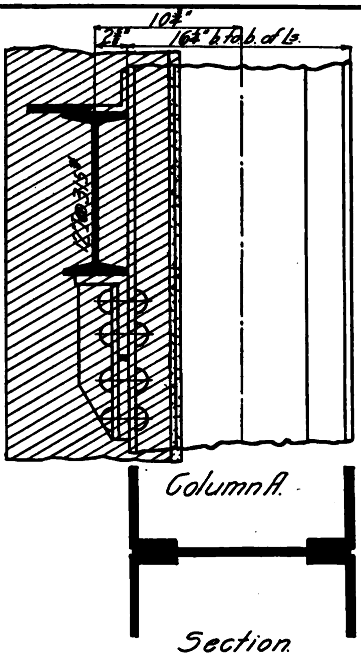


FIG 5c.

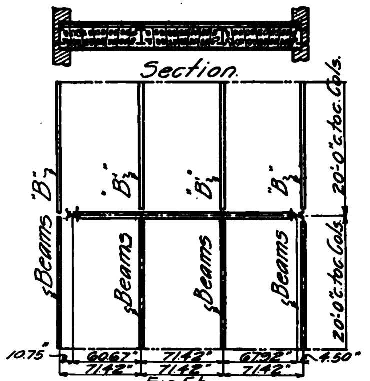


FIG 5b.

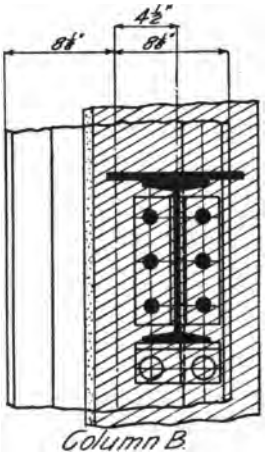


FIG 5d.

It will be noted that no reduction of live load on floors is provided for in the column designs. It is desired to keep girder and column loading identical for the purposes of this discussion.

Allowed Unit Stresses.—

Tension, 16,000 ℔ per sq. in., net.

Compression, 16,000–70 L/r per sq. in. gross.

Bending-tension fibre, 16,000 ℔ per sq. in., net.

Bending-comp. fibre, 16,000–200 L/b per sq. in., gross.

Shear on webs of girders, 10,000 ℔ per sq. in., gross.

L = unsupported length;
r = least radius of gyration;
b = width of girder flange;
L, r, and b in inches.

An increase in the above units of 25 per cent. will be allowed when wind stresses are included, but, if this requires less material than is required for the other stresses, exclusive of wind, at the above units, the wind stresses shall be disregarded.

A rough trial shows that 16 in. may be used for the width of the column web plates Figs. 5, *c*, and 5, *d*, show that the concentrated loads P^A will have an eccentricity of 10.75 in. while the loads P^B will have an eccentricity of 4.50 in. In order that all floor arch spans may be the same the spacing of floor beams is made constant as shown in Fig. 5, *b*.

Values of P^A Due to Wall Load Only.—

$P_4^A = 20.0$ ft. × 5.0 ft. × 125 ℔ = 12,500 ℔.

$P_3^A = P_2^A = P_1^A = P_0^A = 20.0$ ft. × 12.5 ft. × 94 ℔ = 23,500 ℔.

Values of P^B Due to Wall Load Only.—

$P_4^B = 0$.

$P_3^B = P_2^B = P_1^B = P_0^B = 20.0$ ft. × 12.5 ft. × 125 ℔ = 31,225 ℔.

Values of $F^A = F^B$.—

$F_4^A = 20.0$ ft. (5.0 ft. + 6.25 ft.) × 30 ℔ = 6,750 ℔.

$F_3^A = F_2^A = F_1^A = 20.0$ ft. × 12.5 ft. × 30 ℔ = 7,500 ℔.

$F_0^A = \frac{1}{2}F_3^A = 3{,}750$ ℔.

Floor Beam Concentration.—Let W be the concentration from floor beams B, and W', that from beams B'. Then

$$W_4 = \frac{71.42\text{ in.}}{24} \times 20.0\text{ ft.} \times (30\text{ ℔} + 90\text{ ℔}) = 7{,}142\text{ ℔};$$

$$W_4' = 2W_4 = 14{,}284\text{ ℔}.$$

$$W_3 = W_2 = W_1 = \frac{71.42\text{ in.}}{24} \times 20.0\text{ ft.} \times (50\text{ ℔} + 90\text{ ℔}) = 8{,}325\text{ ℔};$$

$$W_3' = 2W_3 = W_2' = W_1' = 16{,}650\text{ ℔}.$$

$$W_0 = \frac{71.42\text{ in.}}{24} \times 20.0\text{ ft.} \times (100\text{ ℔} + 110\text{ ℔}) = 12{,}500\text{ ℔};$$

$$W_0' = 2W_0 = 25{,}000\text{ ℔}.$$

The general formulae for determining column stresses reduce in this case 25 to the following.

$$h^B = \frac{bA^A}{A^A + A^B} = 200 \times \frac{A^A}{A^A + A^B}, \quad h^A = 200 - h^B, \quad p = \frac{Y}{A^A + A^B}.$$

$$X = h^B Y + \Sigma P^B e^B - \Sigma p^A (b + e^A) - 205.26 \Sigma W' + X_w,$$

where X_w is the value of X for wind only; this will be plus if the wind blows on the front and minus if the wind blows on the rear of the building.

$$s = \frac{X}{A^A h^{A^2} + A^B h^{B^2}}, \quad s^A = -h^A s, \quad s^B = h^B s,$$

$$s'^A = \frac{I^A}{I^A + I^B} \times \frac{\Sigma F \frac{c}{2}}{\frac{I^A}{V^A}}, \quad s'^B = \frac{I^B}{I^A + I^B} \times \frac{\Sigma F \frac{c}{2}}{\frac{I^B}{V^B}}.$$

The maximum compressive unit stress in column $A = p + s^A + s'^A$ and occurs for wind blowing on rear.

The maximum compressive unit stress in column $B = p + s^B + s'^B$ and occurs for wind blowing on front.

Data for column design is given in the following table.

Floor.	Total P^A, lbs.	ΣP^A, lbs.	Total P^B, lbs.	ΣP^B, lbs.	W′, lbs.	$\Sigma W'$, lbs.	*Y, lbs.	$F^A = F^B$.	ΣF, lbs.
4	19,642	19,642	7,142	7,142	14,284	14,284	55,352	6,750	6,750
3	31,825	51,467	39,550	46,692	16,650	30,934	160,027	7,500	14,250
2	31,825	83,292	39,550	86,242	16,650	47,584	264,702	7,500	21,750
1	31,825	115,117	39,550	125,792	16,650	64,234	369,377	7,500	29,250
0	36,000	151,117	43,725	169,517	25,000	89,234	499,102	3,750	33,000

* These values are exclusive of column weights and column fireproofing, for which values will be added as the columns are designed. The various trials made in securing the following sections will not be given; in any given case these trials will be governed by the experience of the designer.

Fourth Story—Columns.—

For Column 4–*A* use

$$\left.\begin{array}{l} 4\angle\text{'s, } 3 \times 2\tfrac{1}{2} \times \tfrac{5}{16} \\ \\ 1 \text{ Pl., } 16 \times \tfrac{5}{16} \end{array}\right\} \begin{array}{l} A = 11.52 \text{ sq. in.} \\ I = 461.0 \\ r = 1.15 \text{ in.} \\ \frac{I}{v} = 57.7 \\ w_c = 39.0 \text{ lb} \end{array}$$

Y_4 (from table) =		55,350 lb
Wgt. Col. 4–*A*	= 12.5 ft. × 39.0 lb =	490 lb
Wgt. Col. 4–*B*	= 12.5 ft. × 51.8 lb =	650 lb
F.P. Col. 4–*A*	= 12.5 ft. × 100 lb =	1,250 lb
F.P. Col. 4–*B*	= 12.5 ft. × 100 lb =	1,250 lb
Total value of Y_4 =		58,990 lb

For Column 4–*B* use

$$\left.\begin{array}{l} 4\angle\text{'s, } 5 \times 3\tfrac{1}{2} \times \tfrac{5}{16} \\ \\ 1 \text{ Pl., } 16 \times \tfrac{5}{16} \end{array}\right\} \begin{array}{l} A = 15.23 \text{ sq. in.} \\ I = 660.8 \\ r = 1.94 \text{ in.} \\ \frac{I}{v} = 81.3 \\ w_c = 51.8 \text{ lb} \end{array}$$

$$\Sigma F \frac{c}{2} = 6,750 \times 65.0 = 438,000 \text{ in. lb}$$

$$h^B = 200 \frac{11.52}{11.52 + 15.23} = 86.1 \text{ in.,} \quad h^A = 200 - 86.1 = 113.9 \text{ in.,}$$

$$p = \frac{58,990}{11.52} + 15.23 = 2,205 \text{ lb per sq. in.}$$

$h^B Y$.	$\Sigma P^B e^B$.	$-\Sigma P^A(b+e^A.)$	$-205.26\Sigma W'$.	X_w.	Wind on Front. X.	Wind on Rear. X.
+5,090,000	+32,200	−4,145,000	−2,930,000	±506,500	−1,446,300	−2,459,300

$$s \text{ (wind on rear)} = -\frac{2,459,300}{11.52 \times \overline{113.9^2} + 15.23 \times \overline{86.1^2}}$$

$$= -9.38 \text{ lb per sq. in.}$$

s^A, max. comp., wind on rear $= 113.9 \times 9.38 = +1{,}067$ lb per sq. in.

$$s'^A \text{ max. comp} = \frac{461.0}{461.0 + 660.8} \times \frac{438{,}000}{57.7} = +3{,}125 \text{ lb per sq. in.}$$

$$\begin{aligned} \text{Max. unit. comp. stress in Col. } 4\text{–}A,\ p &= +2{,}205 \text{ lb per sq. in.} \\ s^A &= +1{,}067 \text{ lb per sq. in.} \\ s'^A &= \underline{+3{,}125} \text{ lb per sq. in.} \\ \text{Total} &= +6{,}397 \text{ lb per sq. in.} \end{aligned}$$

Max. allowed unit comp. stress in Col. 4–*A*

$$= 1.25\left(16{,}000 - 70 \times \frac{150}{1.94}\right) = +8{,}380 \text{ lb per sq. in.}$$

This column section will be O.K.

$$s \text{ (wind on front)} = -\frac{1{,}446{,}300}{11.52 \times \overline{113.9}^2 + 15.23 \times \overline{86.1}^2} = -5.51.$$

s^B, max. comp. wind on face $= -86.1 \times 5.51 = -475$ lb per sq. in.

$$s'^B, \text{ max. comp.} = \frac{660.8}{461.0 + 660.8} \times \frac{438{,}000}{81.3} = +3{,}175 \text{ lb per sq. in.}$$

$$\begin{aligned} \text{Max. unit comp. stress in Col. } 4\text{–}B,\ p &= +2{,}205 \text{ lb per sq. in.} \\ s^B &= -\ \ 475 \text{ lb per sq. in.} \\ s'^B &= +\underline{3{,}175} \text{ lb per sq. in.} \\ \text{Total} &= +4{,}905 \text{ lb per sq. in.} \end{aligned}$$

Splice occurs in Col. 3–*B*, hence see third story for allowed and max. stresses.

Third Story—Columns.—

For Column 3–*A* use

$$\left.\begin{array}{r} 4\angle\text{'s, } 6 \times 3\tfrac{1}{2} \times \tfrac{3}{8} \\ \\ \\ 1 \text{ Pl., } 16 \times \tfrac{3}{8} \\ \\ \end{array}\right\} \begin{array}{l} A = 19.73 \text{ sq. in.} \\ I = 878.6 \\ r = 2.46 \text{ in.} \\ \dfrac{I}{v} = 108.1 \\ w_c = 67.2 \text{ lb} \end{array}$$

Y_3 (from table) = 160,027 lb
Col. wgts. and F.P. from above = 3,640 lb
Wgt. col. 3–*A* = 12.5 ft. × 67.2 lb = 840 lb
Wgt. col. 3–*B* = 12.5 ft. × 51.8 lb = 650 lb
F.P. col. 3–*A* = 12.5 ft. × 100 lb = 1,250 lb
F.P. col. 3–*B* = 12.5 ft. × 100 lb = 1,250 lb
Total value of Y_3 = 167,657 lb

For Column 3–*B* use

$$\left.\begin{array}{r} 4\angle\text{'s, } 5 \times 3\tfrac{1}{2} \times \tfrac{5}{16} \\ \\ 1\text{ Pl., } 16 \times \tfrac{5}{16} \end{array}\right\} \begin{array}{lcl} A & = & 15.23 \text{ sq. in.} \\ I & = & 660.8 \quad " \\ r & = & 1.94 \text{ in.} \\ \dfrac{I}{v} & = & 81.3 \\ w_c & = & 51.8 \text{ lb} \end{array}$$

$$\Sigma F\frac{c}{2} = 14{,}250 \times 62.5 = 891{,}500 \text{ in. lb}$$

$$h^B = 200 \times \frac{19.73}{19.73 + 15.23} = 112.8 \text{ in.}$$

$$h^A = 200.0 - 112.8 = 87.2 \text{ in.}$$

$$p = \frac{167{,}657}{19.73 + 15.23} = 4{,}785 \text{ lb per sq. in.}$$

h^BY.	$\Sigma P^B e^B$.	$-\Sigma P^A(b+e^A)$.	$-205.26\Sigma W'$.	X_w.	Wind on Front. X.	Wind on Rear. X.
+18,940,000	+210,000	−10,840,000	−6,345,000	±2,096,000	+4,061,000	−131,000

$$s \text{ (wind on rear)} = -\frac{131{,}000}{19.73 \times \overline{87.2}^2 + 15.23 \times \overline{112.8}^2}$$

$$= -0.381 \text{ lb per sq. in.}$$

s^A, max. comp., wind on rear = 87.2 × 0.381 = + 33 lb per sq. in

$$s'^A\text{, max. comp.} = \frac{878.6}{878.6 + 660.8} \times \frac{891{,}500}{108.1} = +4{,}720 \text{ lb per sq. in.}$$

Max. unit. comp. stress in Col. 3–*A*, p = + 4,785 lb per sq. in.
s^A = + 33 lb per sq. in.
s'^A = + 4,720 lb per sq. in.
Total = + 9,538 lb per sq. in.

Splice occurs in Col. 2–*A*; hence see second story for allowed and max. stresses.

$$s \text{ (wind on front)} = + \frac{4{,}061{,}000}{19.73 \times \overline{87.2^2} + 15.23 \times \overline{112.8^2}}$$

$$= + 11.80 \text{ lb per sq. in.}$$

$$s^B\text{, max. comp., wind on front, } = 112.8 \times 11.80$$

$$= + 1{,}332 \text{ lb per sq. in.}$$

$$s'^B\text{, max. comp.} = \frac{660.8}{878.6 + 660.8} \times \frac{891{,}500}{81.3} = + 4{,}710 \text{ lb per sq. in.}$$

$$\begin{array}{rrl} \text{Max. unit. comp. stress in Col. 3–}B,\ p & = + & 4{,}785 \text{ lb per sq. in.} \\ s^B & = + & 1{,}332 \text{ lb per sq. in.} \\ s'^B & = + & \underline{4{,}710} \text{ lb per sq. in.} \\ \text{Total} & = + & 10{,}827 \text{ lb per sq. in.} \end{array}$$

Max. allowed unit comp. stress in Col. 3–*B*

$$= 1.25\left(16{,}000–70 \times \frac{150}{1.94}\right) = + 13{,}250 \text{ lb per sq. in.}$$

If the next smaller section (4 × 3 × $\frac{5}{16}$ in. ∠'s) were used the allowed stress would be only 11,500 lb per sq. in. and the max. stress would be 12,240 lb per sq. in. instead of the 10,827 lb per sq. in. above. The section used is O.K.

Second Story—Columns.—

For Column 2–*A* use

$$\left.\begin{array}{l} 4\angle\text{'s, } 6 \times 3\frac{1}{2} \times \frac{3}{8} \\ \\ 1 \text{ Pl., } 16 \times \frac{3}{8} \end{array}\right\} \left\{\begin{array}{l} A = 19.73 \text{ sq. in.} \\ I = 878.6 \\ r = 2.46 \text{ in.} \\ \dfrac{I}{v} = 108.1 \\ w_c = 67.2 \text{ lb} \end{array}\right.$$

Y_2 (from table)		=	264,702 lb
Col. wgts. and F.P. from above		=	7,630 lb
Wgt. Col. 2–*A*	= 12.5 ft. × 67.2 lb	=	840 lb
Wgt. Col. 2–*B*	= 12.5 ft. × 83.8 lb	=	1,050 lb

F.P. Col. 2–*A* = 12.5 ft. × 100 lb = 1,250 lb
F.P. Col. 2–*B* = 12.5 ft. × 100 lb = 1,250 lb
Total value of Y_2 = 276,720 lb

For Column 2–*B* use

$$\left.\begin{array}{l} 4\angle\text{'s, } 7 \times 3\tfrac{1}{2} \times \tfrac{7}{16} \\ \\ 1 \text{ Pl., } 16 \times \tfrac{7}{16} \end{array}\right\} \begin{array}{ll} A & = 24.65 \text{ sq. in.} \\ I & = 1{,}122.6 \\ r & = 3.00 \text{ in.} \\ \dfrac{I}{V} & = 138.2 \\ W_c & = 83.8 \text{ lb} \end{array}$$

$$\Sigma F\frac{c}{2} = 21{,}750 \times 62.5 = 1{,}360.000 \text{ in. lb},$$

$$h^B = 200 \times \frac{19.73}{19.73 + 24.65} = 89.0 \text{ in.},$$

$$h^A = 200.0 - 89.0 = 111.0 \text{ in.},$$

$$p = \frac{276.720}{19.73 + 24.65} = 6{,}230 \text{ lb per sq. in.}$$

h^BY.	$\Sigma P^B{}_eB$.	$-\Sigma P^A(b+e^A)$.	$-205.26\Sigma W'$.	X_w.	Wind on Front. X.	Wind on Rear. X.
+24,620,000	+389,000	−17,550,000	−9,768,000	±4,798,000	+2,489,000	−7,107,000

$$s \text{ (wind on rear)} = -\frac{7{,}107{,}000}{19.73 \times \overline{111.0^2} + 24.65 \times \overline{89.0^2}}$$

$$= -\ 16.23 \text{ lb per sq. in.}$$

$$s^A\text{, max. comp., wind on rear,} = 111.0 \times 16.23$$

$$= +\ 1{,}802 \text{ lb per sq. in.}$$

$$s'^A\text{, max. comp.} = \frac{878.6}{878.6 + 1{,}122.6} \times \frac{1{,}360{,}000}{108.1}$$

$$= +5{,}525 \text{ lb per sq. in.}$$

Max. unit. comp. stress in Col. 2–*A*, p = + 6,230 lb per sq. in.
s^A = + 1,802 lb per sq. in.
s'^A = + 5,525 lb per sq. in.
Total = + 13,557 lb per sq. in.

Max. allowed unit comp. stress in Col. 2–*A*

$$= 1.25\left(16{,}000 - 70 \times \frac{150}{2.46}\right) = +\ 14{,}700 \text{ lb per sq. in.}$$

The section used can not be reduced.

$$s \text{ (wind on front)} = +\frac{2{,}489{,}000}{19.73 \times \overline{111.0}^2 + 24.65 \times \overline{89.0}^2}$$

$$= +\ 5.68 \text{ lb per sq. in.}$$

s^B, max. comp.-wind on front $= 89.0 \times 5.68 = +\ 506$ lb per sq. in.

$$s'^B, \text{ max. comp.} = \frac{1{,}122.6}{878.6 + 1122.6} \times \frac{1{,}360{,}000}{138.2}$$

$$= +\ 5520 \text{ lb per sq. in.}$$

Max. unit comp. stress in Col. 2–*B*, p = + 6,230 lb per sq. in.

s^B = + 506 lb per sq. in.

s'^B = + 5,520 lb per sq. in.

Total = +12,256 lb per sq. in.

Splice occurs in Col. 1–*A*; hence see first story for allowed and max. stresses.

First Story—Columns.—

For Column 1–*A* use

4 ∠'s, $7 \times 3\frac{1}{2} \times \frac{5}{8}$

1 Pl., $16 \times \frac{5}{8}$

A = 34.73 sq. in.

I = 1,550.9

r = 3.08 in.

$\frac{I}{v}$ = 190.9

w_c = 118.0 lb

Y_1 (from table) = 369,377 lb

Col. wgts. and F.P. from above = 12,020 lb

Wgt. Col. 1–*A* = 12.5 ft. × 118.0 lb = 1,480 lb

Wgt. Col. 1–*B* = 12.5 ft. × 83.8 lb = 1,050 lb

F.P. Col. 1–*A* = 12.5 ft. × 100 lb = 1,250 lb

F.P. Col. 1–*B* = 12.5 ft. × 100 lb = 1,250 lb

Total value of Y_1 = 386,430 lb

For Column 1–*B* use

$$\left.\begin{array}{l} 4\angle\text{'s},\ 7 \times 3\tfrac{1}{2} \times \tfrac{7}{16} \\ \\ 1\,\text{Pl.},\ 16 \times \tfrac{7}{16} \end{array}\right|\begin{array}{lcl} A & = & 24.65 \text{ sq. in.} \\ I & = & 1{,}122.6 \\ r & = & 3.00 \text{ in.} \\ \dfrac{I}{v} & = & 138.2 \\ w_c & = & 83.8 \text{ lb} \end{array}$$

$$\Sigma F \frac{c}{2} = 29{,}250 \times 62.5 = 1{,}827{,}000 \text{ in. lb}$$

$$h^B = 200 \times \frac{34.73}{34.73 + 24.65} = 117.0 \text{ in.}$$

$$h^A = 200.0 - 117.0 = 83.0 \text{ in.}$$

$$p = \frac{386{,}430}{34.73 + 24.65} = 6{,}510 \text{ lb per sq. in.}$$

h^BY.	$\Sigma P^B e^B$.	$-\Sigma P^A(b+e^A)$.	$-205.26\Sigma W'$.	X_w.	Wind on Front. X.	Wind on Rear. X.
+45,230,000	+565,000	−24,270,000	−13,175,000	±8,622,500	+16,972,500	−272,500

$$s \text{ (wind on rear)} = -\frac{272{,}500}{34.73 \times \overline{83.0^2} + 24.65 \times \overline{117.0^2}}$$

$$= -\ 0.472 \text{ lb per sq. in.}$$

s^A, max. comp., wind on rear $= 83.0 \times 0.472 = +\ 39$ lb per sq. in.

$$s'^A, \text{ max. comp.} = \frac{1{,}550.9}{1550.9 + 1{,}122.6} \times \frac{1{,}827{,}000}{190.9}$$

$$= +\ 5{,}550 \text{ lb per sq. in.}$$

Max. unit. comp. stress in Col. 1–*A*, p = + 6,510 lb per sq. in.
s^A = + 39 lb per sq. in.
s'^A = + 5,550 lb per sq. in.
Total = +12,099 lb per sq. in.

1–*A* and 0–*A* are the same member, hence see 0 story for allowed and max. stresses.

$$s \text{ (wind on front)} = +\frac{16{,}972{,}500}{34.73 \times \overline{83.0^2} + 24.65 \times \overline{117.0^2}}$$

$$= +\ 29.4 \text{ lb per sq. in.}$$

s^B, max. comp., wind on front = 117.0×29.4

$$= +\ 3{,}445 \text{ ℔ per sq, in.}$$

$$s'^B\text{, max. comp.} = \frac{1{,}122.6}{1{,}550.9 + 1122.6} \times \frac{1{,}827{,}000}{138.2}$$

$$= +\ 5{,}550 \text{ ℔ per sq. in.}$$

$$\begin{aligned}
\text{Max. unit. comp. stress in Col. 1–}B,\ p &= +\ 6{,}510 \text{ ℔ per sq. in.}\\
s^B &= +\ 3{,}445 \text{ ℔ per sq. in.}\\
s'^B &= \underline{+\ 5{,}550} \text{ ℔ per sq. in.}\\
\text{Total} &= +15{,}505 \text{ ℔ per sq. in.}
\end{aligned}$$

Max. allowed unit comp. stress in Col. 1–*B*

$$= 1.25\left(16{,}000 - 70 \times \frac{150}{3.00}\right) = +\ 15{,}620 \text{ ℔ per sq. in.}$$

The section used is O.K.

Basement Story—Columns.—
For Column 0–*A* use

$$\left.\begin{array}{l} 4\angle\text{'s, } 7 \times 3\tfrac{1}{2} \times \tfrac{5}{8} \\ \\ 1 \text{ Pl., } 16 \times \tfrac{5}{8} \end{array}\right\} \begin{array}{lcl} A &=& 34.73 \text{ sq. in.}\\ I &=& 1{,}550.9\\ r &=& 3.08 \text{ in.}\\ \dfrac{I}{v} &=& 190.9\\ w_c &=& 118.0 \text{ ℔} \end{array}$$

Y_0 (from table) =		499,102 ℔
Col. wgts. and F.P. from above	=	17,050 ℔
Wgt. Col. 0–*A*	= 12.5 ft. × 118.0 ℔ =	1,480 ℔
Wgt. Col. 0–*B*	= 12.5 ft. × 118.0 ℔ =	1,480 ℔
F.P. Col. 0–*A*	= 12.5 ft. × 100.0 ℔ =	1,250 ℔
F.P. Col. 0–*B*	= 12.5 ft. × 100.0 ℔ =	1,250 ℔
Total value of Y_0	=	521,610 ℔

For Column o–*B* use

$$\left.\begin{array}{l} 4\angle\text{'s, } 7 \times 3\tfrac{1}{2} \times \tfrac{5}{8} \\ \\ 1 \text{ Pl., } 16 \times \tfrac{5}{8} \end{array}\right\} \begin{array}{lcl} A &=& 34.73 \text{ sq. in.} \\ I &=& 1{,}550.9 \\ r &=& 3.08 \text{ in.} \\ \dfrac{I}{v} &=& 190.9 \\ w_c &=& 118.0 \text{ lb} \end{array}$$

$$\Sigma F \frac{c}{2} = 33{,}000 \times 62.5 = 2{,}062{,}500 \text{ in. lb}$$

$$h^B = 200 \times \frac{34.73}{34.73 + 34.73} = 100.0 \text{ in.}$$

$$h^A = 200.0 - 100.0 = 100.0 \text{ in.}$$

$$p = \frac{521{,}610}{34.73 + 34.73} = 7{,}510 \text{ lb per sq. in.}$$

$h^B Y$.	$\Sigma P^B e^B$.	$-\Sigma P^A(b+e^A)$.	$-205.26\Sigma W'$.	X_w.	Wind on Front. X.	Wind on Rear. X.
+52,161,000	+763,000	−31,870,000	−18,300,000	±13,295,000	+16,049,000	−10,541,000

$$s \text{ (wind on rear)} = -\frac{10{,}541{,}000}{34.73 \times \overline{100}^2 + 34.73 \times \overline{100}^2}$$

$$= -15.175 \text{ lb per sq. in.}$$

$$s^A, \text{ max. comp., wind on rear} = 100 \times 15.17$$

$$= +1{,}517 \text{ lb per sq. in.}$$

$$s'^A, \text{ max. comp.} = \frac{1{,}550.9}{1{,}550.9 + 1{,}550.9} \times \frac{2{,}062{,}500}{190.9}$$

$$= +5{,}405 \text{ lb per sq. in.}$$

$$\begin{array}{rcl} \text{Max. unit comp. stress in Col. o–}A,\ p &=& +\ 7{,}510 \text{ lb per sq. in.} \\ s^A &=& +\ 1{,}517 \text{ lb per sq. in.} \\ s'^A &=& \underline{+\ 5{,}405} \text{ lb per sq. in.} \\ \text{Total} &=& +14{,}432 \text{ lb per sq. in.} \end{array}$$

Max. allowed unit comp. stress on Col. o–*A*

$$= 1.25\left(16{,}000 - 70 \times \frac{150}{3.08}\right) = +16{,}150 \text{ lb per sq. in.}$$

If the next smaller section ($\frac{9}{16}$ in. material) were used the allowed stress would be 16,150 ℔ per sq. in. and the max. stress would be 16,340 ℔ per sq. in. instead of the 14,432 ℔ per sq. in. above. The section used is O.K.

$$s \text{ (wind on front)} = + \frac{16{,}049{,}000}{34.73 \times \overline{100}^2 + 34.73 \times \overline{100}^2}$$

$$= + 23.11 \text{ ℔ per sq. in.}$$

$$s^B, \text{ max. comp. wind on front} = 100 \times 23.11$$

$$= + 2{,}311 \text{ ℔ per sq. in.}$$

$$s'^B, \text{ max. comp.} = \frac{1{,}550.9}{1550.9 + 1550.9} \times \frac{2{,}062{,}500}{190.9}$$

$$= + 5{,}405 \text{ ℔ per sq. in.}$$

$$\begin{aligned} \text{Max. unit. comp. stress in Col. o–}B,\ p &= +7{,}510 \text{ ℔ per sq. in.} \\ s^B &= +\ 2{,}311 \text{ ℔ per sq. in.} \\ s'^B &= \underline{+\ 5{,}405} \text{ ℔ per sq. in.} \\ \text{Total} &= +15{,}226 \text{ ℔ per sq. in.} \end{aligned}$$

Max. allowed unit comp. stress on Col. o–B

$$= 1.25\left(16{,}000 - 70 \times \frac{150}{3.08}\right) = + 16{,}150 \text{ ℔ per sq. in.}$$

The section used is O.K.

Girder Design.—The general formulae for determining girder flange stresses reduce in this case to the following.

Girder A_n, top flange compression

$$= S^A_n \frac{c_n}{2d_n} + S^A_{n+1} \frac{c_{n+1}+2d_n}{2d_n} + \tfrac{1}{2}F^A_n + (V^A_n - V^A_{n+1})\frac{x}{d_n} - P^A_n \frac{(e^A_n + x)}{d_n}$$

$$- W' \frac{(x - 60.67)}{d_n} \times 760.67 - \frac{W'(x - 132.09)}{d_n} \times 7132.09$$

Girder A_n, bottom flange tension

$$= S^A_n \frac{c_n + 2d_n}{2d_n} + S^A_{n+1} \frac{c_{n+1}}{2d_n} - \tfrac{1}{2}F^A_n + (V^A_n - V^A_{n+1})\frac{x}{d_n} - P^A_n \frac{(e^A_n + x)}{d_n}$$

$$- W' \frac{(x - 60.67)}{d_n} \times 760.67 - \frac{W'(x - 132.09)}{d_n} \times 7132.09$$

The formulae apply for wind blowing on either front or rear when the corresponding values of S^A and V^A are used. For wind on the rear F^A is of course zero.

Values of V and S are given in the following paragraphs.

$$\left.\begin{aligned}
S_0^A &= \frac{1{,}550.9}{1{,}550.9 + 1{,}550.9} \times 33{,}000 = 16{,}500 \text{ lb} \\
S_1^A &= \frac{1{,}550.9}{1{,}550.9 + 1122.6} \times 29{,}250 = 16{,}970 \text{ lb} \\
S_2^A &= \frac{878.6}{878.6 + 1122.6} \times 21{,}750 = 9{,}553 \text{ lb} \\
S_3^A &= \frac{878.6}{878.6 + 660.8} \times 14{,}250 = 8{,}135 \text{ lb} \\
S_4^A &= \frac{461.0}{461.0 + 660.8} \times 6{,}750 = 2{,}773 \text{ lb} \\
S_0^B &= 33{,}000 - 16{,}500 = 16{,}500 \text{ lb} \\
S_1^B &= 29{,}250 - 16{,}970 = 12{,}280 \text{ lb} \\
S_2^B &= 21{,}750 - 9{,}553 = 12{,}197 \text{ lb} \\
S_3^B &= 14{,}250 - 8{,}135 = 6{,}115 \text{ lb} \\
S_4^B &= 6{,}750 - 2{,}773 = 3{,}977 \text{ lb}
\end{aligned}\right\} \begin{array}{l}\text{+ for wind on} \\ \text{front; − for} \\ \text{wind on rear.}\end{array}$$

The total axial stress in a column equals the area of the column times $(p + s)$.

The values of V are as follows: For wind on rear (Cols. A, max.).

$$\begin{aligned}
V_0^A &= 34.73 \times 9{,}027 = 313{,}500 \text{ lb} & V_0^A - V_1^A &= 86{,}100 \text{ lb} \\
V_1^A &= 34.73 \times 6{,}549 = 227{,}400 \text{ lb} & V_1^A - V_2^A &= 69{,}000 \text{ lb} \\
V_2^A &= 19.73 \times 8{,}032 = 158{,}400 \text{ lb} & V_2^A - V_3^A &= 63{,}270 \text{ lb} \\
V_3^A &= 19.73 \times 4{,}818 = 95{,}130 \text{ lb} & V_3^A - V_4^A &= 57{,}420 \text{ lb} \\
V_4^A &= 11.52 \times 3{,}272 = 37{,}710 \text{ lb} & V_4^A &= 37{,}710 \text{ lb}
\end{aligned}$$

$$\begin{aligned}
V_0^B &= Y_0 - V_0^A = 521{,}610 - 313{,}500 = 208{,}110 \text{ lb} \\
V_1^B &= Y_1 - V_1^A = 386{,}430 - 227{,}400 = 159{,}030 \text{ lb} \\
V_2^B &= Y_2 - V_2^A = 276{,}720 - 158{,}400 = 118{,}320 \text{ lb} \\
V_3^B &= Y_3 - V_3^A = 167{,}657 - 95{,}130 = 72{,}527 \text{ lb} \\
V_4^B &= Y_4 - V_4^A = 58{,}990 - 37{,}710 = 21{,}280 \text{ lb}
\end{aligned}$$

For Wind on Front (Cols. *B*, Max.).

$$\begin{aligned}
V_0^B &= 34.73 \times 9{,}821 = 341{,}000 \text{ ℔} \\
V_1^B &= 24.65 \times 9{,}955 = 245{,}400 \text{ ℔} \\
V_2^B &= 24.65 \times 6{,}736 = 166{,}000 \text{ ℔} \\
V_3^B &= 15.23 \times 6{,}117 = 93{,}230 \text{ ℔} \\
V_4^B &= 15.23 \times 1{,}730 = 26{,}360 \text{ ℔}
\end{aligned}$$

$$\begin{aligned}
V_0^A &= Y_0 - V_0^B = 521{,}610 - 341{,}000 = 180{,}610 \text{ ℔} \\
V_1^A &= Y_1 - V_1^B = 386{,}430 - 245{,}400 = 141{,}030 \text{ ℔} \\
V_2^A &= Y_2 - V_2^B = 276{,}720 - 166{,}000 = 110{,}720 \text{ ℔} \\
V_3^A &= Y_3 - V_3^B = 167{,}657 - 93{,}230 = 74{,}427 \text{ ℔} \\
V_4^A &= Y_4 - V_4^B = 58{,}990 - 26{,}360 = 32{,}630 \text{ ℔}
\end{aligned}$$

$$\begin{aligned}
V_0^A - V_1^A &= 39{,}580 \text{ ℔} \\
V_1^A - V_2^A &= 30{,}310 \text{ ℔} \\
V_2^A - V_3^A &= 36{,}293 \text{ ℔} \\
V_3^A - V_4^A &= 41{,}797 \text{ ℔} \\
V_4^A \quad &= 32{,}630 \text{ ℔}
\end{aligned}$$

The following table gives in a condensed form all the calculations of girder flange stresses and the positions and amounts of maximum stresses of both kinds in both top and bottom flanges.

Girder.	Flange.	Surface Facing Wind.	Value of x for Max. Stress.	S^A_n Term.	S^A_{n+1} Term.	$\frac{1}{2}F^A_n$.	$\left(V^A_n - V^A_{n+1}\right)$ Term.	P^A_n Term.	W' Terms.	Stresses, lbs.
a–4	Bottom	Front	200.00	−11,780	—	+3,380	−326,300	+207,000	+148,100	+ 20,400
		Front	60.67	+11,780	—	−3,380	+ 99,000	− 70,100	0	− 37,300
		Rear	0.00	+11,780	—	0	0	+ 10,600	0	+ 22,380
		Rear	132.09	−11,780	—	0	+249,100	−140,300	− 51,000	− 46,020
	Top	Front	60.67	+ 9,020	—	+3,380	+ 99,000	− 70,100	0	+ 41,300
		Front	200.00	− 9,020	—	−3,380	−326,300	+207,000	+148,100	− 16,400
		Rear	132.09	− 9,020	—	0	+249,100	−140,300	− 51,000	+ 48,780
		Rear	0.00	+ 9,020	—	0	0	+ 10,600	0	− 19,620
a–3	Bottom	Front	200.00	−28,480	− 7,215	+3,750	−334,376	+268,000	+138,100	+ 39,779
		Front	60.67	+28,480	+ 7,215	−3,750	+101,400	− 91,000	0	− 42,345
		Rear	0.00	+28,480	+ 7,215	0	0	+ 13,650	0	+ 49,345
		Rear	132.09	−28,480	− 7,215	0	+303,500	−182,000	− 47,500	− 38,305
	Top	Front	60.67	+20,330	+ 9,990	+3,750	+101,400	− 91,000	0	+ 44,470
		Front	200.00	−20,330	− 9,990	−3,750	−334,376	+268,000	+138,100	− 37,654
		Rear	132.09	−20,330	− 9,990	0	+303,500	−182,000	− 47,500	+ 43,680
		Rear	0.00	+20,330	+ 9,990	0	0	+ 13,650	0	− 43,970
a–2	Bottom	Front	200.00	−33,430	−20,330	+3,750	−290,344	+268,000	+138,100	+ 65,746
		Front	60.67	+33,430	+20,330	−3,750	+ 88,100	− 91,000	0	− 47,110
		Rear	0.00	+33,430	+20,330	0	0	+ 13,650	0	+ 67,410
		Rear	132.09	−33,430	−20,330	0	+334,500	−182,000	− 47,500	− 51,240
	Top	Front	60.67	+23,880	+28,480	+3,750	+ 88,100	− 91,000	0	+ 53,210
		Front	200.00	−23,880	−28,480	−3,750	−290,344	+268,000	+138,100	− 59,646
		Rear	132.09	−23,880	−28,480	0	+334,500	−182,000	− 47,500	+ 52,640
		Rear	0.00	+23,880	+28,480	0	0	+ 13,650	0	− 66,010

Girder.	Flange.	Surface Facing Wind.	Value of X for Max. Stress.	S_n^A Term.	S_{n+1}^A Term.	$\frac{1}{2}F_n^A$.	$(V_n^A - V_{n+1}^A)$ Term.	P_n^A. Term.	W' Terms.	Stresses, lbs.
a–1	Bottom	Front	200.00	−59,400	−23,880	+3,750	−242,480	+268,000	+138,100	+ 84,090
		Front	60.67	+59,400	+23,880	−3,750	+ 73,550	− 91,000	0	− 62,080
		Rear	0.00	+59,400	+23,880	0	0	+ 13,650	0	+ 96,930
		Rear	200.00	−59,400	−23,880	0	+552,000	−268,000	−138,100	− 62,620
	Top	Front	60.67	+42,430	+33,430	+3,750	+ 73,550	− 91,000	0	+ 62,160
		Front	200.00	−42,430	−33,430	−3,750	−242,480	+268,000	+138,100	− 84,010
		Rear	200.00	−42,430	−33,430	0	+552,000	−268,000	−138,100	+ 70,040
		Rear	0.00	+42,430	+33,430	0	0	+ 13,650	0	− 89,510
a–0	Bottom	Front	200.00	−57,770	−42,430	+1,875	−316,640	+303,500	+207,300	+ 95,835
		Front	60.67	+57,770	+42,430	−1,875	+ 96,050	−102,800	0	− 91,575
		Rear	0.00	+57,770	+42,430	0	0	+ 15,450	0	+115,650
		Rear	200.00	−57,770	−42,430	0	+688,800	−303,500	−207,300	− 77,800
	Top	Front	60.67	+41,260	+59,400	+1,875	+ 96,050	−102,800	0	+ 95,785
		Front	200.00	−41,260	−59,400	−1,875	−316,640	+303,500	+207,300	− 91,625
		Rear	200.00	−41,260	−59,400	0	+688,800	−303,500	−207,300	+ 77,340
		Rear	0.00	+41,260	+59,400	0	0	+ 15,450	0	−116,110

The design of the girders is given in the following table.

Girder.	Flange.	Max. Stress.		Allowed Tens.	Allowed Comp. Unif.	Max. Area Required.	Make-up of Flanges.	Area Used.	Max. Shear.	Girder Web.
		+	−							
a–0	Top Bottom	95,785 115,650	116,110 91,575	1.25(16,000) = 20,000 net.	16,000 16,000	5.80 net. 7.23 gr.	$2 \angle s, 6 \times 3\frac{1}{2} \times \frac{3}{8}$* $2 \angle s, 6 \times 3\frac{1}{2} \times \frac{3}{8}$*	5.55 net. 6.86 gr.	51,875	$26\frac{1}{2} \times \frac{3}{8}$
a–1	Top Bottom	70,040 96,930	89,510 62,620		1.25(16,000 − 200 L/b) 15,200 15,200	4.48 net. 6.38 gr.	$2 \angle s, 5 \times 3\frac{1}{2} \times \frac{3}{8}$ $2 \angle s, 5 \times 3\frac{1}{2} \times \frac{3}{8}$*	4.79 net. 6.10 gr.	39,850	$26 \times \frac{5}{16}$
a–2	Top Bottom	53,210 67,410	66,010 51,240		15,200 15,200	3.30 net. 4.43 gr.	$2 \angle s, 5 \times 3 \times \frac{5}{16}$ $2 \angle s, 5 \times 3 \times \frac{5}{16}$	3.73 net. 4.82 gr.	33,220	$26 \times \frac{5}{16}$
a–3	Top Bottom	44,470 49,345	43,970 42,345		15,200 13,200	2.93 gr. 3.24 gr.	$2 \angle s, 5 \times 3 \times \frac{5}{16}$† $2 \angle s, 3\frac{1}{2} \times 3 \times \frac{5}{16}$	4.82 gr. 3.88 gr.	27,320	$26\frac{1}{2} \times \frac{5}{16}$
a–4	Top Bottom	48,780 22,380	19,620 46,020		13,200 13,200	3.21 gr. 2.30 net.	$2 \angle s, 3\frac{1}{2} \times 3 \times \frac{5}{16}$ $2 \angle s, 3\frac{1}{2} \times 3 \times \frac{5}{16}$	3.88 gr. 3.33 net.	19,218	$21\frac{1}{2} \times \frac{5}{16}$

* Larger sections should have been used. N. B.—Two holes deducted from each angle for net section.

† Top and bottom flanges should have $3\frac{1}{2} \times 3 \times \frac{5}{16}$ ∠s.

These errors were due to errors in original stresses. They were not noticed until after the design had been used in future chapters and the sections above will not be revised. The areas of cross section, and moments of inertia of the above girders about an axis midway between the centers of gravity of flanges will be required in future chapters and these calculations are given in the following table.

Girder.	Area, 4∠s.	Area, Web.	Total Area.	*d.*	I. Web.	I–4∠s.		Total I.
						Primary.	Ad^2 Secondary.	
A–0	13.72	9.95	23.67	12.5	582	13	2,145	2,740
A–1	12.20	8.13	20.33	12.5	458	13	1,907	2,378
A–2	9.64	8.13	17.77	12.5	458	7	1,507	1,972
A–3	8.70	8.30	17.00	12.5	484	6	1,360	1,850
A–4	7.76	6.71	14.47	10.0	259	6	776	1,041

CHAPTER III.

STRESSES IN PORTAL BRACED BUILDINGS—ENTIRE STRUCTURE ELASTIC.

In this chapter there is developed a method of determining the stresses in a plain plate girder portal braced building for all types of loading on the assumption that the entire structure is elastic. First the general case is developed and then special formulae and methods are given for a structure containing only two columns in a transverse section.

The following assumptions are involved in this discussion:

(*a*) That the connections of the girders to the columns are sufficiently rigid to maintain the column in a straight line for the full depth of the girder and that, as a consequence, the deformation curves of the girders and columns are normal to each other at their intersections.

(*b*) That the curvature of the columns and girders is considered as having no effect on their axial lengths.

(*c*) That the columns at their anchorages will always be maintained in a fixed vertical direction.

(*d*) That the elastic limit of the material will never be exceeded.

(*e*) That the material shall be uniform throughout the structure.

(*f*) The increase of bending moment leverage due to the deflection of columns and girders is neglected.

(*g*) The mid depth lines of girders are considered as their neutral axes.

These assumptions, it is believed, are all rational and non conflicting.

In order to make the method about to be developed more readily understood the following outline is given:

The fundamental differential equation of the elastic line of any member under stress is

$$EI\frac{d^2y}{dx^2} = Mx.$$

The building is considered as made up of individual members

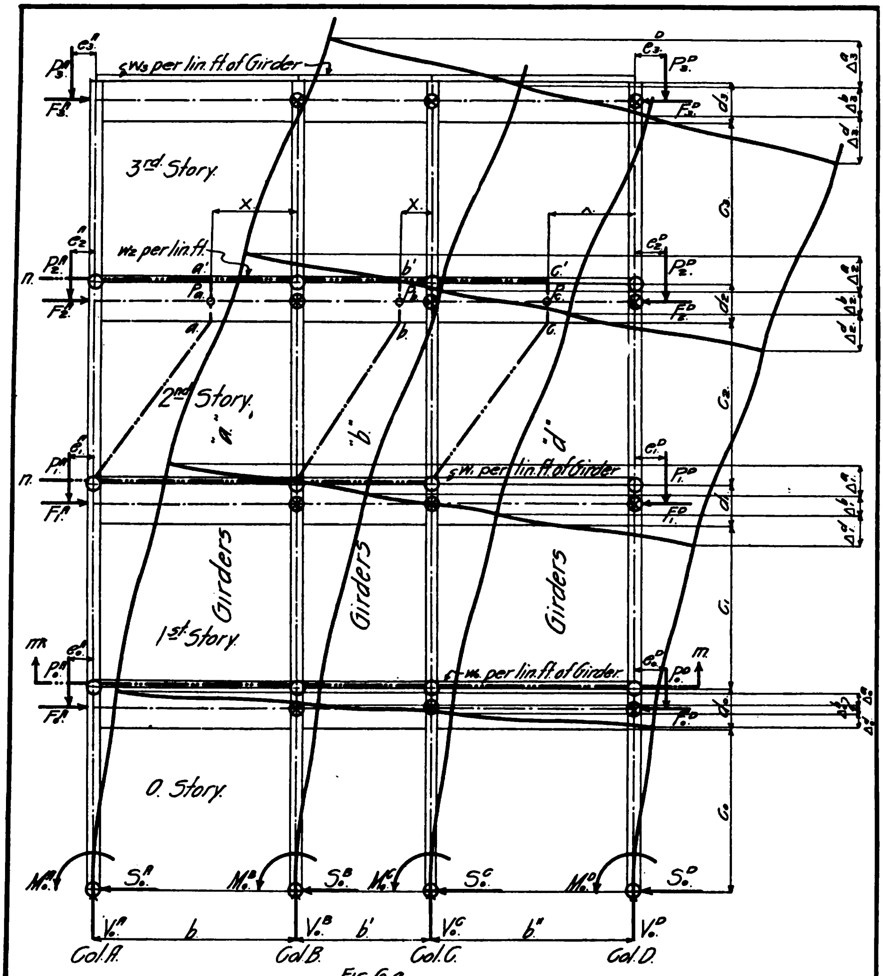


FIG. 6 a

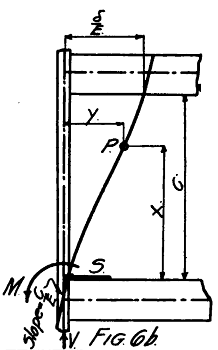


FIG. 6 b.

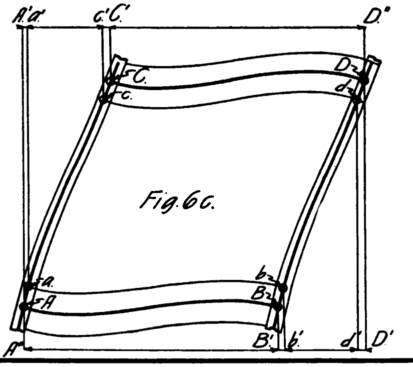


Fig. 6 c.

(columns and girders) rigidly connected to each other, and the above equation successively applied to each member. First, certain conditions of the problem are used in eliminating and evaluating the constants of integration of the elastic lines of all members. Then the remaining unused conditions are developed into three independent equations per panel of the cross section of the building. These equations are verbally expressed as follows:

Equations A are best understood by a reference to Fig. 6, *c*, which represents a distorted panel taken out of the structure. The equation is simply

$$A'D'' = A''D'$$

or

$$A'a' + a'c' + c'C' + C'D'' = A''B' + B'b' + b'd' + d'D',$$

in which $A'a'$, $c'C'$, $B'b'$, and $d'D'$ are equal to the half depth of the girder times the slope of the corresponding column at the points A, C, B, and D respectively; $a'c'$ and $b'd'$ represent the deflection of the columns ac and bd respectively; $A''B'$ and $C'D''$ represent the original unstrained length of the neutral axes of the girders minus the shortening due to axial compression in the girder.

Equations B.—The slope of the elastic line of any girder (inclination to the horizontal) at the end of the girder will be equal to the slope of the elastic line of the column (inclination to the vertical) connecting thereto. There of course exists such a relation for each end of the girder, but one of these conditions has already been used to evaluate a constant of integration.

Equations C.—Considering the origin of coordinates for the elastic line of a girder as located on the neutral axis and at one end of the girder, the vertical deflection of the other end of the girder will be equal to the total difference in the change of length of the two columns connecting to the girder in the distance from the column anchorages to the neutral axis of the girder. These quantities are shown graphically in Fig. 6, *a*, in which they are marked Δ.

Now, starting at the bottom of the building, by means of these equations three of the unknowns of the problem are eliminated from each successive panel until the top of the building is reached. The equations A, B, and C for the top panels may then and not till then be solved for the unknowns of the top floor panels. Substitution may then be made in the lower floors successively until, having reached the bottom panel, all quantities are known. Each panel contains three unknowns in terms of which all other quantities may be expressed and as seen above each panel furnishes the necessary equations for their elimination.

The origins of coordinates are taken as follows:

For All Columns.—Separate origins are taken for each column section; they are located at the foot of the column section (length $= c$). The coordinates are measured vertically and horizontally, x being considered positive upwards and y positive to the right.

For all Girders.—Origins are located on the intersections of the neutral axes with the axis of the column connecting on the right end of the girders. Coordinates are measured horizontally and vertically, x being considered positive to the left and y positive upwards.

All column origins are shown on Fig. 6, *a*; thus ⊕.

All girder origins are shown on Fig. 6, *a*; thus ⊗.

EXPRESSIONS GIVING THE VALUES OF THE VARIOUS PROPERTIES OF THE STRUCTURE UNDER LOAD.

Slopes and Deflections of Columns.—Referring to Fig. 6, *b*, which represents in general a column section from any part of the building, the differential equation of the elastic line of any column section is

$$E\frac{d^2y}{dx^2} = \frac{M}{I} - \frac{S}{I}x$$

(cf. Chapter II). First integration gives

$$E\frac{dy}{dx} = \frac{M}{I}x - \frac{S}{2I}x^2 + K$$

where K is a constant of integration. Second integration gives

$$Ey = \frac{M}{2I}x^2 - \frac{S}{6I}x^3 + Kx + K'$$

where K' is a constant of integration.

But when $x = 0$, $E\,dy/dx = C$ (see Fig. 6, *b*, and "Notation"), hence $K = C$ and

$$E\frac{dy}{dx} = \frac{M}{I}x - \frac{S}{2I}x^2 + C.$$

Now, making $x = c$ in this equation the value of C for the column section above is found to be $Mc/I - Sc^2/2I + C$. For the basement sections $C = 0$, since the columns are here fixed in a vertical direction. By assigning the proper subscripts and exponents to the terms of this formulae values of C for all column sections are written as follows:

For Columns A.

$$C_0^A = 0,$$

$$C_1^A = \frac{M_0^A C_0}{I_0^A} - \frac{S_0^A C_0^2}{2I_0^A},$$

$$C_2^A = \frac{M_1^A C_1}{I_1^A} - \frac{S_1^A C_1^2}{2I_1^A} + C_1^A,$$

$$C_3^A = \frac{M_2^A C_2}{I_2^A} - \frac{S_2^A C_2^2}{2I_2^A} + C_2^A.$$

For Columns B.

$$C_0^B = 0,$$

$$C_1^B = \frac{M_0^B C_0}{I_0^B} - \frac{S_0^B C_0^2}{2I_0^B},$$

$$C_2^B = \frac{M_1^B C_1}{I_1^B} - \frac{S_1^B C_1^2}{2I_1^B} + C_1^B,$$

$$C_3^B = \frac{M_2^B C_2}{I_2^B} - \frac{S_2^B C_2^2}{2I_2^B} + C_2^B.$$

For Columns C.

$$C_0^C = 0,$$

$$C_1^C = \frac{M_0^C C_0}{I_0^C} - \frac{S_0^C C_0^2}{2I_0^C},$$

$$C_2^C = \frac{M_1^C C_1}{I_1^C} - \frac{S_1^C C_1^2}{2I_1^C} + C_1^C,$$

$$C_3^C = \frac{M_2^C C_2}{I_2^C} - \frac{S_2^C C_2^2}{2I_2^C} + C_2^C.$$

For Columns D.

$$C_0^D = 0,$$

$$C_1^D = \frac{M_0^D C_0}{I_0^D} - \frac{S_0^D C_0^2}{2I_0^D},$$

$$C_2^D = \frac{M_1^D C_1}{I_1^D} - \frac{S_1^D C_1^2}{2I_1^D} + C_1^D,$$

$$C_3^D = \frac{M_2^D C_2}{I_2^D} - \frac{S_2^D C_2^2}{2I_2^D} + C_2^D.$$

Now when $x = 0$, $y = 0$, and hence $K' = 0$ and

$$Ey = \frac{M}{2I}x^2 - \frac{S}{6I}x^3 + Cx.$$

In this equation, when $x = c$, $Ey = \delta$ (see "notation") $= Mc^2/2I - sc^3/6I + Cc$. Assigning proper subscripts and exponents as above, values of δ for all column sections are written as follows:

For Columns A.

$$\delta_0^A = \frac{M_0^A C_0^2}{2I_0^A} - \frac{S_0^A C_0^3}{6I_0^A},$$

$$\delta_1^A = \frac{M_1^A C_1^2}{2I_1^A} - \frac{S_1^A C_1^3}{6I_1^A} + C_1^A c_1,$$

For Columns B.

$$\delta_0^B = \frac{M_0^B C_0^2}{2I_0^B} - \frac{S_3^B C_0^3}{6I_0^B},$$

$$\delta_1^B = \frac{M_1^B C_1^2}{2I_1^B} - \frac{S_1^B C_1^3}{6I_1^B} + C_1^B c_1,$$

$$\delta_2^A = \frac{M_2^A C_2^2}{2I_2^A} - \frac{S_2^A C_2^3}{6I_2^A} + C_2^A c_2, \quad \delta_2^B = \frac{M_2^B C_2^2}{2I_2^B} - \frac{S_2^B C_2^3}{6I_2^B} + C_2^B c_2,$$

$$\delta_3^A = \frac{M_3^A C_3^2}{2I_3^A} - \frac{S_3^A C_3^3}{6I_3^A} + C_3^A c_3. \quad \delta_3^B = \frac{M_3^B C_3^2}{2I_3^B} - \frac{S_3^B C_3^3}{6I_3^B} + C_3^B c_3.$$

For Columns *C*. For Columns *D*.

$$\delta_0^C = \frac{M_0^C C_0^2}{2I_0^C} - \frac{S_0^C C_0^3}{6I_0^C}, \quad \delta_0^D = \frac{M_0^D C_0^2}{2I_0^D} - \frac{S_0^D C_0^3}{6I_0^D},$$

$$\delta_1^C = \frac{M_1^C C_1^2}{2I_1^C} - \frac{S_1^C C_1^3}{6I_1^C} + C_1^C c_1, \quad \delta_1^D = \frac{M_1^D C_1^2}{2I_2^D} - \frac{S_1^D C_1^3}{6I_1^D} + C_1^D c_1,$$

$$\delta_2^C = \frac{M_2^C C_2^2}{2I_2^C} - \frac{S_2^C C_2^3}{6I_2^C} + C_2^C c_2, \quad \delta_2^D = \frac{M_2^D C_2^2}{2I_2^D} - \frac{S_2^D C_2^3}{6I_2^D} + C_2^D c_2,$$

$$\delta_3^C = \frac{M_3^C C_3^2}{2I_3^C} - \frac{S_3^C C_3^3}{6I_3^C} + C_3^C c_3. \quad \delta_3^D = \frac{M_3^D C_3^2}{2I_3^D} - \frac{S_3^D C_3^3}{6I_3^D} + C_3^D c_3.$$

Axial Shortening of Girders.—For an axial thrust H the shortening in the length b will equal Hb/AE or E times this shortening $= \lambda = Hb/A$. Values of λ for all girders are given as follows:

For Girders "*a*." For Girders "*b*." For Girders "*d*."

$$\lambda_0^a = \frac{H_0^a b}{A_0^a}, \quad \lambda_0^b = \frac{H_0^b b'}{A_0^b}, \quad \lambda_0^d = \frac{H_0^d b''}{A_0^d},$$

$$\lambda_1^a = \frac{H_0^a b}{A_1^a}, \quad \lambda_1^b = \frac{H_1^b b'}{A_1^b}, \quad \lambda_1^d = \frac{H_1^d b''}{A_1^d},$$

$$\lambda_2^a = \frac{H_2^a b}{A_2^a}, \quad \lambda_2^b = \frac{H_2^b b'}{A_2^b}, \quad \lambda_2^d = \frac{H_2^b b''}{A_2^d},$$

$$\lambda_3^a = \frac{H_3^a b}{A_3^a}. \quad \lambda_3^b = \frac{H_3^b b'}{A_3^b}. \quad \lambda_3^d = \frac{H_3^d b''}{A_3^d}.$$

Axial Stresses in Girders.—Considering sections such as *naa'n*, *nbb'n*, and *ndd'n* and equating the sums of the horizontal forces acting on the sections cut to zero the following values of H may be written.

For Girders "*a*." For Girders "*b*."

$$H_0^a = S_1^A - S_0^A + F_0^A, \quad H_0^b = S_1^A + S_1^B - S_0^A - S_0^B + F_0^A,$$

$$H_1^a = S_2^A - S_1^A + F_1^A, \quad H_1^b = S_2^A + S_2^B - S_1^A - S_1^B + F_1^A,$$

$$H_2^a = S_3^A - S_2^A + F_2^A, \quad H_2^b = S_3^A + S_3^B - S_2^A - S_2^B + F_2^A,$$

$$H_3^a = - S_3^A + F_3^A. \quad H_3^b = - S_3^A - S_3^B + F_3^A.$$

For Girders "*d*."

$$H_0^d = S_1^A + S_1^B + S_1^C - S_0^A - S_0^B - S_0^C + F_0^A,$$

$$H_1^d = S_2^A + S_2^B + S_2^C - S_1^A - S_1^B - S_1^C + F_1^A,$$

$$H_2^d = S_3^A + S_3^B + S_3^C - S_2^A - S_2^B - S_2^C + F_2^A,$$

$$H_3^d = - S_3^A - S_3^B - S_3^C + F_3^A.$$

Axial Stresses in Columns.—In considering the loads on this structure it is to be noted that the wind is taken as acting on each side simultaneously; the purpose of this is to make the equations perfectly general, so that they may be used for wind on either side by merely dropping the terms involving wind on the other side. Passing horizontal planes *mm*, Fig. 6, *a*, through the origins of coordinates for the columns and taking as centre of moments the intersection of these planes with Col. D, let X be the moment about this point of all the given external forces acting on the upper portion of the structure cut by *mm* (positive clockwise) and let Y be the sum of all the vertical forces or components acting downward on this portion of the structure. Now, using this center of moments there is obtained

$$X + V^A(b + b' + b'') + V^B(b' + b'') + V^C b'' - (M^A + M^B + M^C + M^D) = 0$$

or

$$V^A = \frac{(M^A + M^B + M^C + M^D) - V^B(b' + b'') - V^C b'' - X}{b + b' + b''}.$$

Also

$$V^D = Y - V^A - V^B - V^C$$

$$= - \frac{(M^A + M^B + M^C + M^D) - V^B b - V^C(b + b') + Y(b + b' + b'') + X}{b + b' + b''}.$$

Therefore

$$V_0^A = \frac{(M_0^A + M_0^B + M_0^C + M_0^D) - V_0^B(b' + b'') - V_0^C b'' - X_0}{b + b' + b''},$$

$$V_1^A = \frac{(M_1^A + M_1^B + M_1^C + M_1^A) - V_1^B(b' + b'') - V_1^C b'' - X_1}{b + b' + b''},$$

$$V_2^A = \frac{(M_2^A + M_2^B + M_2^C + M_2^D) - V_2^B(b' + b'') - V_2^C b'' - X_2}{b + b' + b''},$$

$$V_3^A = \frac{(M_3^A + M_3^B + M_3^C + M_3^D) - V_3^B(b' + b'') - V_3^C b'' - X_3}{b + b' + b''},$$

$$V_0^D = \frac{\begin{array}{r} -(M_0^A + M_0^B + M_0^C + M_0^D) - V_0^B b - V_0^C(b + b') \\ + Y_0(b + b' + b'') + X_0 \end{array}}{b + b' + b''},$$

$$V_1^D = \frac{\begin{array}{r} -(M_1^A + M_1^B + M_1^C + M_1^D) - V_1^B b - V_1^C(b + b') \\ + Y_1(b + b' + b'') + X_1 \end{array}}{b + b' + b''},$$

$$V_2^D = \frac{\begin{array}{r} -(M_2^A + M_2^B + M_2^C + M_2^D) - V_2^B b - V_2^C(b + b') \\ + Y_2(b + b' + b'') + X_2 \end{array}}{b + b' + b'',},$$

$$V_3^D = \frac{\begin{array}{r} -(M_3^A + M_3^B + M_3^C + M_3^D) - V_3^B b - V_3^C(b + b') \\ + Y_3(b + b' + b'') + X_3 \end{array}}{b + b' + b''.},$$

This shows that, no matter how many columns in a cross section the axial stress in two of them may always be expressed in terms of the others and the other unknowns. V^B and V^C must be solved for in the regular course of solution as outlined.

Axial Shortening of Column Sections.—From centre of floor girder to centre of floor girder. Since the thrust is axial the shortening of the nth story section will equal

$$\frac{V_n(c_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)}{A_n E}$$

or E times the shortening =

$$\alpha_n = \frac{V_n(c_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)}{A_n}.$$

Values of α are given as follows:

For Column *A*.	For Column *B*.
$\alpha_0^A = \frac{V_0^A(c_0 + \frac{1}{2}d_0)}{A_0^A},$	$\alpha_0^B = \frac{V_0^B(c_0 + \frac{1}{2}d_0)/}{A_0^B},$

$$\alpha_1^A = \frac{V_1^A(c_1 + \frac{1}{2}d_0 + \frac{1}{2}d_1)}{A_1^A}, \qquad \alpha_1^B = \frac{V_1^B(c_1 + \frac{1}{2}d_0 + \frac{1}{2}d_1)}{A_1^B},$$

$$\alpha_2^A = \frac{V_2^A(c_2 + \frac{1}{2}d_1 + \frac{1}{2}d_2)}{A_2^A}, \qquad \alpha_2^B = \frac{V_2^B(c_2 + \frac{1}{2}d_1 + \frac{1}{2}d_2)}{A_2^B},$$

$$\alpha_3^A = \frac{V_3^A(c_3 + \frac{1}{2}d_2 + \frac{1}{2}d_3)}{A_3^A}. \qquad \alpha_3^B = \frac{V_3^B(c_3 + \frac{1}{2}d_2 + \frac{1}{2}d_3)}{A_3^B}.$$

For Column *C*. For Column *D*.

$$\alpha_0^C = \frac{V_0^C(c_0 + \frac{1}{2}d_0)}{A_0^C}, \qquad \alpha_0^D = \frac{V_0^D(c_0 + \frac{1}{2}d_0)}{A_0^D},$$

$$\alpha_1^C = \frac{V_1^C(c_1 + \frac{1}{2}d_0 + \frac{1}{2}d_1)}{A_1^C}, \qquad \alpha_1^D = \frac{V_1^D(c_1 + \frac{1}{2}d_0 + \frac{1}{2}d_1)}{A_1^D},$$

$$\alpha_2^C = \frac{V_2^C(c_2 + \frac{1}{2}d_1 + \frac{1}{2}d_2)}{A_2^C}, \qquad \alpha_2^D = \frac{V_2^D(c_2 + \frac{1}{2}d_1 + \frac{1}{2}d_2)}{A_2^D},$$

$$\alpha_3^C = \frac{V_3^C(c_3 + \frac{1}{2}d_2 + \frac{1}{2}d_3)}{A_3^C}. \qquad \alpha_3^D = \frac{V_3(c_3 + \frac{1}{2}d_2 + \frac{1}{2}d_3)}{A_3^D}.$$

Values of Δ.—Δ, to be positive, will equal *E* times the total shortening of the right hand column minus *E* times the total shortening of the left hand column. Thus, starting at the bottom of the building the values of Δ are:

For Girders "*a*."	For Girders "*b*."	For Girders "*d*."
$\Delta_0^a = \alpha_0^B - \alpha_0^A$,	$\Delta_0^b = \alpha_0^C - \alpha_0^B$,	$\Delta_0^d = o_0^D - \alpha_0^C$,
$\Delta_1^a = \alpha_1^B - \alpha_1^A + \Delta_0^a$,	$\Delta_1^b = \alpha_1^C - \alpha_1^B + \Delta_0^b$,	$\Delta_1^d = \alpha_1^D - \alpha_1^C + \Delta_0^d$,
$\Delta_2^a = \alpha_2^B - \alpha_2^A + \Delta_1^a$,	$\Delta_2^b = \alpha_2^C - \alpha_2^B + \Delta_1^b$,	$\Delta_2^d = \alpha_2^D - \alpha_2^C + \Delta_1^d$,
$\Delta_3^a = a_3^B - \alpha_3^A + \Delta_2^a$.	$\Delta_3^b = \alpha_3^C - \alpha_3^B + \Delta_2^b$.	$\Delta_3^d = \alpha_3^D - \alpha_3^C + \Delta_2^d$.

Slopes and Elastic Lines of the Girders.—Referring to Fig. 6, *a*, pass planes *naa'n*, *nbb'n*, and *ncc'n* as was done in Chapter II. Taking moments about the points *Pa*, *Pb*, and *Pd* respectively the following differential equations of the elastic lines of the girders are found:

For girder *a*–2:

$$\begin{aligned}E\frac{d^2y}{dx^2} = \frac{1}{I_2^a}\Big\{&(V_2^A - V_3^A)(b - x) + \Big[S_2^A(c_2 + \tfrac{1}{2}d_2) + S_3^A\frac{d_2}{2} - M_2^A + M_3^A\Big] \\ &- \frac{w_2(b - x)^2}{2} - P_2^A(b + e_2^A - x)\Big\}.\end{aligned}$$

For girder *b*–2:

$$\begin{aligned}E\frac{d^2y}{dx^2} = \frac{1}{I_2^b}\Big\{&(V_2^A - V_3^A)b + (V_2^A - V_3^A + V_2^B - V_3^B)(b' - x) \\ &+ [(S_2^A + S_2^B)(c_2 + \tfrac{1}{2}d_2) + (S_3^A + S_3^B)\frac{d_2}{2} - M_2^A - M_2^B + M_3^A + M_3^B] \\ &- \frac{w_2b^2}{2} - w_2\Big[b(b' - x) + \frac{(b' - x)^2}{2}\Big] - P_2^A(b + e_2^A) - P_2^A(b' - x)\Big\}.\end{aligned}$$

For girder *d*–2:

$$\begin{aligned}E\frac{d^2y}{dx^2} = \frac{1}{I_2^d}\Big\{&(V_2^A - V_3^A)(b + b') + (V_2^A - V_3^A + V_2^B - V_3^B)b' \\ &+ (V_2^A - V_3^A + V_2^B - V_3^B + V_2^C - V_3^C)(b'' - x) \\ &+ \Big[(S_2^A + S_2^B + S_2^C)(c_2 + \tfrac{1}{2}d_2) + (S_3^A + S_3^B + S_3^C)\frac{d_2}{2} \\ &- M_2^A - M_2^B - M_2^C + M_3^A + M_3^B + M_3^C\Big] - \frac{w_2(b + b')^2}{2} \\ &- w_2\Big[(b+b')(b''-x) + \frac{(b'' - x)^2}{2}\Big] - P_2^A(b+b'+e_2^A) - P_2^A(b''-x)\Big\}.\end{aligned}$$

These expressions may be very much simplified by writing them as follows:

For girder *a*–2:

$$E\frac{d^2y}{dx^2} = Q_2^a + R_2^a(b - x) - \frac{w_2(b - x)^2}{2I_2^a}.$$

For girder *b*–2:

$$E\frac{d^2y}{dx^2} = Q_2^b + R_2^b(b' - x) - \frac{w_2(b' - x)^2}{2I_2^b}.$$

For girder *d*–2:

$$E\frac{d^2y}{dx^2} = Q_2^d + R_2^d(b'' - x) - \frac{w_2(b'' - x)^2}{2I_2^b}.$$

The values of Q in these equations are:

$$Q_2^a = \frac{1}{I_2^a}\left\{ S_2^A(c_2 + \tfrac{1}{2}d_2) + S_3^A\frac{d_2}{2} - M_2^A + M_3^A - P_2^A e_2^A \right\},$$

$$Q_2^b = \frac{1}{I_2^b}\left\{ (S_2^A + S_2^B)(c_2 + \tfrac{1}{2}d_2) + (S_3^A + S_3^B)\frac{d_2}{2} \right.$$
$$\left. - M_2^A - M_2^B + M_3^A + M_3^B - P_2^A(b + e_2^A) - \frac{w_2b^2}{2} + (V_2^A - V_3^A)b \right\},$$

$$Q_2^d = \frac{1}{I_2^d}\left\{ (S_2^A + S_2^B + S_2^C)(c_2 + \tfrac{1}{2}d_2) + (S_3^A + S_3^B + S_3^C)\frac{d_2}{2} \right.$$
$$- M_2^A - M_2^B - M_2^C + M_3^A + M_3^B + M_3^C - P_2^A(b + b' + e_2^A) - \frac{w_2(b + b')^2}{2}$$
$$\left. + (V_2^A - V_3^A)(b + b') + (V_2^A - V_3^A + V_2^B - V_3^B)b' \right\}.$$

The values of R are:

$$R_2^a = \frac{1}{I_2^a}\{(V_2^A - V_3^A) - P_2^A\},$$

$$R_2^b = \frac{1}{I_2^b}\{(V_2^A - V_3^A + V_2^B - V_3^B) - P_2^A - w_2b\},$$

$$R_2^d = \frac{1}{I_2^d}\{(V_2^A - V_3^A + V_2^B - V_3^B + V_2^C - V_3^C) - P_2^A - w_2(b + b')\}.$$

Now the differential equations of the elastic lines for any other story may be written by merely changing the subscripts to correspond with the story.

Integrating the differential equations of the elastic line there results:

For girder a–2.

$$E\frac{dy}{dx} = (Q_2^a + R_2^a b)x - R_2^a\frac{x^2}{2} + \frac{w_2(b - x)^3}{6I_2^a} + K_2^a$$

where K_2^a is a constant of integration.

For girder b–2.

$$E\frac{dy}{dx} = (Q_2^b + R_2^b b')x - R_2^b\frac{x^2}{2} + \frac{w_2(b' - x)^3}{6I_2^b} + K_2^b$$

where K_2^b is a constant of integration.

For girder *d*–2.

$$E\frac{dy}{dx} = (Q_2^d + R_2^d b'')x - R_2^b\frac{x^2}{2} + \frac{w_2(b'' - x)^3}{6I_2^d} + K_2^d$$

where K_2^d is a constant of integration.

But when $x = 0$ for girder *a*–2,

$$E\frac{dy}{dx} = C_3^B$$

and therefore

$$K_2^A = C_3^B - \frac{w_2b^3}{6I_2^a}.$$

Also, when $x = 0$ for girder *b*–2,

$$E\frac{dy}{dx} = C_3^C,$$

and therefore

$$K_2^b = C_3^C - \frac{w_2b'^3}{6I_2^b}.$$

Also, when $x = 0$ for girder *d*–2,

$$E\frac{dy}{dx} = C_3^D,$$

and therefore

$$K_2^d = C_3^D - \frac{w_2b''^3}{6I_2^d}.$$

Thus the *slope equations for the elastic lines of the girders* become:
For girder *a*–2,

$$E\frac{dy}{dx} = (Q_2^a + R_2^a b)x - R_2^a\frac{x^2}{2} + \frac{w_2(b - x)^3}{6I_2^a} + C_3^B - \frac{w_2b^3}{6I_3^A}.$$

For girder *b*–2,

$$E\frac{dy}{dx} = (Q_2^b + R_2^b b')x - R_2^b\frac{x^2}{2} + \frac{w_2(b' - x)^3}{6I_2^b} + C_3^C - \frac{w_2b'^3}{6I_2^b}.$$

For girder *d*–2,

$$E\frac{dy}{dx} = (Q_2^d + R_2^d b'')x - R_2^d\frac{x^2}{2} + \frac{w_2(b'' - x)^3}{6I_2^d} + C_3^D - \frac{w_2b''^3}{6I_2^d}.$$

Integrating again there results:

For girder *a*–2,

$$Ey = (Q_2^a + R_2^a b)\frac{x^2}{2} - R_2^a\frac{x^3}{6} - \frac{w_2(b-x)^4}{24I_2^a} + \left(C_3^B - \frac{w_2b^3}{6I_2^a}\right)x + L_2^a,$$

where L_2^a is a constant of integration.

For girder *b*–2,

$$Ey = (Q_2^b + R_2^b b')\frac{x^2}{2} - R_2^b\frac{x^3}{6} - \frac{w_2(b'-x)^4}{24I_2^b} + \left(C_3^C - \frac{w_2b'^3}{6I_2^b}\right)x + L_2^b,$$

where L_2^b is a constant of integration.

For girder *d*–2,

$$Ey = (Q_2^d + R_2^d b'')\frac{x^2}{2} - R_2^d\frac{x^3}{6} - \frac{w_2(b''-x)^4}{24I_2^d} + \left(C_3^D - \frac{w_2b''^3}{6I_2^d}\right)x + L_2^d,$$

where L_2^d is a constant of integration.

But, in all cases, when $x = 0$, $y = 0$ hence

$$L_2^a = \frac{w_2b^4}{24I_2^a}, \quad L_2^b = \frac{w_2b'^4}{24I_2^b},$$

and

$$L_2^d = \frac{w_2b''^4}{24I_2^d}.$$

Thus the *equations of the elastic lines of the girders* become:

For girder *a*–2,

$$Ey = (Q_2^a + R_2^a b)\frac{x^2}{2} - R_2^a\frac{x^3}{6} - \frac{w_2(b-x)^4}{24I_2^a} + \left(C_3^B - \frac{w_2b^3}{6I_2^a}\right)x + \frac{w_2b^4}{24I_2^a}.$$

For girder *b*–2,

$$Ey = (Q_2^b + R_2^b b')\frac{x^2}{2} - R_2^b\frac{x^3}{6} - \frac{w_2(b'-x)^4}{24I_2^b} + \left(C_3^C - \frac{w_2b'^3}{6I_2^b}\right)x + \frac{w_2b'^4}{24I_2^b}.$$

For girder *d*–2,

$$Ey = (Q_2^d + R_2^d b'')\frac{x^2}{2} - R_2^d\frac{x^3}{6} - \frac{w_2(b''-x)^4}{24I_2^d} + \left(C_3^D - \frac{w_2b''^3}{6I_2^d}\right)x + \frac{w_2b''^4}{24I_2^d}.$$

EQUATIONS FOR THE DETERMINATION OF THE UNKNOWNS.

The properties and elastic lines of all members of the structure have now been given. The equations whose solution determines all the unknowns are as follows:

Equation A.—Referring back to page 38, equation *A*, in general, is given as follows:

$$A'a' + a'c' + c'C' + C'D'' = A''B' + B'b' + b'd' + d'D'.$$

But

$$A'a',\ c'C',\ B'b',\ \text{and}\ \ d'D' = \frac{C}{E} \times \frac{d}{2};$$

$$a'c'\ \ \text{and}\ \ b'd' = \frac{\delta}{E};$$

and

$$A''B'\ \ \text{and}\ \ C'D'' = b - \frac{\lambda}{E}.$$

There will be such an equation for each panel of the cross section and, in order that any one of these equations may be readily referred to, the equations for panels included between Columns *A* and *B* will be known as equations A^a; for the panels included between Columns *B* and *C* they will be known as equations A^b; for those between Columns *C* and *D* they will be known as equations A^d. These equations are as follows:

Equations A^a.

$$\delta_0^A - \delta_0^B + \frac{d_0}{2}(C_1^A - C_1^B) - \lambda_0^a = 0,$$

$$\delta_1^A - \delta_1^B + \frac{d_1}{2}(C_2^A - C_2^B) + \frac{d_0}{2}(C_1^A - C_1^B) + \lambda_0^a - \lambda_1^a = 0,$$

$$\delta_2^A - \delta_2^B + \frac{d_2}{2}(C_3^A - C_3^B) + \frac{d_1}{2}(C_2^A - C_2^B) + \lambda_1^a - \lambda_2^a = 0,$$

$$\delta_3^A - \delta_3^B + \frac{d_3}{2}(C_4^A - C_4^B) + \frac{d_2}{2}(C_3^A - C_3^B) + \lambda_2^a - \lambda_3^a = 0.$$

Equations A^b.

$$\delta_0^B - \delta_0^C + \frac{d_0}{2}(C_1^B - C_1^C) - \lambda_0^b = 0,$$

$$\delta_1^B - \delta_1^C + \frac{d_1}{2}(C_2^B - C_2^C) + \frac{d_0}{2}(C_1^B - C_1^C) + \lambda_0^b - \lambda_1^b = 0,$$

$$\delta_2^B - \delta_2^C + \frac{d_2}{2}(C_3^B - C_3^C) + \frac{d_1}{2}(C_2^B - C_2^C) + \lambda_1^b - \lambda_2^b = 0,$$

$$\delta_3^B - \delta_3^C + \frac{d_3}{2}(C_4^B - C_4^C) + \frac{d_2}{2}(C_3^B - C_3^C) + \lambda_2^b - \lambda_3^b = 0.$$

Equations A^d.

$$\delta_0^C - \delta_0^D + \frac{d_0}{2}(C_1^C - C_1^D) - \lambda_0^d = 0,$$

$$\delta_1^C - \delta_1^D + \frac{d_1}{2}(C_2^C - C_2^D) + \frac{d_0}{2}(C_1^C + C_2^D) + \lambda_0^d - \lambda_1^d = 0,$$

$$\delta_2^C - \delta_2^D + \frac{d_2}{2}(C_3^C - C_3^D) + \frac{d_1}{2}(C_2^C - C_2^D) + \lambda_1^d - \lambda_2^d = 0,$$

$$\delta_3^C - \delta_3^D + \frac{d_3}{2}(C_4^C - C_4^D) + \frac{d_2}{2}(C_3^C - C_3^D) + \lambda_2^d - \lambda_3^d = 0.$$

Equations B.—When x is made equal to b, b', and b'' respectively in the *slope equations of the elastic lines of the girders* above $E \cdot dy/dx$ equals C^A, C^B, and C^C respectively. Now, designating equations B as above, *i. e.*, B^a for girders "a," B^b for girders "b" and B^d for girders "d," these equations may be expressed thus:

Equations B^a ($x = b$).	Equations B^b ($x = b'$).	Equations B^d ($x = b''$).
$E\frac{dy}{dx} = C_1^A,$	$E\frac{dy}{dx} = C_1^B,$	$E\frac{dy}{dx} = C_1^C,$
$E\frac{dy}{dx} = C_2^A,$	$E\frac{dy}{dx} = C_2^B,$	$E\frac{dy}{dx} = C_2^C,$
$E\frac{dy}{dx} = C_3^A,$	$E\frac{dy}{dx} = C_3^B,$	$E\frac{dy}{dx} = C_3^C,$
$E\frac{dy}{dx} = C_4^A.$	$E\frac{dy}{dx} = C_4^B.$	$E\frac{dy}{dx} = C_4^C.$

Equations C.—When x is made equal to b, b', and b'' respectively in the *equations of the elastic lines of the girders* above Ey equals Δ^a, Δ^b, and Δ^d respectively. Now, designating equations C as above, *i. e.*, C^a for girders "a," C^b for girders "b" and C^d for girders "d" these equations may be expressed thus:

Equations C^a ($x = b$).	Equations C^b ($x = b'$).	Equations C^d ($x = b''$).
$Ey = \Delta_0^a,$	$Ey = \Delta_0^b,$	$Ey = \Delta_0^d,$
$Ey = \Delta_1^a,$	$Ey = \Delta_1^b,$	$Ey = \Delta_1^d,$
$Ey = \Delta_2^a,$	$Ey = \Delta_2^b,$	$Ey = \Delta_2^d,$
$Ey = \Delta_3^a.$	$Ey = \Delta_3^b.$	$Ey = \Delta_3^d.$

Now, if the expressions already given for the different terms comprised in these three sets of equations be substituted in the equations there will result, even when reduced to as simple forms as possible, very long and forbidding equations. A systematic method of handling these combined expressions and the three sets of equations A, B, and C must be devised before a solution can be comprehensively considered. Each story contains *nine* unknowns, or three per panel; these are M^A, S^A, V^B, M^B, S^B, V^C, and M^C, S^C, M^D. In addition to keeping these *nine unknowns* in the equations it is found much simpler to include also the dependent quantities C^A, C^B, C^C, C^D, and Δ^a, Δ^b, Δ^d. The reason for this will be understood as the method becomes clear. Now, each of the *expressions* as well as each of the *equations A, B and C* involve more or less of these terms. The coefficients of these terms will in many cases be long expressions of known quantities and these coefficients will be represented in this discussion by the letters of the alphabet and the values of these coefficients may be readily written by reference to the expressions in which they occur. The *most general equation for the nth story,* which will apply to *all expressions* as well as *all equations* is:

$$\begin{aligned} Z_n = {} & a_n^Z M_n^A + b_n^Z M_n^B + c_n^Z M_n^C + d_n^Z M_n^D + e_n^Z M_{n+1}^A + f_n^Z M_{n+1}^B \\ & + g_n^Z M_{n+1}^C + h_n^Z M_{n+1}^D + i_n^Z S_n^A + j_n^Z S_n^B + k_n^Z S_n^C + l_n^Z S_{n+1}^A + m_n^Z S_{n+1}^B \\ & + n_n^Z S_{n+1}^C + o_n^Z V_n^B + p_n^Z V_n^C + q_n^Z V_{n+1}^B + r_n^Z V_{n+1}^C + s_n^Z C_n^A + t_n^Z C_n^B \\ & + u_n^Z C_n^C + v_n^Z C_n^D + w_n^Z \Delta_n^a + x_n^Z \Delta_n^b + y_n^Z \Delta_n^d + P_n^Z S_{n-1}^A + Q_n^Z S_{n-1}^B \\ & + R_n^Z S_{n-1}^C + (z_n^Z = \text{Constant Term}). \end{aligned}$$

In this equation Z_n represents the terms whose value is to be expressed in terms of the unknowns. Thus, for the expression for C_2^B, $Z_n = C_2^B$ and the exponents "Z" used with all coefficients shown above would be $C - B$; therefore, referring back to the expressions for C (page 40)

$$C_2^B = b_1^{C-B} M_1^B + j_1^{C-B} S_1^B + C_1^B$$

in which, of course,

$$b_1^{C-B} = \frac{c_1}{l^B}$$

and

$$j_1^{C-B} = -\frac{c_1^2}{2I_1^B}.$$

For equations A, B, and C, Z_n always equals zero and for the exponents "z" the letters A, B, and C will be used respectively. It will be noted that S_n^D does not appear in the general equation above. This quantity is eliminated by writing

$$S_n^D = \Sigma F - S_n^A - S_n^B - S_n^C$$

since the sum of the horizontal column shears "S" in any story must equal the total wind shear on a section of the building cutting these columns. The equations of the form given above will be called "symbolic" equations and these equations are now given for all expressions and equations of the structure in Plates, 1, 2 and 3.

By comparing the expressions and equations, as already given, with these symbolic forms the values of the coefficients in the symbolic equations are readily found. It will be noticed that in the general expression covering all symbolic equations the value of the subscript n of Z_n will be the same as that of the same subscript occurring with all terms of the equation except as above shown for the expressions for values of C, for which it might have been more properly written Z_{n+1}. The values of the symbolic coefficients are given in Plates 4, 5, 6, 7, and 8.

It is well now to review the process of calculation for determining the stresses in a building up to this point. First the values of w, P, F, X, Y, c, d, b, e, A and I must be calculated and tabulated for each story; also tabulate values of ΣF on the upper portion of the building for each story. Use the proper subscripts and exponents in designating these quantities. Now make blank computation schedules to conform with the blanks containing values of the symbolic coefficients as given in Plates 4, 5, 6, 7, and 8, only, under each coefficient, as a_n say, provide as many columns as there are stories in the structure, *i. e.*, as many as there are values of n. Start at the top of each column and, working downward, fill in each blank, making the calculations indicated in the corresponding blank of Plates 4, 5, 6, 7, and 8, with a Thacher slide rule or instrument

SYMBOLIC EQUATIONS

Values of C

$C_0^A = 0.$

$C_1^A = a_0^{C-A} \cdot M_0^A + i_0^{C-A} \cdot S_0^A.$

$C_2^A = a_1^{C-A} \cdot M_1^A + i_1^{C-A} \cdot S_1^A + C_1^A.$

$C_3^A = a_2^{C-A} \cdot M_2^A + i_2^{C-A} \cdot S_2^A + C_2^A$

$C_4^A = a_3^{C-A} \cdot M_3^A + i_3^{C-A} \cdot S_3^A + C_3^A$

$C_0^B = 0$

$C_1^B = b_0^{C-B} \cdot M_0^B + j_0^{C-B} \cdot S_0^B$

$C_2^B = b_1^{C-B} \cdot M_1^B + j_1^{C-B} \cdot S_1^B + C_1^B$

$C_3^B = b_2^{C-B} \cdot M_2^B + j_2^{C-B} \cdot S_2^B + C_2^B$

$C_4^B = b_3^{C-B} \cdot M_3^B + j_3^{C-B} \cdot S_3^B + C_3^B$

$C_0^C = 0$

$C_1^C = c_0^{C-C} \cdot M_0^C + k_0^{C-C} \cdot S_0^C$

$C_2^C = c_1^{C-C} \cdot M_1^C + k_1^{C-C} \cdot S_1^C + C_1^C$

$C_3^C = c_2^{C-C} \cdot M_2^C + k_2^{C-C} \cdot S_2^C + C_2^C$

$C_4^C = c_3^{C-C} \cdot M_3^C + k_3^{C-C} \cdot S_3^C + C_3^C$

$C_0^D = 0$

$C_1^D = d_0^{C-D} \cdot M_0^D + i_0^{C-D} \cdot S_0^A + j_0^{C-D} \cdot S_0^B + k_0^{C-D} \cdot S_0^C + z_0^{C-D}$

$C_2^D = d_1^{C-D} \cdot M_1^D + i_1^{C-D} \cdot S_1^A + j_1^{C-D} \cdot S_1^B + k_1^{C-D} \cdot S_1^C + z_1^{C-D} + C_1^D$

$C_3^D = d_2^{C-D} \cdot M_2^D + i_2^{C-D} \cdot S_2^A + j_2^{C-D} \cdot S_2^B + k_2^{C-D} \cdot S_2^C + z_2^{C-D} + C_2^D$

$C_4^D = d_3^{C-D} \cdot M_3^D + i_3^{C-D} \cdot S_3^A + j_3^{C-D} \cdot S_3^B + k_3^{C-D} \cdot S_3^C + z_3^{C-D} + C_3^D$

Values of δ

$\delta_0^A = a_0^{\delta-A} \cdot M_0^A + i_0^{\delta-A} \cdot S_0^A$

$\delta_1^A = a_1^{\delta-A} \cdot M_1^A + i_1^{\delta-A} \cdot S_1^A + s_1^{\delta-A} \cdot C_1^A$

$\delta_2^A = a_2^{\delta-A} \cdot M_2^A + i_2^{\delta-A} \cdot S_2^A + s_2^{\delta-A} \cdot C_2^A$

$\delta_3^A = a_3^{\delta-A} \cdot M_3^A + i_3^{\delta-A} \cdot S_3^A + s_3^{\delta-A} \cdot C_3^A$

$\delta_0^B = b_0^{\delta-B} \cdot M_0^B + j_0^{\delta-B} \cdot S_0^B$

$\delta_1^B = b_1^{\delta-B} \cdot M_1^B + j_1^{\delta-B} \cdot S_1^B + t_1^{\delta-B} \cdot C_1^B$

$\delta_2^B = b_2^{\delta-B} \cdot M_2^B + j_2^{\delta-B} \cdot S_2^B + t_2^{\delta-B} \cdot C_2^B$

$\delta_3^B = b_3^{\delta-B} \cdot M_3^B + j_3^{\delta-B} \cdot S_3^B + t_3^{\delta-B} \cdot C_3^B$

$\delta_0^C = c_0^{\delta-C} \cdot M_0^C + k_0^{\delta-C} \cdot S_0^C$

$\delta_1^C = c_1^{\delta-C} \cdot M_1^C + k_1^{\delta-C} \cdot S_1^C + u_1^{\delta-C} \cdot C_1^C$

$\delta_2^C = c_2^{\delta-C} \cdot M_2^C + k_2^{\delta-C} \cdot S_2^C + u_2^{\delta-C} \cdot C_2^C$

$\delta_3^C = c_3^{\delta-C} \cdot M_3^C + k_3^{\delta-C} \cdot S_3^C + u_3^{\delta-C} \cdot C_3^C$

$\delta_0^D = d_0^{\delta-D} \cdot M_0^D + i_0^{\delta-D} \cdot S_0^A + j_0^{\delta-D} \cdot S_0^B + k_0^{\delta-D} \cdot S_0^C + z_0^{\delta-D}$

$\delta_1^D = d_1^{\delta-D} \cdot M_1^D + i_1^{\delta-D} \cdot S_1^A + j_1^{\delta-D} \cdot S_1^B + k_1^{\delta-D} \cdot S_1^C + z_1^{\delta-D} + v_1^{\delta-D} \cdot C_1^D$

$\delta_2^D = d_2^{\delta-D} \cdot M_2^D + i_2^{\delta-D} \cdot S_2^A + j_2^{\delta-D} \cdot S_2^B + k_2^{\delta-D} \cdot S_2^C + z_2^{\delta-D} + v_2^{\delta-D} \cdot C_2^D$

$\delta_3^D = d_3^{\delta-D} \cdot M_3^D + i_3^{\delta-D} \cdot S_3^A + j_3^{\delta-D} \cdot S_3^B + k_3^{\delta-D} \cdot S_3^C + z_3^{\delta-D} + v_3^{\delta-D} \cdot C_3^D$

Values of H.

$H_0^a = l_0^{H-a} \cdot S_1^A + i_0^{H-a} \cdot S_0^A + z_0^{H-a}$

$H_1^a = l_1^{H-a} \cdot S_2^A + i_1^{H-a} \cdot S_1^A + z_1^{H-a}$

$H_2^a = l_2^{H-a} \cdot S_3^A + i_2^{H-a} \cdot S_2^A + z_2^{H-a}$

$H_3^a = \qquad i_3^{H-a} \cdot S_3^A + z_3^{H-a}$

$H_0^b = l_0^{H-b} \cdot S_1^A + m_0^{H-b} \cdot S_1^B + i_0^{H-b} \cdot S_0^A + j_0^{H-b} \cdot S_0^B + z_0^{H-b}$

$H_1^b = l_1^{H-b} \cdot S_2^A + m_1^{H-b} \cdot S_2^B + i_1^{H-b} \cdot S_1^A + j_1^{H-b} \cdot S_1^B + z_1^{H-b}$

$H_2^b = l_2^{H-b} \cdot S_3^A + m_2^{H-b} \cdot S_3^B + i_2^{H-b} \cdot S_2^A + j_2^{H-b} \cdot S_2^B + z_2^{H-b}$

$H_3^b = \qquad i_3^{H-b} \cdot S_3^A + j_3^{H-b} \cdot S_3^B + z_3^{H-b}$

$H_0^d = l_0^{H-d} \cdot S_1^A + m_0^{H-d} \cdot S_1^B + n_0^{H-d} \cdot S_1^C + i_0^{H-d} \cdot S_0^A + j_0^{H-d} \cdot S_0^B + k_0^{H-d} \cdot S_0^C + z_0^{H-d}$

$H_1^d = l_1^{H-d} \cdot S_2^A + m_1^{H-d} \cdot S_2^B + n_1^{H-d} \cdot S_2^C + i_1^{H-d} \cdot S_1^A + j_1^{H-d} \cdot S_1^B + k_1^{H-d} \cdot S_1^C + z_1^{H-d}$

$H_2^d = l_2^{H-d} \cdot S_3^A + m_2^{H-d} \cdot S_3^B + n_2^{H-d} \cdot S_3^C + i_2^{H-d} \cdot S_2^A + j_2^{H-d} \cdot S_2^B + k_2^{H-d} \cdot S_2^C + z_2^{H-d}$

$H_3^d = \qquad i_3^{H-d} \cdot S_3^A + j_3^{H-d} \cdot S_3^B + k_3^{H-d} \cdot S_3^C + z_3^{H-d}$

Values of λ

$\lambda_0^a = l_0^{\lambda-a} \cdot S_1^A + i_0^{\lambda-a} \cdot S_0^A + z_0^{\lambda-a}$

$\lambda_1^a = l_1^{\lambda-a} \cdot S_2^A + i_1^{\lambda-a} \cdot S_1^A + z_1^{\lambda-a}$

$\lambda_2^a = l_2^{\lambda-a} \cdot S_3^A + i_2^{\lambda-a} \cdot S_2^A + z_2^{\lambda-a}$

$\lambda_3^a = \qquad i_3^{\lambda-a} \cdot S_3^A + z_3^{\lambda-a}$

$\lambda_0^b = l_0^{\lambda-b} \cdot S_1^A + m_0^{\lambda-b} \cdot S_1^B + i_0^{\lambda-b} \cdot S_0^A + j_0^{\lambda-b} \cdot S_0^B + z_0^{\lambda-b}$

$\lambda_1^b = l_1^{\lambda-b} \cdot S_2^A + m_1^{\lambda-b} \cdot S_2^B + i_1^{\lambda-b} \cdot S_1^A + j_1^{\lambda-b} \cdot S_1^B + z_1^{\lambda-b}$

$\lambda_2^b = l_2^{\lambda-b} \cdot S_3^A + m_2^{\lambda-b} \cdot S_3^B + i_2^{\lambda-b} \cdot S_2^A + j_2^{\lambda-b} \cdot S_2^B + z_2^{\lambda-b}$

$\lambda_3^b = \qquad i_3^{\lambda-b} \cdot S_3^A + j_3^{\lambda-b} \cdot S_3^B + z_3^{\lambda-b}$

$\lambda_0^d = l_0^{\lambda-d} \cdot S_1^A + m_0^{\lambda-d} \cdot S_1^B + n_0^{\lambda-d} \cdot S_1^C + i_0^{\lambda-d} \cdot S_0^A + j_0^{\lambda-d} \cdot S_0^B + k_0^{\lambda-d} \cdot S_0^C + z_0^{\lambda-d}$

$\lambda_1^d = l_1^{\lambda-d} \cdot S_2^A + m_1^{\lambda-d} \cdot S_2^B + n_1^{\lambda-d} \cdot S_2^C + i_1^{\lambda-d} \cdot S_1^A + j_1^{\lambda-d} \cdot S_1^B + k_1^{\lambda-d} \cdot S_1^C + z_1^{\lambda-d}$

$\lambda_2^d = l_2^{\lambda-d} \cdot S_3^A + m_2^{\lambda-d} \cdot S_3^B + n_2^{\lambda-d} \cdot S_3^C + i_2^{\lambda-d} \cdot S_2^A + j_2^{\lambda-d} \cdot S_2^B + k_2^{\lambda-d} \cdot S_2^C + z_2^{\lambda-d}$

$\lambda_3^d = \qquad i_3^{\lambda-d} \cdot S_3^A + j_3^{\lambda-d} \cdot S_3^B + k_3^{\lambda-d} \cdot S_3^C + z_3^{\lambda-d}$

Values of V

$V_0^A = a_0^{V-A} \cdot M_0^A + b_0^{V-A} \cdot M_0^B + c_0^{V-A} \cdot M_0^C + d_0^{V-A} \cdot M_0^D + o_0^{V-A} \cdot V_0^B + p_0^{V-A} \cdot V_0^C + z_0^{V-A}$

$V_1^A = a_1^{V-A} \cdot M_1^A + b_1^{V-A} \cdot M_1^B + c_1^{V-A} \cdot M_1^C + d_1^{V-A} \cdot M_1^D + o_1^{V-A} \cdot V_1^B + p_1^{V-A} \cdot V_1^C + z_1^{V-A}$

$V_2^A = a_2^{V-A} \cdot M_2^A + b_2^{V-A} \cdot M_2^B + c_2^{V-A} \cdot M_2^C + d_2^{V-A} \cdot M_2^D + o_2^{V-A} \cdot V_2^B + p_2^{V-A} \cdot V_2^C + z_2^{V-A}$

$V_3^A = a_3^{V-A} \cdot M_3^A + b_3^{V-A} \cdot M_3^B + c_3^{V-A} \cdot M_3^C + d_3^{V-A} \cdot M_3^D + o_3^{V-A} \cdot V_3^B + p_3^{V-A} \cdot V_3^C + z_3^{V-A}$

$V_0^D = a_0^{V-D} \cdot M_0^A + b_0^{V-D} \cdot M_0^B + c_0^{V-D} \cdot M_0^C + d_0^{V-D} \cdot M_0^D + o_0^{V-D} \cdot V_0^B + p_0^{V-D} \cdot V_0^C + z_0^{V-D}$

$V_1^D = a_1^{V-D} \cdot M_1^A + b_1^{V-D} \cdot M_1^B + c_1^{V-D} \cdot M_1^C + d_1^{V-D} \cdot M_1^D + o_1^{V-D} \cdot V_1^B + p_1^{V-D} \cdot V_1^C + z_1^{V-D}$

$V_2^D = a_2^{V-D} \cdot M_2^A + b_2^{V-D} \cdot M_2^B + c_2^{V-D} \cdot M_2^C + d_2^{V-D} \cdot M_2^D + o_2^{V-D} \cdot V_2^B + p_2^{V-D} \cdot V_2^C + z_2^{V-D}$

$V_3^D = a_3^{V-D} \cdot M_3^A + b_3^{V-D} \cdot M_3^B + c_3^{V-D} \cdot M_3^C + d_3^{V-D} \cdot M_3^D + o_3^{V-D} \cdot V_3^B + p_3^{V-D} \cdot V_3^C + z_3^{V-D}$

Values of α.

$\alpha_0^A = a_0^{\alpha-A} \cdot M_0^A + b_0^{\alpha-A} \cdot M_0^B + c_0^{\alpha-A} \cdot M_0^C + d_0^{\alpha-A} \cdot M_0^D + o_0^{\alpha-A} \cdot V_0^B + p_0^{\alpha-A} \cdot V_0^C + z_0^{\alpha-A}$

$\alpha_1^A = a_1^{\alpha-A} \cdot M_1^A + b_1^{\alpha-A} \cdot M_1^B + c_1^{\alpha-A} \cdot M_1^C + d_1^{\alpha-A} \cdot M_1^D + o_1^{\alpha-A} \cdot V_1^B + p_1^{\alpha-A} \cdot V_1^C + z_1^{\alpha-A}$

$\alpha_2^A = a_2^{\alpha-A} \cdot M_2^A + b_2^{\alpha-A} \cdot M_2^B + c_2^{\alpha-A} \cdot M_2^C + d_2^{\alpha-A} \cdot M_2^D + o_2^{\alpha-A} \cdot V_2^B + p_2^{\alpha-A} \cdot V_2^C + z_2^{\alpha-A}$

$\alpha_3^A = a_3^{\alpha-A} \cdot M_3^A + b_3^{\alpha-A} \cdot M_3^B + c_3^{\alpha-A} \cdot M_3^C + d_3^{\alpha-A} \cdot M_3^D + o_3^{\alpha-A} \cdot V_3^B + p_3^{\alpha-A} \cdot V_3^C + z_3^{\alpha-A}$

$\alpha_0^B = o_0^{\alpha-B} \cdot V_0^B$

$\alpha_1^B = o_1^{\alpha-B} \cdot V_1^B$

$\alpha_2^B = o_2^{\alpha-B} \cdot V_2^B$

$\alpha_3^B = o_3^{\alpha-B} \cdot V_3^B$

$\alpha_0^C = p_0^{\alpha-C} \cdot V_0^C$

$\alpha_1^C = p_1^{\alpha-C} \cdot V_1^C$

$\alpha_2^C = p_2^{\alpha-C} \cdot V_2^C$

$\alpha_3^C = p_3^{\alpha-C} \cdot V_3^C$

$\alpha_0^D = a_0^{\alpha-D} \cdot M_0^A + b_0^{\alpha-D} \cdot M_0^B + c_0^{\alpha-D} \cdot M_0^C + d_0^{\alpha-D} \cdot M_0^D + o_0^{\alpha-D} \cdot V_0^B + p_0^{\alpha-D} \cdot V_0^C + z_0^{\alpha-D}$

$\alpha_1^D = a_1^{\alpha-D} \cdot M_1^A + b_1^{\alpha-D} \cdot M_1^B + c_1^{\alpha-D} \cdot M_1^C + d_1^{\alpha-D} \cdot M_1^D + o_1^{\alpha-D} \cdot V_1^B + p_1^{\alpha-D} \cdot V_1^C + z_1^{\alpha-D}$

$\alpha_2^D = a_2^{\alpha-D} \cdot M_2^A + b_2^{\alpha-D} \cdot M_2^B + c_2^{\alpha-D} \cdot M_2^C + d_2^{\alpha-D} \cdot M_2^D + o_2^{\alpha-D} \cdot V_2^B + p_2^{\alpha-D} \cdot V_2^C + z_2^{\alpha-D}$

$\alpha_3^D = a_3^{\alpha-D} \cdot M_3^A + b_3^{\alpha-D} \cdot M_3^B + c_3^{\alpha-D} \cdot M_3^C + d_3^{\alpha-D} \cdot M_3^D + o_3^{\alpha-D} \cdot V_3^B + p_3^{\alpha-D} \cdot V_3^C + z_3^{\alpha-D}$

Values of Δ.

$\Delta_0^a = a_0^{\Delta-a} \cdot M_0^A + b_0^{\Delta-a} \cdot M_0^B + c_0^{\Delta-a} \cdot M_0^C + d_0^{\Delta-a} \cdot M_0^D + o_0^{\Delta-a} \cdot V_0^B + p_0^{\Delta-a} \cdot V_0^C + z_0^{\Delta-a}$

$\Delta_1^a = a_1^{\Delta-a} \cdot M_1^A + b_1^{\Delta-a} \cdot M_1^B + c_1^{\Delta-a} \cdot M_1^C + d_1^{\Delta-a} \cdot M_1^D + o_1^{\Delta-a} \cdot V_1^B + p_1^{\Delta-a} \cdot V_1^C + z_1^{\Delta-a} + \Delta_0^a$

$\Delta_2^a = a_2^{\Delta-a} \cdot M_2^A + b_2^{\Delta-a} \cdot M_2^B + c_2^{\Delta-a} \cdot M_2^C + d_2^{\Delta-a} \cdot M_2^D + o_2^{\Delta-a} \cdot V_2^B + p_2^{\Delta-a} \cdot V_2^C + z_2^{\Delta-a} + \Delta_1^a$

$\Delta_3^a = a_3^{\Delta-a} \cdot M_3^A + b_3^{\Delta-a} \cdot M_3^B + c_3^{\Delta-a} \cdot M_3^C + d_3^{\Delta-a} \cdot M_3^D + o_3^{\Delta-a} \cdot V_3^B + p_3^{\Delta-a} \cdot V_3^C + z_3^{\Delta-a} + \Delta_2^a$

$\Delta_0^b = o_0^{\Delta-b} \cdot V_0^B + p_0^{\Delta-b} \cdot V_0^C$

$\Delta_1^b = o_1^{\Delta-b} \cdot V_1^B + p_1^{\Delta-b} \cdot V_1^C + \Delta_0^b$

$\Delta_2^b = o_2^{\Delta-b} \cdot V_2^B + p_2^{\Delta-b} \cdot V_2^C + \Delta_1^b$

$\Delta_3^b = o_3^{\Delta-b} \cdot V_3^B + p_3^{\Delta-b} \cdot V_3^C + \Delta_2^b$

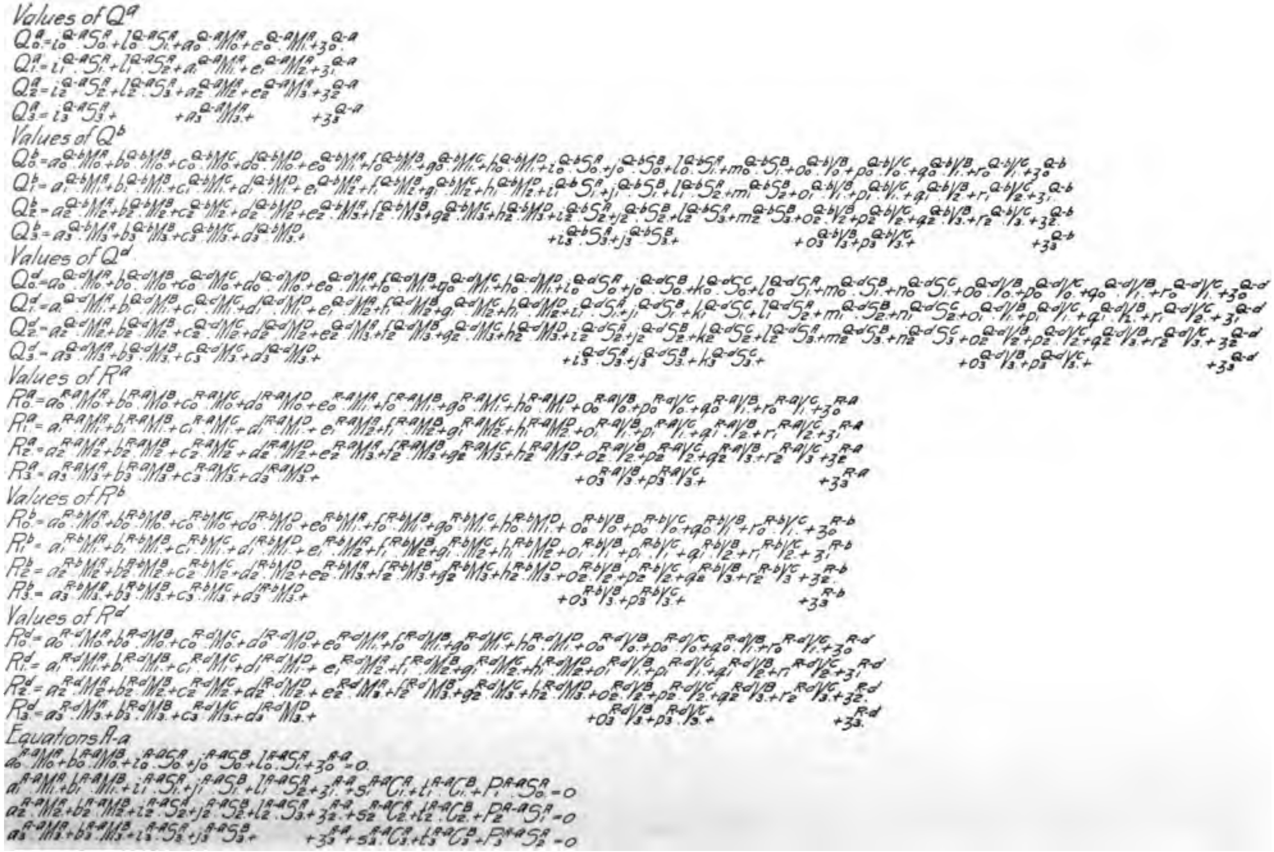
$\Delta_0^d = a_0^{\Delta-d} \cdot M_0^A + b_0^{\Delta-d} \cdot M_0^B + c_0^{\Delta-d} \cdot M_0^C + d_0^{\Delta-d} \cdot M_0^D + o_0^{\Delta-d} \cdot V_0^B + p_0^{\Delta-d} \cdot V_0^C + z_0^{\Delta-d}$

$\Delta_1^d = a_1^{\Delta-d} \cdot M_1^A + b_1^{\Delta-d} \cdot M_1^B + c_1^{\Delta-d} \cdot M_1^C + d_1^{\Delta-d} \cdot M_1^D + o_1^{\Delta-d} \cdot V_1^B + p_1^{\Delta-d} \cdot V_1^C + z_1^{\Delta-d} + \Delta_0^d$

$\Delta_2^d = a_2^{\Delta-d} \cdot M_2^A + b_2^{\Delta-d} \cdot M_2^B + c_2^{\Delta-d} \cdot M_2^C + d_2^{\Delta-d} \cdot M_2^D + o_2^{\Delta-d} \cdot V_2^B + p_2^{\Delta-d} \cdot V_2^C + z_2^{\Delta-d} + \Delta_1^d$

$\Delta_3^d = a_3^{\Delta-d} \cdot M_3^A + b_3^{\Delta-d} \cdot M_3^B + c_3^{\Delta-d} \cdot M_3^C + d_3^{\Delta-d} \cdot M_3^D + o_3^{\Delta-d} \cdot V_3^B + p_3^{\Delta-d} \cdot V_3^C + z_3^{\Delta-d} + \Delta_2^d$

PLATE I.



Values of Q^a

$$Q_0^a = i_0 \cdot {}^{Q-a}S_0^A + l_0 \cdot {}^{Q-a}S_1^A + a_0 \cdot {}^{Q-a}M_0^A + e_0 \cdot {}^{Q-a}M_1^A + z_0^{Q-a}$$

$$Q_1^a = i_1 \cdot {}^{Q-a}S_1^A + l_1 \cdot {}^{Q-a}S_2^A + a_1 \cdot {}^{Q-a}M_1^A + e_1 \cdot {}^{Q-a}M_2^A + z_1^{Q-a}$$

$$Q_2^a = i_2 \cdot {}^{Q-a}S_2^A + l_2 \cdot {}^{Q-a}S_3^A + a_2 \cdot {}^{Q-a}M_2^A + e_2 \cdot {}^{Q-a}M_3^A + z_2^{Q-a}$$

$$Q_3^a = i_3 \cdot {}^{Q-a}S_3^A + \qquad + a_3 \cdot {}^{Q-a}M_3^A + \qquad + z_3^{Q-a}$$

Values of Q^b

$$Q_0^b = a_0 \cdot {}^{Q-b}M_0^A + b_0 \cdot {}^{Q-b}M_0^B + c_0 \cdot {}^{Q-b}M_0^C + d_0 \cdot {}^{Q-b}M_0^D + e_0 \cdot {}^{Q-b}M_1^A + f_0 \cdot {}^{Q-b}M_1^B + g_0 \cdot {}^{Q-b}M_1^C + h_0 \cdot {}^{Q-b}M_1^D + i_0 \cdot {}^{Q-b}S_0^A + j_0 \cdot {}^{Q-b}S_0^B + l_0 \cdot {}^{Q-b}S_1^A + m_0 \cdot {}^{Q-b}S_1^B + o_0 \cdot {}^{Q-b}V_0^B + p_0 \cdot {}^{Q-b}V_0^C + q_0 \cdot {}^{Q-b}V_1^B + r_0 \cdot {}^{Q-b}V_1^C + z_0^{Q-b}$$

$$Q_1^b = a_1 \cdot {}^{Q-b}M_1^A + b_1 \cdot {}^{Q-b}M_1^B + c_1 \cdot {}^{Q-b}M_1^C + d_1 \cdot {}^{Q-b}M_1^D + e_1 \cdot {}^{Q-b}M_2^A + f_1 \cdot {}^{Q-b}M_2^B + g_1 \cdot {}^{Q-b}M_2^C + h_1 \cdot {}^{Q-b}M_2^D + i_1 \cdot {}^{Q-b}S_1^A + j_1 \cdot {}^{Q-b}S_1^B + l_1 \cdot {}^{Q-b}S_2^A + m_1 \cdot {}^{Q-b}S_2^B + o_1 \cdot {}^{Q-b}V_1^B + p_1 \cdot {}^{Q-b}V_1^C + q_1 \cdot {}^{Q-b}V_2^B + r_1 \cdot {}^{Q-b}V_2^C + z_1^{Q-b}$$

$$Q_2^b = a_2 \cdot {}^{Q-b}M_2^A + b_2 \cdot {}^{Q-b}M_2^B + c_2 \cdot {}^{Q-b}M_2^C + d_2 \cdot {}^{Q-b}M_2^D + e_2 \cdot {}^{Q-b}M_3^A + f_2 \cdot {}^{Q-b}M_3^B + g_2 \cdot {}^{Q-b}M_3^C + h_2 \cdot {}^{Q-b}M_3^D + i_2 \cdot {}^{Q-b}S_2^A + j_2 \cdot {}^{Q-b}S_2^B + l_2 \cdot {}^{Q-b}S_3^A + m_2 \cdot {}^{Q-b}S_3^B + o_2 \cdot {}^{Q-b}V_2^B + p_2 \cdot {}^{Q-b}V_2^C + q_2 \cdot {}^{Q-b}V_3^B + r_2 \cdot {}^{Q-b}V_3^C + z_2^{Q-b}$$

$$Q_3^b = a_3 \cdot {}^{Q-b}M_3^A + b_3 \cdot {}^{Q-b}M_3^B + c_3 \cdot {}^{Q-b}M_3^C + d_3 \cdot {}^{Q-b}M_3^D + \qquad + i_3 \cdot {}^{Q-b}S_3^A + j_3 \cdot {}^{Q-b}S_3^B + \qquad + o_3 \cdot {}^{Q-b}V_3^B + p_3 \cdot {}^{Q-b}V_3^C + \qquad + z_3^{Q-b}$$

Values of Q^d

$$Q_0^d = a_0 \cdot {}^{Q-d}M_0^A + b_0 \cdot {}^{Q-d}M_0^B + c_0 \cdot {}^{Q-d}M_0^C + d_0 \cdot {}^{Q-d}M_0^D + e_0 \cdot {}^{Q-d}M_1^A + f_0 \cdot {}^{Q-d}M_1^B + g_0 \cdot {}^{Q-d}M_1^C + h_0 \cdot {}^{Q-d}M_1^D + i_0 \cdot {}^{Q-d}S_0^A + j_0 \cdot {}^{Q-d}S_0^B + k_0 \cdot {}^{Q-d}S_0^C + l_0 \cdot {}^{Q-d}S_1^A + m_0 \cdot {}^{Q-d}S_1^B + n_0 \cdot {}^{Q-d}S_1^C + o_0 \cdot {}^{Q-d}V_0^B + p_0 \cdot {}^{Q-d}V_0^C + q_0 \cdot {}^{Q-d}V_1^B + r_0 \cdot {}^{Q-d}V_1^C + z_0^{Q-d}$$

$$Q_1^d = a_1 \cdot {}^{Q-d}M_1^A + b_1 \cdot {}^{Q-d}M_1^B + c_1 \cdot {}^{Q-d}M_1^C + d_1 \cdot {}^{Q-d}M_1^D + e_1 \cdot {}^{Q-d}M_2^A + f_1 \cdot {}^{Q-d}M_2^B + g_1 \cdot {}^{Q-d}M_2^C + h_1 \cdot {}^{Q-d}M_2^D + i_1 \cdot {}^{Q-d}S_1^A + j_1 \cdot {}^{Q-d}S_1^B + k_1 \cdot {}^{Q-d}S_1^C + l_1 \cdot {}^{Q-d}S_2^A + m_1 \cdot {}^{Q-d}S_2^B + n_1 \cdot {}^{Q-d}S_2^C + o_1 \cdot {}^{Q-d}V_1^B + p_1 \cdot {}^{Q-d}V_1^C + q_1 \cdot {}^{Q-d}V_2^B + r_1 \cdot {}^{Q-d}V_2^C + z_1^{Q-d}$$

$$Q_2^d = a_2 \cdot {}^{Q-d}M_2^A + b_2 \cdot {}^{Q-d}M_2^B + c_2 \cdot {}^{Q-d}M_2^C + d_2 \cdot {}^{Q-d}M_2^D + e_2 \cdot {}^{Q-d}M_3^A + f_2 \cdot {}^{Q-d}M_3^B + g_2 \cdot {}^{Q-d}M_3^C + h_2 \cdot {}^{Q-d}M_3^D + i_2 \cdot {}^{Q-d}S_2^A + j_2 \cdot {}^{Q-d}S_2^B + k_2 \cdot {}^{Q-d}S_2^C + l_2 \cdot {}^{Q-d}S_3^A + m_2 \cdot {}^{Q-d}S_3^B + n_2 \cdot {}^{Q-d}S_3^C + o_2 \cdot {}^{Q-d}V_2^B + p_2 \cdot {}^{Q-d}V_2^C + q_2 \cdot {}^{Q-d}V_3^B + r_2 \cdot {}^{Q-d}V_3^C + z_2^{Q-d}$$

$$Q_3^d = a_3 \cdot {}^{Q-d}M_3^A + b_3 \cdot {}^{Q-d}M_3^B + c_3 \cdot {}^{Q-d}M_3^C + d_3 \cdot {}^{Q-d}M_3^D + \qquad + i_3 \cdot {}^{Q-d}S_3^A + j_3 \cdot {}^{Q-d}S_3^B + k_3 \cdot {}^{Q-d}S_3^C + \qquad + o_3 \cdot {}^{Q-d}V_3^B + p_3 \cdot {}^{Q-d}V_3^C + \qquad + z_3^{Q-d}$$

Values of R^a

$$R_0^a = a_0 \cdot {}^{R-a}M_0^A + b_0 \cdot {}^{R-a}M_0^B + c_0 \cdot {}^{R-a}M_0^C + d_0 \cdot {}^{R-a}M_0^D + e_0 \cdot {}^{R-a}M_1^A + f_0 \cdot {}^{R-a}M_1^B + g_0 \cdot {}^{R-a}M_1^C + h_0 \cdot {}^{R-a}M_1^D + o_0 \cdot {}^{R-a}V_0^B + p_0 \cdot {}^{R-a}V_0^C + q_0 \cdot {}^{R-a}V_1^B + r_0 \cdot {}^{R-a}V_1^C + z_0^{R-a}$$

$$R_1^a = a_1 \cdot {}^{R-a}M_1^A + b_1 \cdot {}^{R-a}M_1^B + c_1 \cdot {}^{R-a}M_1^C + d_1 \cdot {}^{R-a}M_1^D + e_1 \cdot {}^{R-a}M_2^A + f_1 \cdot {}^{R-a}M_2^B + g_1 \cdot {}^{R-a}M_2^C + h_1 \cdot {}^{R-a}M_2^D + o_1 \cdot {}^{R-a}V_1^B + p_1 \cdot {}^{R-a}V_1^C + q_1 \cdot {}^{R-a}V_2^B + r_1 \cdot {}^{R-a}V_2^C + z_1^{R-a}$$

$$R_2^a = a_2 \cdot {}^{R-a}M_2^A + b_2 \cdot {}^{R-a}M_2^B + c_2 \cdot {}^{R-a}M_2^C + d_2 \cdot {}^{R-a}M_2^D + e_2 \cdot {}^{R-a}M_3^A + f_2 \cdot {}^{R-a}M_3^B + g_2 \cdot {}^{R-a}M_3^C + h_2 \cdot {}^{R-a}M_3^D + o_2 \cdot {}^{R-a}V_2^B + p_2 \cdot {}^{R-a}V_2^C + q_2 \cdot {}^{R-a}V_3^B + r_2 \cdot {}^{R-a}V_3^C + z_2^{R-a}$$

$$R_3^a = a_3 \cdot {}^{R-a}M_3^A + b_3 \cdot {}^{R-a}M_3^B + c_3 \cdot {}^{R-a}M_3^C + d_3 \cdot {}^{R-a}M_3^D + \qquad + o_3 \cdot {}^{R-a}V_3^B + p_3 \cdot {}^{R-a}V_3^C + \qquad + z_3^{R-a}$$

Values of R^b

$$R_0^b = a_0 \cdot {}^{R-b}M_0^A + b_0 \cdot {}^{R-b}M_0^B + c_0 \cdot {}^{R-b}M_0^C + d_0 \cdot {}^{R-b}M_0^D + e_0 \cdot {}^{R-b}M_1^A + f_0 \cdot {}^{R-b}M_1^B + g_0 \cdot {}^{R-b}M_1^C + h_0 \cdot {}^{R-b}M_1^D + o_0 \cdot {}^{R-b}V_0^B + p_0 \cdot {}^{R-b}V_0^C + q_0 \cdot {}^{R-b}V_1^B + r_0 \cdot {}^{R-b}V_1^C + z_0^{R-b}$$

$$R_1^b = a_1 \cdot {}^{R-b}M_1^A + b_1 \cdot {}^{R-b}M_1^B + c_1 \cdot {}^{R-b}M_1^C + d_1 \cdot {}^{R-b}M_1^D + e_1 \cdot {}^{R-b}M_2^A + f_1 \cdot {}^{R-b}M_2^B + g_1 \cdot {}^{R-b}M_2^C + h_1 \cdot {}^{R-b}M_2^D + o_1 \cdot {}^{R-b}V_1^B + p_1 \cdot {}^{R-b}V_1^C + q_1 \cdot {}^{R-b}V_2^B + r_1 \cdot {}^{R-b}V_2^C + z_1^{R-b}$$

$$R_2^b = a_2 \cdot {}^{R-b}M_2^A + b_2 \cdot {}^{R-b}M_2^B + c_2 \cdot {}^{R-b}M_2^C + d_2 \cdot {}^{R-b}M_2^D + e_2 \cdot {}^{R-b}M_3^A + f_2 \cdot {}^{R-b}M_3^B + g_2 \cdot {}^{R-b}M_3^C + h_2 \cdot {}^{R-b}M_3^D + o_2 \cdot {}^{R-b}V_2^B + p_2 \cdot {}^{R-b}V_2^C + q_2 \cdot {}^{R-b}V_3^B + r_2 \cdot {}^{R-b}V_3^C + z_2^{R-b}$$

$$R_3^b = a_3 \cdot {}^{R-b}M_3^A + b_3 \cdot {}^{R-b}M_3^B + c_3 \cdot {}^{R-b}M_3^C + d_3 \cdot {}^{R-b}M_3^D + \qquad + o_3 \cdot {}^{R-b}V_3^B + p_3 \cdot {}^{R-b}V_3^C + \qquad + z_3^{R-b}$$

Values of R^d

$$R_0^d = a_0 \cdot {}^{R-d}M_0^A + b_0 \cdot {}^{R-d}M_0^B + c_0 \cdot {}^{R-d}M_0^C + d_0 \cdot {}^{R-d}M_0^D + e_0 \cdot {}^{R-d}M_1^A + f_0 \cdot {}^{R-d}M_1^B + g_0 \cdot {}^{R-d}M_1^C + h_0 \cdot {}^{R-d}M_1^D + o_0 \cdot {}^{R-d}V_0^B + p_0 \cdot {}^{R-d}V_0^C + q_0 \cdot {}^{R-d}V_1^B + r_0 \cdot {}^{R-d}V_1^C + z_0^{R-d}$$

$$R_1^d = a_1 \cdot {}^{R-d}M_1^A + b_1 \cdot {}^{R-d}M_1^B + c_1 \cdot {}^{R-d}M_1^C + d_1 \cdot {}^{R-d}M_1^D + e_1 \cdot {}^{R-d}M_2^A + f_1 \cdot {}^{R-d}M_2^B + g_1 \cdot {}^{R-d}M_2^C + h_1 \cdot {}^{R-d}M_2^D + o_1 \cdot {}^{R-d}V_1^B + p_1 \cdot {}^{R-d}V_1^C + q_1 \cdot {}^{R-d}V_2^B + r_1 \cdot {}^{R-d}V_2^C + z_1^{R-d}$$

$$R_2^d = a_2 \cdot {}^{R-d}M_2^A + b_2 \cdot {}^{R-d}M_2^B + c_2 \cdot {}^{R-d}M_2^C + d_2 \cdot {}^{R-d}M_2^D + e_2 \cdot {}^{R-d}M_3^A + f_2 \cdot {}^{R-d}M_3^B + g_2 \cdot {}^{R-d}M_3^C + h_2 \cdot {}^{R-d}M_3^D + o_2 \cdot {}^{R-d}V_2^B + p_2 \cdot {}^{R-d}V_2^C + q_2 \cdot {}^{R-d}V_3^B + r_2 \cdot {}^{R-d}V_3^C + z_2^{R-d}$$

$$R_3^d = a_3 \cdot {}^{R-d}M_3^A + b_3 \cdot {}^{R-d}M_3^B + c_3 \cdot {}^{R-d}M_3^C + d_3 \cdot {}^{R-d}M_3^D + \qquad + o_3 \cdot {}^{R-d}V_3^B + p_3 \cdot {}^{R-d}V_3^C + \qquad + z_3^{R-d}$$

Equations A-a

$$a_0 \cdot {}^{A-a}M_0^A + b_0 \cdot {}^{A-a}M_0^B + i_0 \cdot {}^{A-a}S_0^A + j_0 \cdot {}^{A-a}S_0^B + l_0 \cdot {}^{A-a}S_1^A + z_0^{A-a} = 0.$$

$$a_1 \cdot {}^{A-a}M_1^A + b_1 \cdot {}^{A-a}M_1^B + i_1 \cdot {}^{A-a}S_1^A + j_1 \cdot {}^{A-a}S_1^B + l_1 \cdot {}^{A-a}S_2^A + z_1^{A-a} + s_1 \cdot {}^{A-a}C_1^A + t_1 \cdot {}^{A-a}C_1^B + P_1 \cdot {}^{A-a}S_0^A = 0$$

$$a_2 \cdot {}^{A-a}M_2^A + b_2 \cdot {}^{A-a}M_2^B + i_2 \cdot {}^{A-a}S_2^A + j_2 \cdot {}^{A-a}S_2^B + l_2 \cdot {}^{A-a}S_3^A + z_2^{A-a} + s_2 \cdot {}^{A-a}C_2^A + t_2 \cdot {}^{A-a}C_2^B + P_2 \cdot {}^{A-a}S_1^A = 0$$

$$a_3 \cdot {}^{A-a}M_3^A + b_3 \cdot {}^{A-a}M_3^B + i_3 \cdot {}^{A-a}S_3^A + j_3 \cdot {}^{A-a}S_3^B + \qquad + z_3^{A-a} + s_3 \cdot {}^{A-a}C_3^A + t_3 \cdot {}^{A-a}C_3^B + P_3 \cdot {}^{A-a}S_2^A = 0$$

PLATE 2.

Equations A-b.

$$b_0^{A-b}M_0^B+c_0^{A-b}M_0^C+i_0^{A-b}S_0^A+j_0^{A-b}S_0^B+k_0^{A-b}S_0^C+l_0^{A-b}S_1^A+m_0^{A-b}S_1^B+z_0^{A-b}=0$$

$$b_1^{A-b}M_1^B+c_1^{A-b}M_1^C+i_1^{A-b}S_1^A+j_1^{A-b}S_1^B+k_1^{A-b}S_1^C+l_1^{A-b}S_2^A+m_1^{A-b}S_2^B+z_1^{A-b}+t_1^{A-b}C_1^B+u_1^{A-b}C_1^C+P_1^{A-b}S_0^A+Q_1^{A-b}S_0^B=0$$

$$b_2^{A-b}M_2^B+c_2^{A-b}M_2^C+i_2^{A-b}S_2^A+j_2^{A-b}S_2^B+k_2^{A-b}S_2^C+l_2^{A-b}S_3^A+m_2^{A-b}S_3^B+z_2^{A-b}+t_2^{A-b}C_2^B+u_2^{A-b}C_2^C+P_2^{A-b}S_1^A+Q_2^{A-b}S_1^B=0$$

$$b_3^{A-b}M_3^B+c_3^{A-b}M_3^C+i_3^{A-b}S_3^A+j_3^{A-b}S_3^B+k_3^{A-b}S_3^C+z_3^{A-b}+t_3^{A-b}C_3^B+u_3^{A-b}C_3^C+P_3^{A-b}S_2^A+Q_3^{A-b}S_2^B=0$$

Equations A-d.

$$c_0^{A-d}M_0^C+d_0^{A-d}M_0^D+i_0^{A-d}S_0^A+j_0^{A-d}S_0^B+k_0^{A-d}S_0^C+l_0^{A-d}S_1^A+m_0^{A-d}S_1^B+n_0^{A-d}S_1^C+z_0^{A-d}=0$$

$$c_1^{A-d}M_1^C+d_1^{A-d}M_1^D+i_1^{A-d}S_1^A+j_1^{A-d}S_1^B+k_1^{A-d}S_1^C+l_1^{A-d}S_2^A+m_1^{A-d}S_2^B+n_1^{A-d}S_2^C+z_1^{A-d}+u_1^{A-d}C_1^C+v_1^{A-d}C_1^D+P_1^{A-d}S_0^A+Q_1^{A-d}S_0^B+R_1^{A-d}S_0^C=0$$

$$c_2^{A-d}M_2^C+d_2^{A-d}M_2^D+i_2^{A-d}S_2^A+j_2^{A-d}S_2^B+k_2^{A-d}S_2^C+l_2^{A-d}S_3^A+m_2^{A-d}S_3^B+n_2^{A-d}S_3^C+z_2^{A-d}+u_2^{A-d}C_2^C+v_2^{A-d}C_2^D+P_2^{A-d}S_1^A+Q_2^{A-d}S_1^B+R_2^{A-d}S_1^C=0$$

$$c_3^{A-d}M_3^C+d_3^{A-d}M_3^D+i_3^{A-d}S_3^A+j_3^{A-d}S_3^B+k_3^{A-d}S_3^C+z_3^{A-d}+u_3^{A-d}C_3^C+v_3^{A-d}C_3^D+P_3^{A-d}S_2^A+Q_3^{A-d}S_2^B+R_3^{A-d}S_2^C=0$$

Equations B-a

$$a_0^{B-a}M_0^A+b_0^{B-a}M_0^B+c_0^{B-a}M_0^C+d_0^{B-a}M_0^D+e_0^{B-a}M_1^A+f_0^{B-a}M_1^B+g_0^{B-a}M_1^C+h_0^{B-a}M_1^D+i_0^{B-a}S_0^A+j_0^{B-a}S_0^B+l_0^{B-a}S_1^A+o_0^{B-a}V_0^B+p_0^{B-a}V_0^C+q_0^{B-a}V_1^B+r_0^{B-a}V_1^C+z_0^{B-a}=0$$

$$a_1^{B-a}M_1^A+b_1^{B-a}M_1^B+c_1^{B-a}M_1^C+d_1^{B-a}M_1^D+e_1^{B-a}M_2^A+f_1^{B-a}M_2^B+g_1^{B-a}M_2^C+h_1^{B-a}M_2^D+i_1^{B-a}S_1^A+j_1^{B-a}S_1^B+l_1^{B-a}S_2^A+o_1^{B-a}V_1^B+p_1^{B-a}V_1^C+q_1^{B-a}V_2^B+r_1^{B-a}V_2^C+z_1^{B-a}+s_1^{B-a}C_1^A+t_1^{B-a}C_1^B=0$$

$$a_2^{B-a}M_2^A+b_2^{B-a}M_2^B+c_2^{B-a}M_2^C+d_2^{B-a}M_2^D+e_2^{B-a}M_3^A+f_2^{B-a}M_3^B+g_2^{B-a}M_3^C+h_2^{B-a}M_3^D+i_2^{B-a}S_2^A+j_2^{B-a}S_2^B+l_2^{B-a}S_3^A+o_2^{B-a}V_2^B+p_2^{B-a}V_2^C+q_2^{B-a}V_3^B+r_2^{B-a}V_3^C+z_2^{B-a}+s_2^{B-a}C_2^A+t_2^{B-a}C_2^B=0$$

$$a_3^{B-a}M_3^A+b_3^{B-a}M_3^B+c_3^{B-a}M_3^C+d_3^{B-a}M_3^D+i_3^{B-a}S_3^A+j_3^{B-a}S_3^B+o_3^{B-a}V_3^B+p_3^{B-a}V_3^C+z_3^{B-a}+s_3^{B-a}C_3^A+t_3^{B-a}C_3^B=0$$

Equations B-b

$$a_0^{B-b}M_0^A+b_0^{B-b}M_0^B+c_0^{B-b}M_0^C+d_0^{B-b}M_0^D+e_0^{B-b}M_1^A+f_0^{B-b}M_1^B+g_0^{B-b}M_1^C+h_0^{B-b}M_1^D+i_0^{B-b}S_0^A+j_0^{B-b}S_0^B+k_0^{B-b}S_0^C+l_0^{B-b}S_1^A+m_0^{B-b}S_1^B+o_0^{B-b}V_0^B+p_0^{B-b}V_0^C+q_0^{B-b}V_1^B+r_0^{B-b}V_1^C+z_0^{B-b}=0$$

$$a_1^{B-b}M_1^A+b_1^{B-b}M_1^B+c_1^{B-b}M_1^C+d_1^{B-b}M_1^D+e_1^{B-b}M_2^A+f_1^{B-b}M_2^B+g_1^{B-b}M_2^C+h_1^{B-b}M_2^D+i_1^{B-b}S_1^A+j_1^{B-b}S_1^B+k_1^{B-b}S_1^C+l_1^{B-b}S_2^A+m_1^{B-b}S_2^B+o_1^{B-b}V_1^B+p_1^{B-b}V_1^C+q_1^{B-b}V_2^B+r_1^{B-b}V_2^C+z_1^{B-b}+t_1^{B-b}C_1^B+u_1^{B-b}C_1^C=0$$

$$a_2^{B-b}M_2^A+b_2^{B-b}M_2^B+c_2^{B-b}M_2^C+d_2^{B-b}M_2^D+e_2^{B-b}M_3^A+f_2^{B-b}M_3^B+g_2^{B-b}M_3^C+h_2^{B-b}M_3^D+i_2^{B-b}S_2^A+j_2^{B-b}S_2^B+k_2^{B-b}S_2^C+l_2^{B-b}S_3^A+m_2^{B-b}S_3^B+o_2^{B-b}V_2^B+p_2^{B-b}V_2^C+q_2^{B-b}V_3^B+r_2^{B-b}V_3^C+z_2^{B-b}+t_2^{B-b}C_2^B+u_2^{B-b}C_2^C=0$$

$$a_3^{B-b}M_3^A+b_3^{B-b}M_3^B+c_3^{B-b}M_3^C+d_3^{B-b}M_3^D+i_3^{B-b}S_3^A+j_3^{B-b}S_3^B+k_3^{B-b}S_3^C+o_3^{B-b}V_3^B+p_3^{B-b}V_3^C+z_3^{B-b}+t_3^{B-b}C_3^B+u_3^{B-b}C_3^C=0$$

Equations B-d

$$a_0^{B-d}M_0^A+b_0^{B-d}M_0^B+c_0^{B-d}M_0^C+d_0^{B-d}M_0^D+e_0^{B-d}M_1^A+f_0^{B-d}M_1^B+g_0^{B-d}M_1^C+h_0^{B-d}M_1^D+i_0^{B-d}S_0^A+j_0^{B-d}S_0^B+k_0^{B-d}S_0^C+l_0^{B-d}S_1^A+m_0^{B-d}S_1^B+n_0^{B-d}S_1^C+o_0^{B-d}V_0^B+p_0^{B-d}V_0^C+q_0^{B-d}V_1^B+r_0^{B-d}V_1^C+z_0^{B-d}=0$$

$$a_1^{B-d}M_1^A+b_1^{B-d}M_1^B+c_1^{B-d}M_1^C+d_1^{B-d}M_1^D+e_1^{B-d}M_2^A+f_1^{B-d}M_2^B+g_1^{B-d}M_2^C+h_1^{B-d}M_2^D+i_1^{B-d}S_1^A+j_1^{B-d}S_1^B+k_1^{B-d}S_1^C+l_1^{B-d}S_2^A+m_1^{B-d}S_2^B+n_1^{B-d}S_2^C+o_1^{B-d}V_1^B+p_1^{B-d}V_1^C+q_1^{B-d}V_2^B+r_1^{B-d}V_2^C+z_1^{B-d}+u_1^{B-d}C_1^C+v_1^{B-d}C_1^D=0$$

$$a_2^{B-d}M_2^A+b_2^{B-d}M_2^B+c_2^{B-d}M_2^C+d_2^{B-d}M_2^D+e_2^{B-d}M_3^A+f_2^{B-d}M_3^B+g_2^{B-d}M_3^C+h_2^{B-d}M_3^D+i_2^{B-d}S_2^A+j_2^{B-d}S_2^B+k_2^{B-d}S_2^C+l_2^{B-d}S_3^A+m_2^{B-d}S_3^B+n_2^{B-d}S_3^C+o_2^{B-d}V_2^B+p_2^{B-d}V_2^C+q_2^{B-d}V_3^B+r_2^{B-d}V_3^C+z_2^{B-d}+u_2^{B-d}C_2^C+v_2^{B-d}C_2^D=0$$

$$a_3^{B-d}M_3^A+b_3^{B-d}M_3^B+c_3^{B-d}M_3^C+d_3^{B-d}M_3^D+i_3^{B-d}S_3^A+j_3^{B-d}S_3^B+k_3^{B-d}S_3^C+o_3^{B-d}V_3^B+p_3^{B-d}V_3^C+z_3^{B-d}+u_3^{B-d}C_3^C+v_3^{B-d}C_3^D=0$$

Equations C-a

$$a_0^{C-a}M_0^A+b_0^{C-a}M_0^B+c_0^{C-a}M_0^C+d_0^{C-a}M_0^D+e_0^{C-a}M_1^A+f_0^{C-a}M_1^B+g_0^{C-a}M_1^C+h_0^{C-a}M_1^D+i_0^{C-a}S_0^A+j_0^{C-a}S_0^B+l_0^{C-a}S_1^A+o_0^{C-a}V_0^B+p_0^{C-a}V_0^C+q_0^{C-a}V_1^B+r_0^{C-a}V_1^C+z_0^{C-a}=0$$

$$a_1^{C-a}M_1^A+b_1^{C-a}M_1^B+c_1^{C-a}M_1^C+d_1^{C-a}M_1^D+e_1^{C-a}M_2^A+f_1^{C-a}M_2^B+g_1^{C-a}M_2^C+h_1^{C-a}M_2^D+i_1^{C-a}S_1^A+j_1^{C-a}S_1^B+l_1^{C-a}S_2^A+o_1^{C-a}V_1^B+p_1^{C-a}V_1^C+q_1^{C-a}V_2^B+r_1^{C-a}V_2^C+z_1^{C-a}+w_1^{C-a}\Delta_0^a+t_1^{C-a}C_1^B=0$$

$$a_2^{C-a}M_2^A+b_2^{C-a}M_2^B+c_2^{C-a}M_2^C+d_2^{C-a}M_2^D+e_2^{C-a}M_3^A+f_2^{C-a}M_3^B+g_2^{C-a}M_3^C+h_2^{C-a}M_3^D+i_2^{C-a}S_2^A+j_2^{C-a}S_2^B+l_2^{C-a}S_3^A+o_2^{C-a}V_2^B+p_2^{C-a}V_2^C+q_2^{C-a}V_3^B+r_2^{C-a}V_3^C+z_2^{C-a}+w_2^{C-a}\Delta_1^a+t_2^{C-a}C_2^B=0$$

$$a_3^{C-a}M_3^A+b_3^{C-a}M_3^B+c_3^{C-a}M_3^C+d_3^{C-a}M_3^D+i_3^{C-a}S_3^A+j_3^{C-a}S_3^B+o_3^{C-a}V_3^B+p_3^{C-a}V_3^C+z_3^{C-a}+w_3^{C-a}\Delta_2^a+t_3^{C-a}C_3^B=0$$

Equations C-b

$$a_0^{C-b}M_0^A+b_0^{C-b}M_0^B+c_0^{C-b}M_0^C+d_0^{C-b}M_0^D+e_0^{C-b}M_1^A+f_0^{C-b}M_1^B+g_0^{C-b}M_1^C+h_0^{C-b}M_1^D+i_0^{C-b}S_0^A+j_0^{C-b}S_0^B+k_0^{C-b}S_0^C+l_0^{C-b}S_1^A+m_0^{C-b}S_1^B+o_0^{C-b}V_0^B+p_0^{C-b}V_0^C+q_0^{C-b}V_1^B+r_0^{C-b}V_1^C+z_0^{C-b}=0$$

$$a_1^{C-b}M_1^A+b_1^{C-b}M_1^B+c_1^{C-b}M_1^C+d_1^{C-b}M_1^D+e_1^{C-b}M_2^A+f_1^{C-b}M_2^B+g_1^{C-b}M_2^C+h_1^{C-b}M_2^D+i_1^{C-b}S_1^A+j_1^{C-b}S_1^B+k_1^{C-b}S_1^C+l_1^{C-b}S_2^A+m_1^{C-b}S_2^B+o_1^{C-b}V_1^B+p_1^{C-b}V_1^C+q_1^{C-b}V_2^B+r_1^{C-b}V_2^C+z_1^{C-b}+x_1^{C-b}\Delta_0^b+u_1^{C-b}C_1^C=0$$

$$a_2^{C-b}M_2^A+b_2^{C-b}M_2^B+c_2^{C-b}M_2^C+d_2^{C-b}M_2^D+e_2^{C-b}M_3^A+f_2^{C-b}M_3^B+g_2^{C-b}M_3^C+h_2^{C-b}M_3^D+i_2^{C-b}S_2^A+j_2^{C-b}S_2^B+k_2^{C-b}S_2^C+l_2^{C-b}S_3^A+m_2^{C-b}S_3^B+o_2^{C-b}V_2^B+p_2^{C-b}V_2^C+q_2^{C-b}V_3^B+r_2^{C-b}V_3^C+z_2^{C-b}+x_2^{C-b}\Delta_1^b+u_2^{C-b}C_2^C=0$$

$$a_3^{C-b}M_3^A+b_3^{C-b}M_3^B+c_3^{C-b}M_3^C+d_3^{C-b}M_3^D+i_3^{C-b}S_3^A+j_3^{C-b}S_3^B+k_3^{C-b}S_3^C+o_3^{C-b}V_3^B+p_3^{C-b}V_3^C+z_3^{C-b}+x_3^{C-b}\Delta_2^b+u_3^{C-b}C_3^C=0$$

Equations C-d

$$a_0^{C-d}M_0^A+b_0^{C-d}M_0^B+c_0^{C-d}M_0^C+d_0^{C-d}M_0^D+e_0^{C-d}M_1^A+f_0^{C-d}M_1^B+g_0^{C-d}M_1^C+h_0^{C-d}M_1^D+i_0^{C-d}S_0^A+j_0^{C-d}S_0^B+k_0^{C-d}S_0^C+l_0^{C-d}S_1^A+m_0^{C-d}S_1^B+n_0^{C-d}S_1^C+o_0^{C-d}V_0^B+p_0^{C-d}V_0^C+q_0^{C-d}V_1^B+r_0^{C-d}V_1^C+z_0^{C-d}=0$$

$$a_1^{C-d}M_1^A+b_1^{C-d}M_1^B+c_1^{C-d}M_1^C+d_1^{C-d}M_1^D+e_1^{C-d}M_2^A+f_1^{C-d}M_2^B+g_1^{C-d}M_2^C+h_1^{C-d}M_2^D+i_1^{C-d}S_1^A+j_1^{C-d}S_1^B+k_1^{C-d}S_1^C+l_1^{C-d}S_2^A+m_1^{C-d}S_2^B+n_1^{C-d}S_2^C+o_1^{C-d}V_1^B+p_1^{C-d}V_1^C+q_1^{C-d}V_2^B+r_1^{C-d}V_2^C+z_1^{C-d}+y_1^{C-d}\Delta_0^d+v_1^{C-d}C_1^D=0$$

$$a_2^{C-d}M_2^A+b_2^{C-d}M_2^B+c_2^{C-d}M_2^C+d_2^{C-d}M_2^D+e_2^{C-d}M_3^A+f_2^{C-d}M_3^B+g_2^{C-d}M_3^C+h_2^{C-d}M_3^D+i_2^{C-d}S_2^A+j_2^{C-d}S_2^B+k_2^{C-d}S_2^C+l_2^{C-d}S_3^A+m_2^{C-d}S_3^B+n_2^{C-d}S_3^C+o_2^{C-d}V_2^B+p_2^{C-d}V_2^C+q_2^{C-d}V_3^B+r_2^{C-d}V_3^C+z_2^{C-d}+y_2^{C-d}\Delta_1^d+v_2^{C-d}C_2^D=0$$

$$a_3^{C-d}M_3^A+b_3^{C-d}M_3^B+c_3^{C-d}M_3^C+d_3^{C-d}M_3^D+i_3^{C-d}S_3^A+j_3^{C-d}S_3^B+k_3^{C-d}S_3^C+o_3^{C-d}V_3^B+p_3^{C-d}V_3^C+z_3^{C-d}+y_3^{C-d}\Delta_2^d+v_3^{C-d}C_3^D=0$$

PLATE. 3

VALUES OF THE COEFFICIENTS IN THE SYMBOLIC EQUATIONS

	a_n	b_n	C_n	d_n
C-A	$C_n \div I_n^A$	0	0	0
C-B	0	$C_n \div I_n^B$	0	0
C-C	0	0	$C_n \div I_n^C$	0
C-D	0	0	0	$C_n \div I_n^D$
δ-A	$C_n^2 \div 2I_n^A$	0	0	0
δ-B	0	$C_n^2 \div 2I_n^B$	0	0
δ-C	0	0	$C_n^2 \div 2I_n^C$	0
δ-D	0	0	0	$C_n^2 \div 2I_n^D$
H-a	0	0	0	0
H-b	0	0	0	0
H-d	0	0	0	0
λ-a	0	0	0	0
λ-b	0	0	0	0
λ-d	0	0	0	0
V-A	$1 \div (b+b'+b'')$	$1 \div (b+b'+b'')$	$1 \div (b+b'+b'')$	$1 \div (b+b'+b'')$
V-D	$-1 \div (b+b'+b'')$	$-1 \div (b+b'+b'')$	$-1 \div (b+b'+b'')$	$-1 \div (b+b'+b'')$
α-A	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)a_n^{V-A} \div A_n^A$	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)b_n^{V-A} \div A_n^A$	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)C_n^{V-A} \div A_n^A$	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)d_n^{V-A} \div A_n^A$
α-B	0	0	0	0
α-C	0	0	0	0
α-D	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)a_n^{V-D} \div A_n^D$	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)b_n^{V-D} \div A_n^D$	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)C_n^{V-D} \div A_n^D$	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)d_n^{V-D} \div A_n^D$
Δ-a	$-a_n^{\alpha-A}$	$-b_n^{\alpha-A}$	$-C_n^{\alpha-A}$	$-d_n^{\alpha-A}$
Δ-b	0	0	0	0
Δ-d	$a_n^{\alpha-D}$	$b_n^{\alpha-D}$	$C_n^{\alpha-D}$	$d_n^{\alpha-D}$
Q-a	$-1 \div I_n^a$	0	0	0
Q-b	$(-1+b.a_n^{V-a}) \div I_n^b$	$(-1+b.b_n^{V-a}) \div I_n^b$	$b.C_n^{V-a} \div I_n^b$	$b.d_n^{V-a} \div I_n^b$
Q-d	$[-1+(b+2b')a_n^{V-a}] \div I_n^d$	$[-1+(b+2b')b_n^{V-a}] \div I_n^d$	$[-1+(b+2b')C_n^{V-a}] \div I_n^d$	$(b+2b')d_n^{V-a} \div I_n^d$
R-a	$a_n^{V-a} \div I_n^a$	$b_n^{V-a} \div I_n^a$	$C_n^{V-a} \div I_n^a$	$d_n^{V-a} \div I_n^a$
R-b	$a_n^{V-a} \div I_n^b$	$b_n^{V-a} \div I_n^b$	$C_n^{V} \div I_n^b$	$d_n^{V-a} \div I_n^b$
R-d	$a_n^{V-a} \div I_n^d$	$b_n^{V-a} \div I_n^d$	$C_n^{V-a} \div I_n^d$	$d_n^{V-a} \div I_n^d$
A-a	$a_n^{\delta-A}+\frac{1}{2}d_n.a_n^{C-A}$	$-b_n^{\delta-B}-\frac{1}{2}d_n.b_n^{C-B}$	0	0
A-b	0	$b_n^{\delta-B}+\frac{1}{2}d_n.b_n^{C-B}$	$-C_n^{\delta-C}-\frac{1}{2}d_n.C_n^{C-C}$	0
A-d	0	0	$C_n^{\delta-C}+\frac{1}{2}d_n.C_n^{C-C}$	$-d_n^{\delta-D}-\frac{1}{2}d_n.d_n^{C-D}$
B-a	$(a_n^{Q-a}+b.a_n^{R-a})2b-\frac{1}{2}b^2.a_n^{R-a}-a_n^{C-A}$	$\frac{1}{2}b^2.b_n^{R-a}+b_n^{C-B}$	$\frac{1}{2}b^2.C_n^{R-a}$	$\frac{1}{2}b^2.d_n^{R-a}$
B-b	$(a_n^{Q-b}+b'.a_n^{R-b})b'-\frac{1}{2}b'^2.a_n^{R-b}$	$(b_n^{Q-b}+b'.b_n^{R-b})b'-\frac{1}{2}b'^2.b_n^{R-b}-b_n^{C-B}$	$(C_n^{Q-b}+b'.C_n^{R-b})b'-\frac{1}{2}b'^2.C_n^{R-b}+C_n^{C-C}$	$(d_n^{Q-b}+b'.d_n^{R-b})b'-\frac{1}{2}b'^2.d_n^{R-b}$
B-d	$(a_n^{Q-d}+b''.a_n^{R-d})b''-\frac{1}{2}b''^2.a_n^{R-d}$	$(b_n^{Q-d}+b''.b_n^{R-d})b''-\frac{1}{2}b''^2.b_n^{R-d}$	$(C_n^{Q-d}+b''.C_n^{R-d})b''-\frac{1}{2}b''^2.C_n^{R-d}-C_n^{C-C}$	$(d_n^{Q-d}+b''.d_n^{R-d})b''-\frac{1}{2}b''^2.d_n^{R-d}+d_n^{C-D}$
C-a	$\frac{1}{2}(a_n^{Q-a}+b.a_n^{R-a})b^2-\frac{1}{6}b^3.a_n^{R-a}-a_n^{\Delta-a}$	$\frac{1}{3}b^3.b_n^{R-a}-b_n^{\Delta-a}+b.b_n^{C-B}$	$\frac{1}{3}b^3.C_n^{R-a}-C_n^{\Delta-a}$	$\frac{1}{3}b^3.d_n^{R-a}-d_n^{\Delta-a}$
C-b	$\frac{1}{2}(a_n^{Q-b}+b'.a_n^{R-b})b'^2-\frac{1}{6}b'^3.a_n^{R-b}$	$\frac{1}{2}(b_n^{Q-b}+b'.b_n^{R-b})b'^2-\frac{1}{6}b'^3.b_n^{R-b}$	$\frac{1}{2}(C_n^{Q-b}+b'.C_n^{R-b})b'^2-\frac{1}{6}b'^3.C_n^{R-b}+b'.C_n^{C-C}$	$\frac{1}{2}(d_n^{Q-b}+b'.d_n^{R-b})b'^2-\frac{1}{6}b'^3.d_n^{R-b}$
C-d	$\frac{1}{2}(a_n^{Q-d}+b''.a_n^{R-d})b''^2-\frac{1}{6}b''^3.a_n^{R-d}-a_n^{\Delta-d}$	$\frac{1}{2}(b_n^{Q-d}+b''.b_n^{R-d})b''^2-\frac{1}{6}b''^3.b_n^{R-d}-b_n^{\Delta-d}$	$\frac{1}{2}(C_n^{Q-d}+b''.C_n^{R-d})b''^2-\frac{1}{6}b''^3.C_n^{R-d}-C_n^{\Delta-d}$	$\frac{1}{2}(d_n^{Q-d}+b''.d_n^{R-d})b''^2-\frac{1}{6}b''^3.d_n^{R-d}+b''.d_n^{C-D}-d_n^{\Delta-d}$

PLATE 4.

VALUES OF THE COEFFICIENTS IN THE SYMBOLIC EQUATIONS.

	e_n	f_n	g_n	h_n	i_n
C-A	0	0	0	0	$-c_n^2 \div 2I_n^A$
C-B	0	0	0	0	0
C-C	0	0	0	0	0
C-D	0	0	0	0	$c_n^2 \div 2I_n^D$
δ-A	0	0	0	0	$-c_n^3 \div 6I_n^A$
δ-B	0	0	0	0	0
δ-C	0	0	0	0	0
δ-D	0	0	0	0	$c_n^3 \div 6I_n^D$
H-a	0	0	0	0	-1
H-b	0	0	0	0	-1
H-d	0	0	0	0	-1
λ-a	0	0	0	0	$-b \div A_n^a$
λ-b	0	0	0	0	$-b' \div A_n^b$
λ-d	0	0	0	0	$-b'' \div A_n^d$
V-A	0	0	0	0	0
V-D	0	0	0	0	0
α-A	0	0	0	0	0
α-B	0	0	0	0	0
α-C	0	0	0	0	0
α-D	0	0	0	0	0
Δ-a	0	0	0	0	0
Δ-b	0	0	0	0	0
Δ-d	0	0	0	0	0
Q-a	$1 \div I_n^a$	0	0	0	$(c_n + \frac{1}{2}d_n) \div I_n^a$
Q-b	$(1 - b'.a_{n+1}^{V-A}) \div I_n^b$	$(1 - b'.b_{n+1}^{V-A}) \div I_n^b$	$-b'.c_{n+1}^{V-A} \div I_n^b$	$-b'.d_{n+1}^{V-A} \div I_n^b$	$(c_n + \frac{1}{2}d_n) \div I_n^b$
Q-d	$[1-(b+2b')a_{n+1}^{V-A}] \div I_n^d$	$[1-(b+2b')b_{n+1}^{V-A}] \div I_n^d$	$[1-(b+2b')c_{n+1}^{V-A}] \div I_n^d$	$-(b+2b')d_{n+1}^{V-A} \div I_n^d$	$(c_n + \frac{1}{2}d_n) \div I_n^d$
R-a	$-a_{n+1}^{V-A} \div I_n^a$	$-b_{n+1}^{V-A} \div I_n^a$	$-c_{n+1}^{V-A} \div I_n^a$	$-d_{n+1}^{V-A} \div I_n^a$	0
R-b	$-a_{n+1}^{V-A} \div I_n^b$	$-b_{n+1}^{V-A} \div I_n^b$	$-c_{n+1}^{V-A} \div I_n^b$	$-d_{n+1}^{V-A} \div I_n^b$	0
R-d	$-a_{n+1}^{V-A} \div I_n^d$	$-b_{n+1}^{V-A} \div I_n^d$	$-c_{n+1}^{V-A} \div I_n^d$	$-d_{n+1}^{V-A} \div I_n^d$	0
A-a	0	0	0	0	$i_n^{\delta-A} + \frac{1}{2}d_n.i_n^{C-A} - i_n^{\lambda-a} + i_{n-1}^{\lambda-a}$
A-b	0	0	0	0	$-i_n^{\lambda-b} + i_{n-1}^{\lambda-b}$
A-d	0	0	0	0	$-i_n^{\delta-D} - \frac{1}{2}d_n.i_n^{C-D} - i_n^{\lambda-d} + i_{n-1}^{\lambda-d}$
B-a	$(e_n^{Q-a} + b.e_n^{R-a})b - \frac{1}{2}b^2.e_n^{R-a}$	$\frac{1}{2}b^2.f_n^{R-a}$	$\frac{1}{2}b^2.g_n^{R-a}$	$\frac{1}{2}b^2.h_n^{R-a}$	$b.i_n^{Q-a} - i_n^{C-A}$
B-b	$(e_n^{Q-b} + b'.e_n^{R-b})b' - \frac{1}{2}b'^2.e_n^{R-b}$	$(f_n^{Q-b} + b'.f_n^{R-b})b' - \frac{1}{2}b'^2.f_n^{R-b}$	$(g_n^{Q-b} + b'.g_n^{R-b})b' - \frac{1}{2}b'^2.g_n^{R-b}$	$(h_n^{Q-b} + b'.h_n^{R-b})b' - \frac{1}{2}b'^2.h_n^{R-b}$	$b'.i_n^{Q-b}$
B-d	$(e_n^{Q-d} + b''.e_n^{R-d})b'' - \frac{1}{2}b''^2.e_n^{R-d}$	$(f_n^{Q-d} + b''.f_n^{R-d})b'' - \frac{1}{2}b''^2.f_n^{R-d}$	$(g_n^{Q-d} + b''.g_n^{R-d})b'' - \frac{1}{2}b''^2.g_n^{R-d}$	$(h_n^{Q-d} + b''.h_n^{R-d})b'' - \frac{1}{2}b''^2.h_n^{R-d}$	$b''.i_n^{Q-d} + i_n^{C-D}$
C-a	$\frac{1}{2}(e_n^{Q-a} + b.e_n^{R-a})b^2 - \frac{1}{6}b^3.e_n^{R-a}$	$\frac{1}{3}b^3.f_n^{R-a}$	$\frac{1}{3}b^3.g_n^{R-a}$	$\frac{1}{3}b^3.h_n^{R-a}$	$\frac{1}{2}b^2.i_n^{Q-a}$
C-b	$\frac{1}{2}(e_n^{Q-b} + b'.e_n^{R-b})b'^2 - \frac{1}{6}b'^3.e_n^{R-b}$	$\frac{1}{2}(f_n^{Q-b} + b'.f_n^{R-b})b'^2 - \frac{1}{6}b'^3.f_n^{R-b}$	$\frac{1}{2}(g_n^{Q-b} + b'.g_n^{R-b})b'^2 - \frac{1}{6}b'^3.g_n^{R-b}$	$\frac{1}{2}(h_n^{Q-b} + b'.h_n^{R-b})b'^2 - \frac{1}{6}b'^3.h_n^{R-b}$	$\frac{1}{2}b'^2.i_n^{Q-b}$
C-d	$\frac{1}{2}(e_n^{Q-d} + b''.e_n^{R-d})b''^2 - \frac{1}{6}b''^3.e_n^{R-d}$	$\frac{1}{2}(f_n^{Q-d} + b''.f_n^{R-d})b''^2 - \frac{1}{6}b''^3.f_n^{R-d}$	$\frac{1}{2}(g_n^{Q-d} + b''.g_n^{R-d})b''^2 - \frac{1}{6}b''^3.g_n^{R-d}$	$\frac{1}{2}(h_n^{Q-d} + b''.h_n^{R-d})b''^2 - \frac{1}{6}b''^3.h_n^{R-d}$	$\frac{1}{2}b''^2.i_n^{Q-d} + b''.i_n^{C-D}$

PLATE 5.

VALUES OF THE COEFFICIENTS IN THE SYMBOLIC EQUATIONS

	j_n	k_n	l_n	m_n	n_n	o_n
C-A	0	0	0	0	0	0
C-B	$-c_n^2\div 2I_n^B$	0	0	0	0	0
C-C	0	$-c_n^2\div 2I_n^C$	0	0	0	0
C-D	$c_n^2\div 2I_n^D$	$c_n^2\div 2I_n^D$	0	0	0	0
δ-A	0	0	0	0	0	0
δ-B	$-c_n^3\div 6I_n^B$	0	0	0	0	0
δ-C	0	$-c_n^3\div 6I_n^C$	0	0	0	0
δ-D	$c_n^3\div 6I_n^D$	$c_n^3\div 6I_n^D$	0	0	0	0
H-a	0	0	1	0	0	0
H-b	-1	0	1	1	0	0
H-d	-1	-1	1	1	1	0
λ-a	0	0	$b\div A_n^a$	0	0	0
λ-b	$-b'\div A_n^b$	0	$b'+A_n^b$	$b'\div A_n^b$	0	0
λ-d	$-b''\div A_n^d$	$-b''\div A_n^d$	$b''\div A_n^d$	$b''\div A_n^d$	$b''\div A_n^d$	0
V-A	0	0	0	0	0	$-(b'+b'')\div(b+b'+b'')$
V-D	0	0	0	0	0	$-b\div(b+b'+b'')$
α-A	0	0	0	0	0	$(c_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)o_n^{V-A}\div A_n^A$
α-B	0	0	0	0	0	$(c_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)\div A_n^B$
α-C	0	0	0	0	0	0
α-D	0	0	0	0	0	$(c_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)o_n^{V-D}\div A_n^D$
Δ-a	0	0	0	0	0	$o_n^{\alpha-B}-o_n^{\alpha-A}$
Δ-b	0	0	0	0	0	$-o_n^{\alpha-B}$
Δ-d	0	0	0	0	0	$o_n^{\alpha-D}$
Q-a	0	0	$d_n\div 2I_n^a$	0	0	0
Q-b	$(c_n+\frac{1}{2}d_n)\div I_n^b$	0	$d_n\div 2I_n^b$	$d_n\div 2I_n^b$	0	$b.o_n^{V-A}\div I_n^b$
Q-d	$(c_n+\frac{1}{2}d_n)\div I_n^d$	$(c_n+\frac{1}{2}d_n)\div I_n^d$	$d_n\div 2I_n^d$	$d_n\div 2I_n^d$	$d_n\div 2I_n^d$	$[b'+(b+2b')o_n^{V-A}]\div I_n^d$
R-a	0	0	0	0	0	$o_n^{V-A}\div I_n^a$
R-b	0	0	0	0	0	$(1+o_n^{V-A})\div I_n^b$
R-d	0	0	0	0	0	$(1+o_n^{V-A})\div I_n^d$
A-a	$-j_n^{\delta-B}-\frac{1}{2}d_n.j_n^{C-B}$	0	$-l_n^{\lambda-a}$	0	0	0
A-b	$j_n^{\delta-B}+\frac{1}{2}d_n.j_n^{C-B}-j_n^{\lambda-b}+m_{n-1}^{\lambda-b}$	$-k_n^{\delta-C}-\frac{1}{2}d_n.k_n^{C-C}$	$-l_n^{\lambda-b}$	$-m_n^{\lambda-b}$	0	0
A-d	$-j_n^{\delta-D}-\frac{1}{2}d_n.j_n^{C-D}-j_n^{\lambda-d}+m_{n-1}^{\lambda-d}$	$k_n^{\delta-C}-k_n^{\delta-D}+\frac{1}{2}d_n(k_n^{C-C}-k_n^{C-D})-k_n^{\lambda-d}+n_{n-1}^{\lambda-d}$	$-l_n^{\lambda-d}$	$-m_n^{\lambda-d}$	$-n_n^{\lambda-d}$	0
B-a	j_n^{C-B}	0	$b.l_n^{Q-a}$	0	0	$\frac{1}{2}b^2.o_n^{R-a}$
B-b	$b'.j_n^{Q-b}-j_n^{C-B}$	k_n^{C-C}	$b'.l_n^{Q-b}$	$b'.m_n^{Q-b}$	0	$(o_n^{Q-b}+b'.o_n^{R-b})b'-\frac{1}{2}b'^2.o_n^{R-b}$
B-d	$b''.j_n^{Q-d}+j_n^{C-D}$	$b''.k_n^{Q-d}+k_n^{C-D}-k_n^{C-C}$	$b''.l_n^{Q-d}$	$b''.m_n^{Q-d}$	$b''.n_n^{Q-d}$	$(o_n^{Q-d}+b''.o_n^{R-d})b''-\frac{1}{2}b''^2.o_n^{R-d}$
C-a	$b.j_n^{C-B}$	0	$\frac{1}{2}b^2.l_n^{Q-a}$	0	0	$\frac{1}{3}b^3.o_n^{R-a}-o_n^{\Delta-a}$
C-b	$\frac{1}{2}b'^2.j_n^{Q-b}$	$b'.k_n^{C-C}$	$\frac{1}{2}b'^2.l_n^{Q-b}$	$\frac{1}{2}b'^2.m_n^{Q-b}$	0	$\frac{1}{2}(o_n^{Q-b}+b'.o_n^{R-b})b'^2-\frac{1}{6}b'^3.o_n^{R-b}-o_n^{\Delta-b}$
C-d	$\frac{1}{2}b''^2.j_n^{Q-d}+b''.j_n^{C-D}$	$\frac{1}{2}b''^2.k_n^{Q-d}+b''.k_n^{C-D}$	$\frac{1}{2}b''^2.l_n^{Q-d}$	$\frac{1}{2}b''^2.m_n^{Q-d}$	$\frac{1}{2}b''^2.n_n^{Q-d}$	$\frac{1}{2}(o_n^{Q-d}+b''.o_n^{R-d})b''^2-\frac{1}{6}b''^3.o_n^{R-d}-o_n^{\Delta-d}$

PLATE 6.

VALUES OF THE COEFFICIENTS IN THE SYMBOLIC EQUATIONS.

	p_n	q_n	r_n	s_n	t_n
C-A	0	0	0	1	0
C-B	0	0	0	0	1
C-C	0	0	0	0	0
C-D	0	0	0	0	0
δ-A	0	0	0	c_n	0
δ-B	0	0	0	0	c_n
δ-C	0	0	0	0	0
δ-D	0	0	0	0	0
H-a	0	0	0	0	0
H-b	0	0	0	0	0
H-d	0	0	0	0	0
λ-a	0	0	0	0	0
λ-b	0	0	0	0	0
λ-d	0	0	0	0	0
V-A	$-b''\div(b+b'+b'')$	0	0	0	0
V-D	$-(b+b')\div(b+b'+b'')$	0	0	0	0
α-A	$(c_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)p_n^{V-A}\div A_n^{A}$	0	0	0	0
α-B	0	0	0	0	0
α-C	$(c_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)\div A_n^{C}$	0	0	0	0
α-D	$(c_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)p_n^{V-D}\div A_n^{D}$	0	0	0	0
Δ-a	$-p_n^{\alpha-A}$	0	0	0	0
Δ-b	$p_n^{\alpha-C}$	0	0	0	0
Δ-d	$p_n^{\alpha-D}-p_n^{\alpha-C}$	0	0	0	0
Q-a	0	0	0	0	0
Q-b	$b.p_n^{V-A}\div I_n^{b}$	$-b.O_{n+1}^{V-A}\div I_n^{b}$	$-b.p_{n+1}^{V-A}\div I_n^{b}$	0	0
Q-d	$(b+2b')p_n^{V-A}\div I_n^{d}$	$[-b'-(b+2b')O_{n+1}^{V-A}]\div I_n^{d}$	$-(b+2b')p_{n+1}^{V-A}\div I_n^{d}$	0	0
R-a	$p_n^{V-A}\div I_n^{a}$	$-O_{n+1}^{V-A}\div I_n^{a}$	$-p_{n+1}^{V-A}\div I_n^{a}$	0	0
R-b	$p_n^{V-A}\div I_n^{b}$	$(-1-O_{n+1}^{V-A})\div I_n^{b}$	$-p_{n+1}^{V-A}\div I_n^{b}$	0	0
R-d	$(1+p_n^{V-A})\div I_n^{d}$	$(-1-O_{n+1}^{V-A})\div I_n^{d}$	$(-1-p_{n+1}^{V-A})\div I_n^{d}$	0	0
A-a	0	0	0	$s_n^{\delta-A}+\frac{1}{2}d_n.s_n^{C-A}+\frac{1}{2}d_{n-1}$	$-t_n^{\delta-B}-\frac{1}{2}d_n.t_n^{C-B}-\frac{1}{2}d_{n-1}$
A-b	0	0	0	0	$t_n^{\delta-B}+\frac{1}{2}d_n.t_n^{C-B}+\frac{1}{2}d_{n-1}$
A-d	0	0	0	0	0
B-a	$\frac{1}{2}b^2.p_n^{R-a}$	$\frac{1}{2}b^2.q_n^{R-a}$	$\frac{1}{2}b^2.r_n^{R-a}$	$-s_n^{C-A}$	t_n^{C-B}
B-b	$(p_n^{Q-b}+b'.p_n^{R-b})b'-\frac{1}{2}b'^2.p_n^{R-b}$	$(q_n^{Q-b}+b'.q_n^{R-b})b'-\frac{1}{2}b'^2.q_n^{R-b}$	$(r_n^{Q-b}+b'.r_n^{R-b})b'-\frac{1}{2}b'^2.r_n^{R-b}$	0	$-t_n^{C-B}$
B-d	$(p_n^{Q-d}+b''.p_n^{R-d})b''-\frac{1}{2}b''^2.p_n^{R-d}$	$(q_n^{Q-d}+b''.q_n^{R-d})b''-\frac{1}{2}b''^2.q_n^{R-d}$	$(r_n^{Q-d}+b''.r_n^{R-d})b''-\frac{1}{2}b''^2.r_n^{R-d}$	0	0
C-a	$\frac{1}{3}b^3.p_n^{R-a}-p_n^{\Delta-a}$	$\frac{1}{3}b^3.q_n^{R-a}$	$\frac{1}{3}b^3.r_n^{R-a}$	0	$b.t_n^{C-B}$
C-b	$\frac{1}{2}(p_n^{Q-b}+b'.p_n^{R-b})b'^2-\frac{1}{6}b'^3.p_n^{R-b}-p_n^{\Delta-b}$	$\frac{1}{2}(q_n^{Q-b}+b'.q_n^{R-b})b'^2-\frac{1}{6}b'^3.q_n^{R-b}$	$\frac{1}{2}(r_n^{Q-b}+b'.r_n^{R-b})b'^2-\frac{1}{6}b'^3.r_n^{R-b}$	0	0
C-d	$\frac{1}{2}(p_n^{Q-d}+b''.p_n^{R-d})b''^2-\frac{1}{6}b''^3.p_n^{R-d}-p_n^{\Delta-d}$	$\frac{1}{2}(q_n^{Q-d}+b''.q_n^{R-d})b''^2-\frac{1}{6}b''^3.q_n^{R-d}$	$\frac{1}{2}(r_n^{Q-d}+b''.r_n^{R-d})b''^2-\frac{1}{6}b''^3.r_n^{R-d}$	0	0

PLATE 7.

VALUES OF THE COEFFICIENTS IN THE SYMBOLIC EQUATIONS.

	U_n	V_n	W_n	X_n	Y_n	P_n	Q_n	R_n	$\mathfrak{Z}_n$
C-A	0	0	0	0	0	0	0	0	0
C-B	0	0	0	0	0	0	0	0	0
C-C	1	0	0	0	0	0	0	0	0
C-D	0	1	0	0	0	0	0	0	$-G_n^2.\Sigma F_n+2I_n^D$
δ-A	0	0	0	0	0	0	0	0	0
δ-B	0	0	0	0	0	0	0	0	0
δ-C	G_n.	0	0	0	0	0	0	0	0
δ-D	0	G_n	0	0	0	0	0	0	$-G_n^3\Sigma F_n+6I_n^D$
H-a	0	0	0	0	0	0	0	0	F_n^A
H-b	0	0	0	0	0	0	0	0	F_n^A
H-d	0	0	0	0	0	0	0	0	F_n^A
λ-a	0	0	0	0	0	0	0	0	$b.\mathfrak{Z}_n^{H-a}\div A_n^a$
λ-b	0	0	0	0	0	0	0	0	$b'.\mathfrak{Z}_n^{H-b}\div A_n^b$
λ-d	0	0	0	0	0	0	0	0	$b''.\mathfrak{Z}_n^{H-d}\div A_n^d$
V-A	0	0	0	0	0	0	0	0	$-X_n\div(b+b'+b'')$
V-D	0	0	0	0	0	0	0	0	$Y_n+X_n\div(b+b'+b'')$
α-A	0	0	0	0	0	0	0	0	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)\mathfrak{Z}_n^{V-A}\div A_n^A$
α-B	0	0	0	0	0	0	0	0	0
α-C	0	0	0	0	0	0	0	0	0
α-D	0	0	0	0	0	0	0	0	$(C_n+\frac{1}{2}d_{n-1}+\frac{1}{2}d_n)\mathfrak{Z}_n^{V-D}\div A_n^D$
Δ-a	0	0	1	0	0	0	0	0	$-\mathfrak{Z}_n^{\alpha-A}$
Δ-b	0	0	0	1	0	0	0	0	0
Δ-d	0	0	0	0	1	0	0	0	$\mathfrak{Z}_n^{\alpha-D}$
Q-a	0	0	0	0	0	0	0	0	$-P_n^A.e_n^A\div I_n^a$
Q-b	0	0	0	0	0	0	0	0	$[-P_n^a(b+e_n^a)-\frac{1}{2}b^2W_n+b(\mathfrak{Z}_n^{V-A}-\mathfrak{Z}_{n+1}^{V-A})]\div I_n^b$
Q-d	0	0	0	0	0	0	0	0	$[-P_n^a(b+b'+e_n^a)-\frac{1}{2}(b+b')^2W_n+(b+2b')(\mathfrak{Z}_n^{V-A}-\mathfrak{Z}_{n+1}^{V-A})]\div I_n^d$
R-a	0	0	0	0	0	0	0	0	$[\mathfrak{Z}_n^{V-A}-\mathfrak{Z}_{n+1}^{V-A}-P_n^a]\div I_n^a$
R-b	0	0	0	0	0	0	0	0	$[\mathfrak{Z}_n^{V-A}-\mathfrak{Z}_{n+1}^{V-A}-P_n^a-bW_n]\div I_n^b$
R-d	0	0	0	0	0	0	0	0	$[\mathfrak{Z}_n^{V-A}-\mathfrak{Z}_{n+1}^{V-A}-P_n^a-(b+b')W_n]\div I_n^d$
A-a	0	0	0	0	0	$-b\div A_{n-1}^a$	0	0	$-\mathfrak{Z}_n^{\lambda-a}+\mathfrak{Z}_{n-1}^{\lambda-a}$
A-b	$-U_n^{\delta-C}-\frac{1}{2}d_n.U_n^{C-C}-\frac{1}{2}d_{n-1}$	0	0	0	0	$-b'\div A_{n-1}^b$	$-b'\div A_{n-1}^b$	0	$-\mathfrak{Z}_n^{\lambda-b}+\mathfrak{Z}_{n-1}^{\lambda-b}$
A-d	$U_n^{\delta-C}+\frac{1}{2}d_n.U_n^{C-C}+\frac{1}{2}d_{n-1}$	$-V_n^{\delta-D}-\frac{1}{2}d_n.V_n^{C-D}-\frac{1}{2}d_{n-1}$	0	0	0	$-b''\div A_{n-1}^d$	$-b''\div A_{n-1}^d$	$-b''\div A_{n-1}^d$	$-\mathfrak{Z}_n^{\delta-D}-\frac{1}{2}d_n.\mathfrak{Z}_n^{C-D}-\mathfrak{Z}_n^{\lambda-d}+\mathfrak{Z}_{n-1}^{\lambda-d}$
B-a	0	0	0	0	0	0	0	0	$(\mathfrak{Z}_n^{Q-a}+b.\mathfrak{Z}_n^{R-a})b-\frac{1}{2}b^2.\mathfrak{Z}_n^{R-a}-b^3.W_n\div 6I_n^a$
B-b	U_n^{C-C}	0	0	0	0	0	0	0	$(\mathfrak{Z}_n^{Q-b}+b'.\mathfrak{Z}_n^{R-b})b'-\frac{1}{2}b'^2.\mathfrak{Z}_n^{R-b}-b'^3.W_n\div 6I_n^b$
B-d	$-U_n^{C-C}$	V_n^{C-D}	0	0	0	0	0	0	$(\mathfrak{Z}_n^{Q-d}+b''.\mathfrak{Z}_n^{R-d})b''-\frac{1}{2}b''^2.\mathfrak{Z}_n^{R-d}-b''^3.W_n\div 6I_n^d+\mathfrak{Z}_n^{C-D}$
C-a	0	0	-1	0	0	0	0	0	$(\mathfrak{Z}_n^{Q-a}+b.\mathfrak{Z}_n^{R-a})\frac{1}{2}b^2-\frac{1}{6}b^3.\mathfrak{Z}_n^{R-a}-b^4.W_n\div 8I_n^a-\mathfrak{Z}_n^{\Delta-a}$
C-b	$b'.U_n^{C-C}$	0	0	-1	0	0	0	0	$(\mathfrak{Z}_n^{Q-b}+b'.\mathfrak{Z}_n^{R-b})\frac{1}{2}b'^2-\frac{1}{6}b'^3.\mathfrak{Z}_n^{R-b}-b'^4.W_n\div 8I_n^b$
C-d	0	$b''.V_n^{C-D}$	0	0	-1	0	0	0	$(\mathfrak{Z}_n^{Q-d}+b''.\mathfrak{Z}_n^{R-d})\frac{1}{2}b''^2-\frac{1}{6}b''^3.\mathfrak{Z}_n^{R-d}-b''^4.W_n\div 8I_n^d-\mathfrak{Z}_n^{\Delta-d}+b''.\mathfrak{Z}_n^{C-D}$

PLATE 8.

of at least equal precision. All calculations should be most carefully checked by an independent calculator. It will be noticed, as the calculations for any column of the table are made, that quantities once computed are referred to in succeeding calculations, *e. g.*, in finding the value of a_n^{A-a} the coefficient of M_n^A in equation A_n^a, the quantities a_n^{b-a} and a_n^{C-A} already computed, are here used. These calculations will be found to be quite simple and are not laborious.

Solution of the Equations of Unknowns.—As has been previously mentioned, the unknowns of the 0 or basement story must first be expressed in terms of those of the first story; then those of the first story in terms of those of the second story, and so on for each story until the top story is reached and its solution gives values in arithmetical units, there being no unknowns above the top story.

Now referring to the nine symbolic equations of the 0 story, A_0^a, A_0^b, A_0^d, B_0^a, B_0^b, B_0^d, C_0^a, C_0^b, and C_0^d it will be seen that each equation contains more or less of the nine unknowns of both the 0 and first stories. Let these nine equations be now written as shown in the first nine equations of Plate 9. The coefficients are here represented by the Greek letters with a subscript which refers to the *number of the equation* in which the coefficient occurs, these equations being numbered from 1 to 9. The quantity ω is used to represent the numerical constant together with all the first story unknowns of the equations; this makes the solution much more easily understood, since ω is then treated as though it were a numerical constant itself. Now the solution of these nine simplified equations is essentially the same as the elimination method used by Gauss in the solution of normal equations in the adjustment of observations. The method is given and, it is thought, explains itself, in Plates 9 and 10. The original equations are marked "1," thus *A–a–*1, and in the successive steps of the solution the new forms of the equations are distinguished by numbers corresponding to the number of eliminations made; thus *B–a–*4 represents equation *B–a–*1 from which M_0^A, M_0^B and M_0^C have been eliminated. In the actual solution of the nine equations it is not necessary to write out these successive steps, but merely to make a table in which the *coefficients* in all the equations of Plates 9 and 10 may be

Let the nine equations be written with simpler coefficients as follows:-

A-a-1 $\alpha_1 M_o^A + \beta_1 M_o^B + \gamma_1 M_o^C + \delta_1 M_o^D + \theta_1 S_o^A + \phi_1 S_o^B + \psi_1 S_o^C + \epsilon_1 V_o^B + \lambda_1 V_o^C + \omega_1 = 0.$

A-b-1 $\alpha_2 M_o^A + \beta_2 M_o^B + \gamma_2 M_o^C + \delta_2 M_o^D + \theta_2 S_o^A + \phi_2 S_o^B + \psi_2 S_o^C + \epsilon_2 V_o^B + \lambda_2 V_o^C + \omega_2 = 0$

A-d-1 $\alpha_3 M_o^A + \beta_3 M_o^B + \gamma_3 M_o^C + \delta_3 M_o^D + \theta_3 S_o^A + \phi_3 S_o^B + \psi_3 S_o^C + \epsilon_3 V_o^B + \lambda_3 V_o^C + \omega_3 = 0$

B-a-1 $\alpha_4 M_o^A + \beta_4 M_o^B + \gamma_4 M_o^C + \delta_4 M_o^D + \theta_4 S_o^A + \phi_4 S_o^B + \psi_4 S_o^C + \epsilon_4 V_o^B + \lambda_4 V_o^C + \omega_4 = 0.$

B-b-1 $\alpha_5 M_o^A + \beta_5 M_o^B + \gamma_5 M_o^C + \delta_5 M_o^D + \theta_5 S_o^A + \phi_5 S_o^B + \psi_5 S_o^C + \epsilon_5 V_o^B + \lambda_5 V_o^C + \omega_5 = 0.$

B-d-1 $\alpha_6 M_o^A + \beta_6 M_o^B + \gamma_6 M_o^C + \delta_6 M_o^D + \theta_6 S_o^A + \phi_6 S_o^B + \psi_6 S_o^C + \epsilon_6 V_o^B + \lambda_6 V_o^C + \omega_6 = 0.$

C-a-1 $\alpha_7 M_o^A + \beta_7 M_o^B + \gamma_7 M_o^C + \delta_7 M_o^D + \theta_7 S_o^A + \phi_7 S_o^B + \psi_7 S_o^C + \epsilon_7 V_o^B + \lambda_7 V_o^C + \omega_7 = 0.$

C-b-1 $\alpha_8 M_o^A + \beta_8 M_o^B + \gamma_8 M_o^C + \delta_8 M_o^D + \theta_8 S_o^A + \phi_8 S_o^B + \psi_8 S_o^C + \epsilon_8 V_o^B + \lambda_8 V_o^C + \omega_8 = 0.$

C-d-1 $\alpha_9 M_o^A + \beta_9 M_o^B + \gamma_9 M_o^C + \delta_9 M_o^D + \theta_9 S_o^A + \phi_9 S_o^B + \psi_9 S_o^C + \epsilon_9 V_o^B + \lambda_9 V_o^C + \omega_9 = 0.$

From Equation A-a-1

$$M_o^A = \beta_1' M_o^B + \gamma_1' M_o^C + \delta_1' M_o^D + \theta_1' S_o^A + \phi_1' S_o^B + \psi_1' S_o^C + \epsilon_1' V_o^B + \lambda_1' V_o^C + \omega_1'$$

Substituting this value of M_o^A in the succeeding equations gives

A-b-2 $\beta_2' M_o^B + \gamma_2' M_o^C + \delta_2' M_o^D + \theta_2' S_o^A + \phi_2' S_o^B + \psi_2' S_o^C + \epsilon_2' V_o^B + \lambda_2' V_o^C + \omega_2' = 0.$

A-d-2 $\beta_3' M_o^B + \gamma_3' M_o^C + \delta_3' M_o^D + \theta_3' S_o^A + \phi_3' S_o^B + \psi_3' S_o^C + \epsilon_3' V_o^B + \lambda_3' V_o^C + \omega_3' = 0.$

B-a-2 $\beta_4' M_o^B + \gamma_4' M_o^C + \delta_4' M_o^D + \theta_4' S_o^A + \phi_4' S_o^B + \psi_4' S_o^C + \epsilon_4' V_o^B + \lambda_4' V_o^C + \omega_4' = 0.$

B-b-2 $\beta_5' M_o^B + \gamma_5' M_o^C + \delta_5' M_o^D + \theta_5' S_o^A + \phi_5' S_o^B + \psi_5' S_o^C + \epsilon_5' V_o^B + \lambda_5' V_o^C + \omega_5' = 0$

B-d-2 $\beta_6' M_o^B + \gamma_6' M_o^C + \delta_6' M_o^D + \theta_6' S_o^A + \phi_6' S_o^B + \psi_6' S_o^C + \epsilon_6' V_o^B + \lambda_6' V_o^C + \omega_6' = 0.$

C-a-2 $\beta_7' M_o^B + \gamma_7' M_o^C + \delta_7' M_o^D + \theta_7' S_o^A + \phi_7' S_o^B + \psi_7' S_o^C + \epsilon_7' V_o^B + \lambda_7' V_o^C + \omega_7' = 0.$

C-b-2 $\beta_8' M_o^B + \gamma_8' M_o^C + \delta_8' M_o^D + \theta_8' S_o^A + \phi_8' S_o^B + \psi_8' S_o^C + \epsilon_8' V_o^B + \lambda_8' V_o^C + \omega_8' = 0.$

C-d-2 $\beta_9' M_o^B + \gamma_9' M_o^C + \delta_9' M_o^D + \theta_9' S_o^A + \phi_9' S_o^B + \psi_9' S_o^C + \epsilon_9' V_o^B + \lambda_9' V_o^C + \omega_9' = 0.$

From Equation A-b-2

$$M_o^B = \gamma_2'' M_o^C + \delta_2'' M_o^D + \theta_2'' S_o^A + \phi_2'' S_o^B + \psi_2'' S_o^C + \epsilon_2'' V_o^B + \lambda_2'' V_o^C + \omega_2''$$

Substituting this value of M_o^B in the succeeding equations gives

A-d-3 $\gamma_3'' M_o^C + \delta_3'' M_o^D + \theta_3'' S_o^A + \phi_3'' S_o^B + \psi_3'' S_o^C + \epsilon_3'' V_o^B + \lambda_3'' V_o^C + \omega_3'' = 0.$

B-a-3 $\gamma_4'' M_o^C + \delta_4'' M_o^D + \theta_4'' S_o^A + \phi_4'' S_o^B + \psi_4'' S_o^C + \epsilon_4'' V_o^B + \lambda_4'' V_o^C + \omega_4'' = 0.$

B-b-3 $\gamma_5'' M_o^C + \delta_5'' M_o^D + \theta_5'' S_o^A + \phi_5'' S_o^B + \psi_5'' S_o^C + \epsilon_5'' V_o^B + \lambda_5'' V_o^C + \omega_5'' = 0.$

B-d-3 $\gamma_6'' M_o^C + \delta_6'' M_o^D + \theta_6'' S_o^A + \phi_6'' S_o^B + \psi_6'' S_o^C + \epsilon_6'' V_o^B + \lambda_6'' V_o^C + \omega_6'' = 0.$

C-a-3 $\gamma_7'' M_o^C + \delta_7'' M_o^D + \theta_7'' S_o^A + \phi_7'' S_o^B + \psi_7'' S_o^C + \epsilon_7'' V_o^B + \lambda_7'' V_o^C + \omega_7'' = 0.$

C-b-3 $\gamma_8'' M_o^C + \delta_8'' M_o^D + \theta_8'' S_o^A + \phi_8'' S_o^B + \psi_8'' S_o^C + \epsilon_8'' V_o^B + \lambda_8'' V_o^C + \omega_8'' = 0.$

C-d-3 $\gamma_9'' M_o^C + \delta_9'' M_o^D + \theta_9'' S_o^A + \phi_9'' S_o^B + \psi_9'' S_o^C + \epsilon_9'' V_o^B + \lambda_9'' V_o^C + \omega_9'' = 0.$

From Equation A-d-3

$$M_o^C = \delta_3''' M_o^D + \theta_3''' S_o^A + \phi_3''' S_o^B + \psi_3''' S_o^C + \epsilon_3''' V_o^B + \lambda_3''' V_o^C + \omega_3'''.$$

Substituting this value of M_o^C in the succeeding equations gives

B-a-4 $\delta_4''' M_o^D + \theta_4''' S_o^A + \phi_4''' S_o^B + \psi_4''' S_o^C + \epsilon_4''' V_o^B + \lambda_4''' V_o^C + \omega_4''' = 0.$

B-b-4 $\delta_5''' M_o^D + \theta_5''' S_o^A + \phi_5''' S_o^B + \psi_5''' S_o^C + \epsilon_5''' V_o^B + \lambda_5''' V_o^C + \omega_5''' = 0.$

B-d-4 $\delta_6''' M_o^D + \theta_6''' S_o^A + \phi_6''' S_o^B + \psi_6''' S_o^C + \epsilon_6''' V_o^B + \lambda_6''' V_o^C + \omega_6''' = 0.$

C-a-4 $\delta_7''' M_o^D + \theta_7''' S_o^A + \phi_7''' S_o^B + \psi_7''' S_o^C + \epsilon_7''' V_o^B + \lambda_7''' V_o^C + \omega_7''' = 0.$

C-b-4 $\delta_8''' M_o^D + \theta_8''' S_o^A + \phi_8''' S_o^B + \psi_8''' S_o^C + \epsilon_8''' V_o^B + \lambda_8''' V_o^C + \omega_8''' = 0.$

C-d-4 $\delta_9''' M_o^D + \theta_9''' S_o^A + \phi_9''' S_o^B + \psi_9''' S_o^C + \epsilon_9''' V_o^B + \lambda_9''' V_o^C + \omega_9''' = 0.$

PLATE 9.

From Equation B-a-4

$$M_o^D = \theta_4^{IV} S_o^A + \phi_4^{IV} S_o^B + \psi_4^{IV} S_o^C + \varepsilon_4^{IV} V_o^B \ \lambda_4^{IV} V_o^C + \omega_4^{IV}.$$

Substituting this value of M_o^D in the succeeding equations gives

B-b-5 $\quad \theta_5^{IV} S_o^A + \phi_5^{IV} S_o^B + \psi_5^{IV} S_o^C + \varepsilon_5^{IV} V_o^B + \lambda_5^{IV} V_o^C + \omega_5^{IV} = 0.$

B-d-5 $\quad \theta_6^{IV} S_o^A + \phi_6^{IV} S_o^B + \psi_6^{IV} S_o^C + \varepsilon_6^{IV} V_o^B + \lambda_6^{IV} V_o^C + \omega_6^{IV} = 0.$

C-a-5 $\quad \theta_7^{IV} S_o^A + \phi_7^{IV} S_o^B + \psi_7^{IV} S_o^C + \varepsilon_7^{IV} V_o^B + \lambda_7^{IV} V_o^C + \omega_7^{IV} = 0.$

C-b-5 $\quad \theta_8^{IV} S_o^A + \phi_8^{IV} S_o^B + \psi_8^{IV} S_o^C + \varepsilon_8^{IV} V_o^B + \lambda_8^{IV} V_o^C + \omega_8^{IV} = 0$

C-d-5 $\quad \theta_9^{IV} S_o^A + \phi_9^{IV} S_o^B + \psi_9^{IV} S_o^C + \varepsilon_9^{IV} V_o^B + \lambda_9^{IV} V_o^C + \omega_9^{IV} = 0$

From Equation B-b-5

$$S_o^A = \phi_5^{V} S_o^B + \psi_5^{V} S_o^C + \varepsilon_5^{V} V_o^B + \lambda_5^{V} V_o^C + \omega_5^{V}$$

Substituting this value of S_o^A in the succeeding equations gives

B-d-6 $\quad \phi_6^{V} S_o^B + \psi_6^{V} S_o^C + \varepsilon_6^{V} V_o^B + \lambda_6^{V} V_o^C + \omega_6^{V} = 0.$

C-a-6 $\quad \phi_7^{V} S_o^B + \psi_7^{V} S_o^C + \varepsilon_7^{V} V_o^B + \lambda_7^{V} V_o^C + \omega_7^{V} = 0$

C-b-6 $\quad \phi_8^{V} S_o^B + \psi_8^{V} S_o^C + \varepsilon_8^{V} V_o^B + \lambda_8^{V} V_o^C + \omega_8^{V} = 0.$

C-d-6 $\quad \phi_9^{V} S_o^B + \psi_9^{V} S_o^C + \varepsilon_9^{V} V_o^B + \lambda_9^{V} V_o^C + \omega_9^{V} = 0$

From Equation B-d-6

$$S_o^B = \psi_6^{VI} S_o^C + \varepsilon_6^{VI} V_o^B + \lambda_6^{VI} V_o^C + \omega_6^{VI}$$

Substituting this value of S_o^B in the succeeding equations gives

C-a-7 $\quad \psi_7^{VI} S_o^C + \varepsilon_7^{VI} V_o^B + \lambda_7^{VI} V_o^C + \omega_7^{VI} = 0.$

C-b-7 $\quad \psi_8^{VI} S_o^C + \varepsilon_8^{VI} V_o^B + \lambda_8^{VI} V_o^C + \omega_8^{VI} = 0.$

C-d-7 $\quad \psi_9^{VI} S_o^C + \varepsilon_9^{VI} V_o^B + \lambda_9^{VI} V_o^C + \omega_9^{VI} = 0.$

From Equation C-a-7

$$S_o^C = \varepsilon_7^{VII} V_o^B + \lambda_7^{VII} V_o^C + \omega_7^{VII}$$

Substituting this value of S_o^C in the succeeding equations gives

C-b-8 $\quad \varepsilon_8^{VII} V_o^B + \lambda_8^{VII} V_o^C + \omega_8^{VII} = 0$

C-d-8 $\quad \varepsilon_9^{VII} V_o^B + \lambda_9^{VII} V_o^C + \omega_9^{VII} = 0$

From Equation C-b-8

$$V_o^B = \lambda_8^{VIII} V_o^C + \omega_8^{VIII}$$

Substituting this value of V_o^B in the succeeding equation gives

$$\lambda_9^{VIII} V_o^C + \omega_9^{VIII} = 0$$

$$\text{or} \quad V_o^C = \omega_9^{IX}.$$

PLATE 10.

TABULATED VALUES OF THE COEFFICIENTS IN THE SOLUTION OF THE EQUATIONS FOR THE O STORY.

Equation.	α	β	γ	δ	θ	ϕ	ψ	ε	λ
A-a-1	a_o^{A-a}	b_o^{A-a}	0	0	i_o^{A-a}	j_o^{A-a}	0	0	0
A-b-1	0	b_o^{A-b}	c_o^{A-b}	0	i_o^{A-b}	j_o^{A-b}	k_o^{A-b}	0	0
A-d-1	0	0	c_o^{A-d}	d_o^{A-d}	i_o^{A-d}	j_o^{A-d}	k_o^{A-d}	0	0
B-a-1	a_o^{B-a}	b_o^{B-a}	c_o^{B-a}	d_o^{B-a}	i_o^{B-a}	j_o^{B-a}	0	o_o^{B-a}	p_o^{B-a}
B-b-1	a_o^{B-b}	b_o^{B-b}	c_o^{B-b}	d_o^{B-b}	i_o^{B-b}	j_o^{B-b}	k_o^{B-b}	a_o^{B-b}	p_o^{B-b}
B-d-1	a_o^{B-d}	b_o^{B-d}	c_o^{B-d}	d_o^{B-d}	i_o^{B-d}	j_o^{B-d}	k_o^{B-d}	o_o^{B-d}	p_o^{B-d}
C-a-1	a_o^{C-a}	b_o^{C-a}	c_o^{C-a}	d_o^{C-a}	i_o^{C-a}	j_o^{C-a}	0	o_o^{C-a}	p_o^{C-a}
C-b-1	a_o^{C-b}	b_o^{C-b}	c_o^{C-b}	d_o^{C-b}	i_o^{C-b}	j_o^{C-b}	k_o^{C-b}	o_o^{C-b}	p_o^{C-b}
C-d-1	a_o^{C-d}	b_o^{C-d}	c_o^{C-d}	d_o^{C-d}	i_o^{C-d}	j_o^{C-d}	k_o^{C-d}	o_o^{C-d}	p_o^{C-d}
	$M_o^B=$	$-\beta_1+\alpha_1$	$-\gamma_1\div\alpha_1$	$-\delta_1+\alpha_1$	$-\theta_1\div\alpha_1$	$-\phi_1+\alpha_1$	$-\psi_1\div\alpha_1$	$-\varepsilon_1+\alpha_1$	$-\lambda_1+\alpha_1$
	A-b-2	$\beta_2+\alpha_2.\beta_1'$	$\gamma_2+\alpha_2\gamma_1'$	$\delta_2+\alpha_2.\delta_1'$	$\theta_2+\alpha_2.\theta_1'$	$\phi_2+\alpha_2.\phi_1'$	$\psi_2+\alpha_2.\psi_1'$	$\varepsilon_2+\alpha_2.\varepsilon_1'$	$\lambda_2+\alpha_2\lambda_1'$
	A-d-2	$\beta_3+\alpha_3.\beta_1'$	$\gamma_3+\alpha_3.\gamma_1'$	$\delta_3+\alpha_3.\delta_1'$	$\theta_3+\alpha_3.\theta_1'$	$\phi_3+\alpha_3.\phi_1'$	$\psi_3+\alpha_3.\psi_1'$	$\varepsilon_3+\alpha_3.\varepsilon_1'$	$\lambda_3+\alpha_3\lambda_1'$
	B-a-2	$\beta_4+\alpha_4.\beta_1'$	$\gamma_4+\alpha_4.\gamma_1'$	$\delta_4+\alpha_4.\delta_1'$	$\theta_4+\alpha_4.\theta_1'$	$\phi_4+\alpha_4.\phi_1'$	$\psi_4+\alpha_4.\psi_1'$	$\varepsilon_4+\alpha_4.\varepsilon_1'$	$\lambda_4+\alpha_4.\lambda_1'$
	B-b-2	$\beta_5+\alpha_5.\beta_1'$	$\gamma_5+\alpha_5.\gamma_1'$	$\delta_5+\alpha_5.\delta_1'$	$\theta_5+\alpha_5.\theta_1'$	$\phi_5+\alpha_5.\phi_1'$	$\psi_5+\alpha_5.\psi_1'$	$\varepsilon_5+\alpha_5.\varepsilon_1'$	$\lambda_5+\alpha_5.\lambda_1'$
	B-d-2	$\beta_6+\alpha_6.\beta_1'$	$\gamma_6+\alpha_6.\gamma_1'$	$\delta_6+\alpha_6.\delta_1'$	$\theta_6+\alpha_6.\theta_1'$	$\phi_6+\alpha_6.\phi_1'$	$\psi_6+\alpha_6.\psi_1'$	$\varepsilon_6+\alpha_6.\varepsilon_1'$	$\lambda_6+\alpha_6.\lambda_1'$
	C-a-2	$\beta_7+\alpha_7.\beta_1'$	$\gamma_7+\alpha_7.\gamma_1'$	$\delta_7+\alpha_7.\delta_1'$	$\theta_7+\alpha_7.\theta_1'$	$\phi_7+\alpha_7.\phi_1'$	$\psi_7+\alpha_7.\psi_1'$	$\varepsilon_7+\alpha_7.\varepsilon_1'$	$\lambda_7+\alpha_7.\lambda_1'$
	C-b-2	$\beta_8+\alpha_8.\beta_1'$	$\gamma_8+\alpha_8.\gamma_1'$	$\delta_8+\alpha_8.\delta_1'$	$\theta_8+\alpha_8.\theta_1'$	$\phi_8+\alpha_8.\phi_1'$	$\psi_8+\alpha_8.\psi_1'$	$\varepsilon_8+\alpha_8.\varepsilon_1'$	$\lambda_8+\alpha_8.\lambda_1'$
	C-d-2	$\beta_9+\alpha_9.\beta_1'$	$\gamma_9+\alpha_9.\gamma_1'$	$\delta_9+\alpha_9.\delta_1'$	$\theta_9+\alpha_9.\theta_1'$	$\phi_9+\alpha_9.\phi_1'$	$\psi_9+\alpha_9.\psi_1'$	$\varepsilon_9+\alpha_9.\varepsilon_1'$	$\lambda_9+\alpha_9.\lambda_1'$
		$M_o^B=$	$-\gamma_2'+\beta_2'$	$-\delta_2'\div\beta_2'$	$-\theta_2'\div\beta_2'$	$-\phi_2'+\beta_2'$	$-\psi_2'\div\beta_2'$	$-\varepsilon_2'+\beta_2'$	$-\lambda_2'+\beta_2'$
		A-d-3	$\gamma_3'+\beta_3'.\gamma_2''$	$\delta_3'+\beta_3'.\delta_2''$	$\theta_3'+\beta_3'.\theta_2''$	$\phi_3'+\beta_3'.\phi_2''$	$\psi_3'+\beta_3'.\psi_2''$	$\varepsilon_3'+\beta_3'.\varepsilon_2''$	$\lambda_3'+\beta_3'\lambda_2''$
		B-a-3	$\gamma_4'+\beta_4'.\gamma_2''$	$\delta_4'+\beta_4'.\delta_2''$	$\theta_4'+\beta_4'.\theta_2''$	$\phi_4'+\beta_4'.\phi_2''$	$\psi_4'+\beta_4'.\psi_2''$	$\varepsilon_4'+\beta_4'.\varepsilon_2''$	$\lambda_4'+\beta_4'\lambda_2''$
		B-b-3	$\gamma_5'+\beta_5'.\gamma_2''$	$\delta_5'+\beta_5'.\delta_2''$	$\theta_5'+\beta_5'.\theta_2''$	$\phi_5'+\beta_5'.\phi_2''$	$\psi_5'+\beta_5'.\psi_2''$	$\varepsilon_5'+\beta_5'.\varepsilon_2''$	$\lambda_5'+\beta_5'.\lambda_2''$
		B-d-3	$\gamma_6'+\beta_6'.\gamma_2''$	$\delta_6'+\beta_6'.\delta_2''$	$\theta_6'+\beta_6'.\theta_2''$	$\phi_6'+\beta_6'.\phi_2''$	$\psi_6'+\beta_6'.\psi_2''$	$\varepsilon_6'+\beta_6'.\varepsilon_2''$	$\lambda_6'+\beta_6'.\lambda_2''$
		C-a-3	$\gamma_7'+\beta_7'.\gamma_2''$	$\delta_7'+\beta_7'.\delta_2''$	$\theta_7'+\beta_7'.\theta_2''$	$\phi_7'+\beta_7'.\phi_2''$	$\psi_7'+\beta_7'.\psi_2''$	$\varepsilon_7'+\beta_7'.\varepsilon_2''$	$\lambda_7'+\beta_7'.\lambda_2''$
		C-b-3	$\gamma_8'+\beta_8'.\gamma_2''$	$\delta_8'+\beta_8'.\delta_2''$	$\theta_8'+\beta_8'.\theta_2''$	$\phi_8'+\beta_8'.\phi_2''$	$\psi_8'+\beta_8'.\psi_2''$	$\varepsilon_8'+\beta_8'.\varepsilon_2''$	$\lambda_8'+\beta_8'.\lambda_2''$
		C-d-3	$\gamma_9'+\beta_9'.\gamma_2''$	$\delta_9'+\beta_9'.\delta_2''$	$\theta_9'+\beta_9'.\theta_2''$	$\phi_9'+\beta_9'.\phi_2''$	$\psi_9'+\beta_9'.\psi_2''$	$\varepsilon_9'+\beta_9'.\varepsilon_2''$	$\lambda_9'+\beta_9'.\lambda_2''$
			$M_o^C=$	$-\delta_3''\div\gamma_3''$	$-\theta_3''\div\gamma_3''$	$-\phi_3''\div\gamma_3''$	$-\psi_3''\div\gamma_3''$	$-\varepsilon_3''\div\gamma_3''$	$-\lambda_3''\div\gamma_3''$
			B-a-4	$\delta_4''+\gamma_4''.\delta_3'''$	$\theta_4''+\gamma_4''.\theta_3'''$	$\phi_4''+\gamma_4''.\phi_3'''$	$\psi_4''+\gamma_4''.\psi_3'''$	$\varepsilon_4''+\gamma_4''.\varepsilon_3'''$	$\lambda_4''+\gamma_4''\lambda_3'''$
			B-b-4	$\delta_5''+\gamma_5''.\delta_3'''$	$\theta_5''+\gamma_5''.\theta_3'''$	$\phi_5''+\gamma_5''.\phi_3'''$	$\psi_5''+\gamma_5''.\psi_3'''$	$\varepsilon_5''+\gamma_5''.\varepsilon_3'''$	$\lambda_5''+\gamma_5''\lambda_3'''$
			B-d-4	$\delta_6''+\gamma_6''.\delta_3'''$	$\theta_6''+\gamma_6''.\theta_3'''$	$\phi_6''+\gamma_6''.\phi_3'''$	$\psi_6''+\gamma_6''.\psi_3'''$	$\varepsilon_6''+\gamma_6''.\varepsilon_3'''$	$\lambda_6''+\gamma_6''.\lambda_3'''$
			C-a-4	$\delta_7''+\gamma_7''.\delta_3'''$	$\theta_7''+\gamma_7''.\theta_3'''$	$\phi_7''+\gamma_7''.\phi_3'''$	$\psi_7''+\gamma_7''.\psi_3'''$	$\varepsilon_7''+\gamma_7''.\varepsilon_3'''$	$\lambda_7''+\gamma_7''.\lambda_3'''$
			C-b-4	$\delta_8''+\gamma_8''.\delta_3'''$	$\theta_8''+\gamma_8''\ \theta_3'''$	$\phi_8''+\gamma_8''.\phi_3'''$	$\psi_8''+\gamma_8''.\psi_3'''$	$\varepsilon_8''+\gamma_8''.\varepsilon_3'''$	$\lambda_8''+\gamma_8''.\lambda_3'''$
			C-d-4	$\delta_9''+\gamma_9''.\delta_3'''$	$\theta_9''+\gamma_9''.\theta_3'''$	$\phi_9''+\gamma_9''.\phi_3'''$	$\psi_9''+\gamma_9''.\psi_3'''$	$\varepsilon_9''+\gamma_9''.\varepsilon_3'''$	$\lambda_9''+\gamma_9''.\lambda_3'''$
				$M_o^D=$	$-\theta_4'''\div\delta_4'''$	$-\phi_4'''\div\delta_4'''$	$-\psi_4'''+\delta_4'''$	$-\varepsilon_4'''\div\delta_4'''$	$-\lambda_4'''+\delta_4'''$
				B-b-5	$\theta_5'''+\delta_5'''.\theta_4^{IV}$	$\phi_5'''+\delta_5'''.\phi_4^{IV}$	$\psi_5'''+\delta_5'''.\psi_4^{IV}$	$\varepsilon_5'''+\delta_5'''.\varepsilon_4^{IV}$	$\lambda_5'''+\delta_5'''.\lambda_4^{IV}$
				B-d-5	$\theta_6'''+\delta_6'''.\theta_4^{IV}$	$\phi_6'''+\delta_6'''.\phi_4^{IV}$	$\psi_6'''+\delta_6'''.\psi_4^{IV}$	$\varepsilon_6'''+\delta_6'''.\varepsilon_4^{IV}$	$\lambda_6'''+\delta_6'''.\lambda_4^{IV}$
				C-a-5	$\theta_7'''+\delta_7'''.\theta_4^{IV}$	$\phi_7'''+\delta_7'''.\phi_4^{IV}$	$\psi_7'''+\delta_7'''\ \psi_4^{IV}$	$\varepsilon_7'''+\delta_7'''.\varepsilon_4^{IV}$	$\lambda_7'''+\delta_7'''.\lambda_4^{IV}$
				C-b-5	$\theta_8'''+\delta_8'''.\theta_4^{IV}$	$\phi_8'''+\delta_8'''.\phi_4^{IV}$	$\psi_8'''+\delta_8'''.\psi_4^{IV}$	$\varepsilon_8'''+\delta_8'''.\varepsilon_4^{IV}$	$\lambda_8'''+\delta_8'''.\lambda_4^{IV}$
				C-d-5	$\theta_9'''+\delta_9'''.\theta_4^{IV}$	$\phi_9'''+\delta_9'''.\phi_4^{IV}$	$\psi_9'''+\delta_9'''.\psi_4^{IV}$	$\varepsilon_9'''+\delta_9'''.\varepsilon_4^{IV}$	$\lambda_9'''+\delta_9'''.\lambda_4^{IV}$
					$S_o^A=$	$-\phi_5^{IV}\div\theta_5^{IV}$	$-\psi_5^{IV}\div\theta_5^{IV}$	$-\varepsilon_5^{IV}\div\theta_5^{IV}$	$-\lambda_5^{IV}\div\theta_5^{IV}$
					B-d-6	$\phi_6^{IV}+\theta_6^{IV}.\phi_5^{V}$	$\psi_6^{IV}+\theta_6^{IV}.\psi_5^{V}$	$\varepsilon_6^{IV}+\theta_6^{IV}.\varepsilon_5^{V}$	$\lambda_6^{IV}+\theta_6^{IV}.\lambda_5^{V}$
					C-a-6	$\phi_7^{IV}+\theta_7^{IV}.\phi_5^{V}$	$\psi_7^{IV}+\theta_7^{IV}.\psi_5^{V}$	$\varepsilon_7^{IV}+\theta_7^{IV}.\varepsilon_5^{V}$	$\lambda_7^{IV}+\theta_7^{IV}\lambda_5^{V}$
					C-b-6	$\phi_8^{IV}+\theta_8^{IV}.\phi_5^{V}$	$\psi_8^{IV}+\theta_8^{IV}.\psi_5^{V}$	$\varepsilon_8^{IV}+\theta_8^{IV}.\varepsilon_5^{V}$	$\lambda_8^{IV}+\theta_8^{IV}\lambda_5^{V}$
					C-d-6	$\phi_9^{IV}+\theta_9^{IV}.\phi_5^{V}$	$\psi_9^{IV}+\theta_9^{IV}.\psi_5^{V}$	$\varepsilon_9^{IV}+\theta_9^{IV}.\varepsilon_5^{V}$	$\lambda_9^{IV}+\theta_9^{IV}\lambda_5^{V}$
						$S_o^B=$	$-\psi_6^{V}\div\phi_6^{V}$	$-\varepsilon_6^{V}\div\phi_6^{V}$	$-\lambda_6^{V}\div\phi_6^{V}$
						C-a-7	$\psi_7^{V}+\phi_7^{V}.\psi_6^{VI}$	$\varepsilon_7^{V}+\phi_7^{V}.\varepsilon_6^{VI}$	$\lambda_7^{V}+\phi_7^{V}\lambda_6^{VI}$
						C-b-7	$\psi_8^{V}+\phi_8^{V}.\psi_6^{VI}$	$\varepsilon_8^{V}+\phi_8^{V}.\varepsilon_6^{VI}$	$\lambda_8^{V}+\phi_8^{V}\lambda_6^{VI}$
						C-d-7	$\psi_9^{V}+\phi_9^{V}.\psi_6^{VI}$	$\varepsilon_9^{V}+\phi_9^{V}.\varepsilon_6^{VI}$	$\lambda_9^{V}+\phi_9^{V}\lambda_6^{VI}$
							$S_o^C=$	$-\varepsilon_7^{VI}\div\psi_7^{VI}$	$-\lambda_7^{VI}\div\psi_7^{VI}$
							C-b-8	$\varepsilon_8^{VI}+\psi_8^{VI}.\varepsilon_7^{VII}$	$\lambda_8^{VI}+\psi_8^{VI}.\lambda_7^{VII}$
							C-d-8	$\varepsilon_9^{VI}+\psi_9^{VI}.\varepsilon_7^{VII}$	$\lambda_9^{VI}+\psi_9^{VI}.\lambda_7^{VII}$
								$V_o^B=$	$-\lambda_8^{VII}\div\varepsilon_8^{VII}$
								C-d-9	$\lambda_9^{VII}\div\varepsilon_9^{VII}\lambda_8^{VIII}$
									$V_o^C=$

Note:– This table corresponds to the solution equations given on the preceding pages

PLATE II.

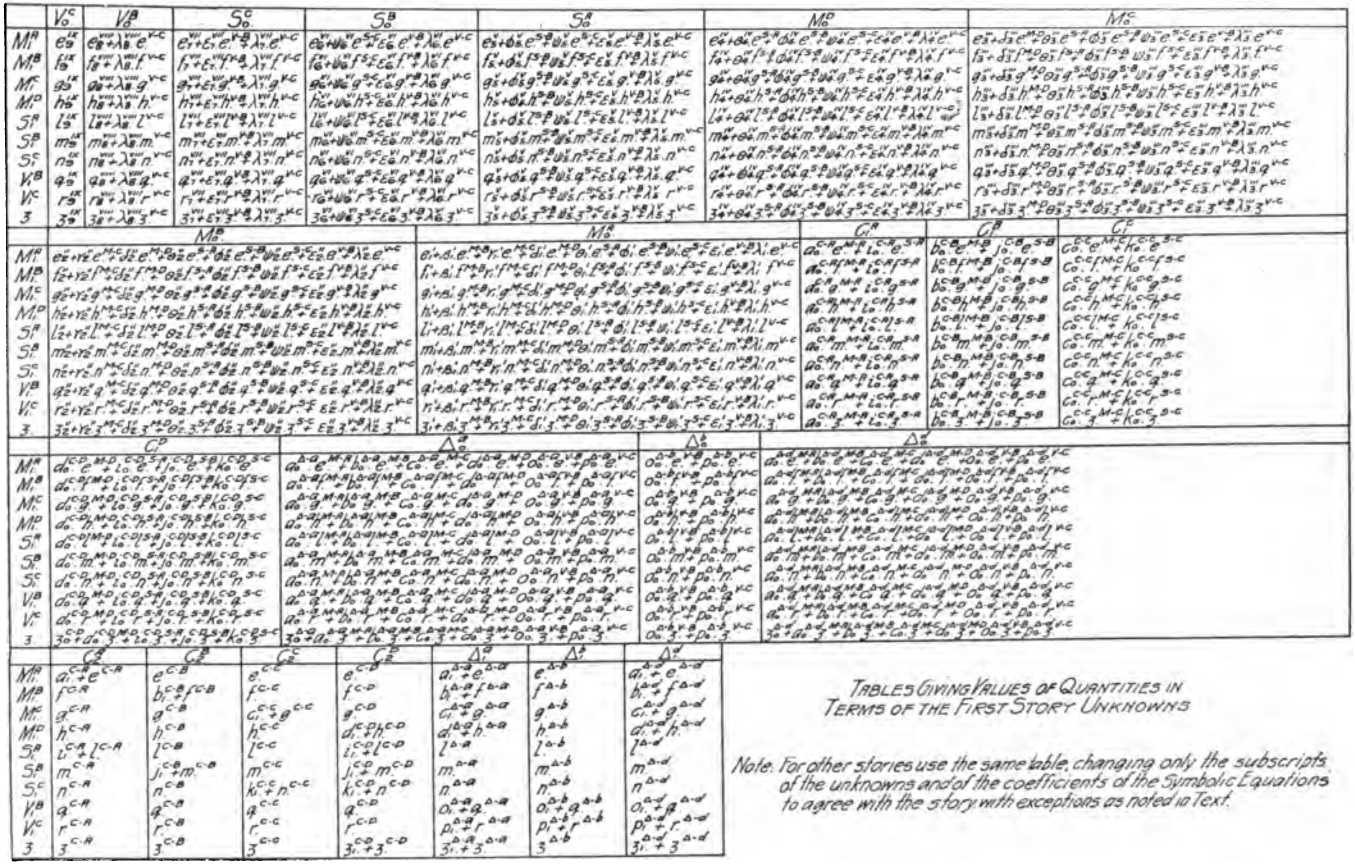
TABULATED VALUES OF THE COEFFICIENTS OF THE TERMS INVOLVED IN THE EXPRESSIONS (ω)

e	f	g	h	l	m	n	q	r	z
0	0	0	0	l_0^{A-a}	0	0	0	0	z_0^{A-a}
0	0	0	0	l_0^{A-b}	m_0^{A-b}	0	0	0	z_0^{A-b}
0	0	0	0	l_0^{A-d}	m_0^{A-d}	n_0^{A-d}	0	0	z_0^{A-d}
e_0^{B-a}	f_0^{B-a}	g_0^{B-a}	h_0^{B-a}	l_0^{B-a}	0	0	q_0^{B-a}	r_0^{B-a}	z_0^{B-a}
e_0^{B-b}	f_0^{B-b}	g_0^{B-b}	h_0^{B-b}	l_0^{B-b}	m_0^{B-b}	0	q_0^{B-b}	r_0^{B-b}	z_0^{B-b}
e_0^{B-d}	f_0^{B-d}	g_0^{B-d}	h_0^{B-d}	l_0^{B-d}	m_0^{B-d}	n_0^{B-d}	q_0^{B-d}	r_0^{B-d}	z_0^{B-d}
e_0^{C-a}	f_0^{C-a}	g_0^{C-a}	h_0^{C-a}	l_0^{C-a}	0	0	q_0^{C-a}	r_0^{C-a}	z_0^{C-a}
e_0^{C-b}	f_0^{C-b}	g_0^{C-b}	h_0^{C-b}	l_0^{C-b}	m_0^{C-b}	0	q_0^{C-b}	r_0^{C-b}	z_0^{C-b}
e_0^{C-d}	f_0^{C-d}	g_0^{C-d}	h_0^{C-d}	l_0^{C-d}	m_0^{C-d}	n_0^{C-d}	q_0^{C-d}	r_0^{C-d}	z_0^{C-d}
$-e_1\div\alpha_1$	$-f_1\div\alpha_1$	$-g_1\div\alpha_1$	$-h_1\div\alpha_1$	$-l_1\div\alpha_1$	$-m_1\div\alpha_1$	$-n_1\div\alpha_1$	$-q_1\div\alpha_1$	$-r_1\div\alpha_1$	$-z_1\div\alpha_1$
$e_2+\alpha_2.e_1'$	$f_2+\alpha_2.f_1'$	$g_2+\alpha_2.g_1'$	$h_2+\alpha_2.h_1'$	$l_2+\alpha_2.l_1'$	$m_2+\alpha_2.m_1'$	$n_2+\alpha_2.n_1'$	$q_2+\alpha_2.q_1'$	$r_2+\alpha_2.r_1'$	$z_2+\alpha_2.z_1'$
$e_3+\alpha_3.e_1'$	$f_3+\alpha_3.f_1'$	$g_3+\alpha_3.g_1'$	$h_3+\alpha_3.h_1'$	$l_3+\alpha_3.l_1'$	$m_3+\alpha_3.m_1'$	$n_3+\alpha_3.n_1'$	$q_3+\alpha_3.q_1'$	$r_3+\alpha_3.r_1'$	$z_3+\alpha_3.z_1'$
$e_4+\alpha_4.e_1'$	$f_4+\alpha_4.f_1'$	$g_4+\alpha_4.g_1'$	$h_4+\alpha_4.h_1'$	$l_4+\alpha_4.l_1'$	$m_4+\alpha_4.m_1'$	$n_4+\alpha_4.n_1'$	$q_4+\alpha_4.q_1'$	$r_4+\alpha_4.r_1'$	$z_4+\alpha_4.z_1'$
$e_5+\alpha_5.e_1'$	$f_5+\alpha_5.f_1'$	$g_5+\alpha_5.g_1'$	$h_5+\alpha_5.h_1'$	$l_5+\alpha_5.l_1'$	$m_5+\alpha_5.m_1'$	$n_5+\alpha_5.n_1'$	$q_5+\alpha_5.q_1'$	$r_5+\alpha_5.r_1'$	$z_5+\alpha_5.z_1'$
$e_6+\alpha_6.e_1'$	$f_6+\alpha_6.f_1'$	$g_6+\alpha_6.g_1'$	$h_6+\alpha_6.h_1'$	$l_6+\alpha_6.l_1'$	$m_6+\alpha_6.m_1'$	$n_6+\alpha_6.n_1'$	$q_6+\alpha_6.q_1'$	$r_6+\alpha_6.r_1'$	$z_6+\alpha_6.z_1'$
$e_7+\alpha_7.e_1'$	$f_7+\alpha_7.f_1'$	$g_7+\alpha_7.g_1'$	$h_7+\alpha_7.h_1'$	$l_7+\alpha_7.l_1'$	$m_7+\alpha_7.m_1'$	$n_7+\alpha_7.n_1'$	$q_7+\alpha_7.q_1'$	$r_7+\alpha_7.r_1'$	$z_7+\alpha_7.z_1'$
$e_8+\alpha_8.e_1'$	$f_8+\alpha_8.f_1'$	$g_8+\alpha_8.g_1'$	$h_8+\alpha_8.h_1'$	$l_8+\alpha_8.l_1'$	$m_8+\alpha_8.m_1'$	$n_8+\alpha_8.n_1'$	$q_8+\alpha_8.q_1'$	$r_8+\alpha_8.r_1'$	$z_8+\alpha_8.z_1'$
$e_9+\alpha_9.e_1'$	$f_9+\alpha_9.f_1'$	$g_9+\alpha_9.g_1'$	$h_9+\alpha_9.h_1'$	$l_9+\alpha_9.l_1'$	$m_9+\alpha_9.m_1'$	$n_9+\alpha_9.n_1'$	$q_9+\alpha_9.q_1'$	$r_9+\alpha_9.r_1'$	$z_9+\alpha_9.z_1'$
$-e_2'\div\beta_2'$	$-f_2'\div\beta_2'$	$-g_2'\div\beta_2'$	$-h_2'\div\beta_2'$	$-l_2'\div\beta_2'$	$-m_2'\div\beta_2'$	$-n_2'\div\beta_2'$	$-q_2'\div\beta_2'$	$-r_2'\div\beta_2'$	$-z_2'\div\beta_2'$
$e_3'+\beta_3'.e_2''$	$f_3'+\beta_3'.f_2''$	$g_3'+\beta_3'.g_2''$	$h_3'+\beta_3'.h_2''$	$l_3'+\beta_3'.l_2''$	$m_3'+\beta_3'.m_2''$	$n_3'+\beta_3'.n_2''$	$q_3'+\beta_3'.q_2''$	$r_3'+\beta_3'.r_2''$	$z_3'+\beta_3'.z_2''$
$e_4'+\beta_4'.e_2''$	$f_4'+\beta_4'.f_2''$	$g_4'+\beta_4'.g_2''$	$h_4'+\beta_4'.h_2''$	$l_4'+\beta_4'.l_2''$	$m_4'+\beta_4'.m_2''$	$n_4'+\beta_4'.n_2''$	$q_4'+\beta_4'.q_2''$	$r_4'+\beta_4'.r_2''$	$z_4'+\beta_4'.z_2''$
$e_5'+\beta_5'.e_2''$	$f_5'+\beta_5'.f_2''$	$g_5'+\beta_5'.g_2''$	$h_5'+\beta_5'.h_2''$	$l_5'+\beta_5'.l_2''$	$m_5'+\beta_5'.m_2''$	$n_5'+\beta_5'.n_2''$	$q_5'+\beta_5'.q_2''$	$r_5'+\beta_5'.r_2''$	$z_5'+\beta_5'.z_2''$
$e_6'+\beta_6'.e_2''$	$f_6'+\beta_6'.f_2''$	$g_6'+\beta_6'.g_2''$	$h_6'+\beta_6'.h_2''$	$l_6'+\beta_6'.l_2''$	$m_6'+\beta_6'.m_2''$	$n_6'+\beta_6'.n_2''$	$q_6'+\beta_6'.q_2''$	$r_6'+\beta_6'.r_2''$	$z_6'+\beta_6'.z_2''$
$e_7'+\beta_7'.e_2''$	$f_7'+\beta_7'.f_2''$	$g_7'+\beta_7'.g_2''$	$h_7'+\beta_7'.h_2''$	$l_7'+\beta_7'.l_2''$	$m_7'+\beta_7'.m_2''$	$n_7'+\beta_7'.n_2''$	$q_7'+\beta_7'.q_2''$	$r_7'+\beta_7'.r_2''$	$z_7'+\beta_7'.z_2''$
$e_8'+\beta_8'.e_2''$	$f_8'+\beta_8'.f_2''$	$g_8'+\beta_8'.g_2''$	$h_8'+\beta_8'.h_2''$	$l_8'+\beta_8'.l_2''$	$m_8'+\beta_8'.m_2''$	$n_8'+\beta_8'.n_2''$	$q_8'+\beta_8'.q_2''$	$r_8'+\beta_8'.r_2''$	$z_8'+\beta_8'.z_2''$
$e_9'+\beta_9'.e_2''$	$f_9'+\beta_9'.f_2''$	$g_9'+\beta_9'.g_2''$	$h_9'+\beta_9'.h_2''$	$l_9'+\beta_9'.l_2''$	$m_9'+\beta_9'.m_2''$	$n_9'+\beta_9'.n_2''$	$q_9'+\beta_9'.q_2''$	$r_9'+\beta_9'.r_2''$	$z_9'+\beta_9'.z_2''$
$-e_3''\div\gamma_3''$	$-f_3''\div\gamma_3''$	$-g_3''\div\gamma_3''$	$-h_3''\div\gamma_3''$	$-l_3''\div\gamma_3''$	$-m_3''\div\gamma_3''$	$-n_3''\div\gamma_3''$	$-q_3''\div\gamma_3''$	$-r_3''\div\gamma_3''$	$-z_3''\div\gamma_3''$
$e_4''+\gamma_4''.e_3'''$	$f_4''+\gamma_4''.f_3'''$	$g_4''+\gamma_4''.g_3'''$	$h_4''+\gamma_4''.h_3'''$	$l_4''+\gamma_4''.l_3'''$	$m_4''+\gamma_4''.m_3'''$	$n_4''+\gamma_4''.n_3'''$	$q_4''+\gamma_4''.q_3'''$	$r_4''+\gamma_4''.r_3'''$	$z_4''+\gamma_4''.z_3'''$
$e_5''+\gamma_5''.e_3'''$	$f_5''+\gamma_5''.f_3'''$	$g_5''+\gamma_5''.g_3'''$	$h_5''+\gamma_5''.h_3'''$	$l_5''+\gamma_5''.l_3'''$	$m_5''+\gamma_5''.m_3'''$	$n_5''+\gamma_5''.n_3'''$	$q_5''+\gamma_5''.q_3'''$	$r_5''+\gamma_5''.r_3'''$	$z_5''+\gamma_5''.z_3'''$
$e_6''+\gamma_6''.e_3'''$	$f_6''+\gamma_6''.f_3'''$	$g_6''+\gamma_6''.g_3'''$	$h_6''+\gamma_6''.h_3'''$	$l_6''+\gamma_6''.l_3'''$	$m_6''+\gamma_6''.m_3'''$	$n_6''+\gamma_6''.n_3'''$	$q_6''+\gamma_6''.q_3'''$	$r_6''+\gamma_6''.r_3'''$	$z_6''+\gamma_6''.z_3'''$
$e_7''+\gamma_7''.e_3'''$	$f_7''+\gamma_7''.f_3'''$	$g_7''+\gamma_7''.g_3'''$	$h_7''+\gamma_7''.h_3'''$	$l_7''+\gamma_7''.l_3'''$	$m_7''+\gamma_7''.m_3'''$	$n_7''+\gamma_7''.n_3'''$	$q_7''+\gamma_7''.q_3'''$	$r_7''+\gamma_7''.r_3'''$	$z_7''+\gamma_7''.z_3'''$
$e_8''+\gamma_8''.e_3'''$	$f_8''+\gamma_8''.f_3'''$	$g_8''+\gamma_8''.g_3'''$	$h_8''+\gamma_8''.h_3'''$	$l_8''+\gamma_8''.l_3'''$	$m_8''+\gamma_8''.m_3'''$	$n_8''+\gamma_8''.n_3'''$	$q_8''+\gamma_8''.q_3'''$	$r_8''+\gamma_8''.r_3'''$	$z_8''+\gamma_8''.z_3'''$
$e_9''+\gamma_9''.e_3'''$	$f_9''+\gamma_9''.f_3'''$	$g_9''+\gamma_9''.g_3'''$	$h_9''+\gamma_9''.h_3'''$	$l_9''+\gamma_9''.l_3'''$	$m_9''+\gamma_9''.m_3'''$	$n_9''+\gamma_9''.n_3'''$	$q_9''+\gamma_9''.q_3'''$	$r_9''+\gamma_9''.r_3'''$	$z_9''+\gamma_9''.z_3'''$
$-e_4'''\div\delta_4'''$	$-f_4'''\div\delta_4'''$	$-g_4'''\div\delta_4'''$	$-h_4'''\div\delta_4'''$	$-l_4'''\div\delta_4'''$	$-m_4'''\div\delta_4'''$	$-n_4'''\div\delta_4'''$	$-q_4'''\div\delta_4'''$	$-r_4'''\div\delta_4'''$	$-z_4'''\div\delta_4'''$
$e_5'''+\delta_5'''.e_4^{iv}$	$f_5'''+\delta_5'''.f_4^{iv}$	$g_5'''+\delta_5'''.g_4^{iv}$	$h_5'''+\delta_5'''.h_4^{iv}$	$l_5'''+\delta_5'''.l_4^{iv}$	$m_5'''+\delta_5'''.m_4^{iv}$	$n_5'''+\delta_5'''.n_4^{iv}$	$q_5'''+\delta_5'''.q_4^{iv}$	$r_5'''+\delta_5'''.r_4^{iv}$	$z_5'''+\delta_5'''.z_4^{iv}$
$e_6'''+\delta_6'''.e_4^{iv}$	$f_6'''+\delta_6'''.f_4^{iv}$	$g_6'''+\delta_6'''.g_4^{iv}$	$h_6'''+\delta_6'''.h_4^{iv}$	$l_6'''+\delta_6'''.l_4^{iv}$	$m_6'''+\delta_6'''.m_4^{iv}$	$n_6'''+\delta_6'''.n_4^{iv}$	$q_6'''+\delta_6'''.q_4^{iv}$	$r_6'''+\delta_6'''.r_4^{iv}$	$z_6'''+\delta_6'''.z_4^{iv}$
$e_7'''+\delta_7'''.e_4^{iv}$	$f_7'''+\delta_7'''.f_4^{iv}$	$g_7'''+\delta_7'''.g_4^{iv}$	$h_7'''+\delta_7'''.h_4^{iv}$	$l_7'''+\delta_7'''.l_4^{iv}$	$m_7'''+\delta_7'''.m_4^{iv}$	$n_7'''+\delta_7'''.n_4^{iv}$	$q_7'''+\delta_7'''.q_4^{iv}$	$r_7'''+\delta_7'''.r_4^{iv}$	$z_7'''+\delta_7'''.z_4^{iv}$
$e_8'''+\delta_8'''.e_4^{iv}$	$f_8'''+\delta_8'''.f_4^{iv}$	$g_8'''+\delta_8'''.g_4^{iv}$	$h_8'''+\delta_8'''.h_4^{iv}$	$l_8'''+\delta_8'''.l_4^{iv}$	$m_8'''+\delta_8'''.m_4^{iv}$	$n_8'''+\delta_8'''.n_4^{iv}$	$q_8'''+\delta_8'''.q_4^{iv}$	$r_8'''+\delta_8'''.r_4^{iv}$	$z_8'''+\delta_8'''.z_4^{iv}$
$e_9'''+\delta_9'''.e_4^{iv}$	$f_9'''+\delta_9'''.f_4^{iv}$	$g_9'''+\delta_9'''.g_4^{iv}$	$h_9'''+\delta_9'''.h_4^{iv}$	$l_9'''+\delta_9'''.l_4^{iv}$	$m_9'''+\delta_9'''.m_4^{iv}$	$n_9'''+\delta_9'''.n_4^{iv}$	$q_9'''+\delta_9'''.q_4^{iv}$	$r_9'''+\delta_9'''.r_4^{iv}$	$z_9'''+\delta_9'''.z_4^{iv}$
$-e_5^{iv}\div\theta_5^{iv}$	$f_5^{iv}\div\theta_5^{iv}$	$-g_5^{iv}\div\theta_5^{iv}$	$-h_5^{iv}\div\theta_5^{iv}$	$-l_5^{iv}\div\theta_5^{iv}$	$-m_5^{iv}\div\theta_5^{iv}$	$-n_5^{iv}\div\theta_5^{iv}$	$-q_5^{iv}\div\theta_5^{iv}$	$-r_5^{iv}\div\theta_5^{iv}$	$-z_5^{iv}\div\theta_5^{iv}$
$e_6^{iv}+\theta_6^{iv}.e_5^{v}$	$f_6^{iv}+\theta_6^{iv}.f_5^{v}$	$g_6^{iv}+\theta_6^{iv}.g_5^{v}$	$h_6^{iv}+\theta_6^{iv}.h_5^{v}$	$l_6^{iv}+\theta_6^{iv}.l_5^{v}$	$m_6^{iv}+\theta_6^{iv}.m_5^{v}$	$n_6^{iv}+\theta_6^{iv}.n_5^{v}$	$q_6^{iv}+\theta_6^{iv}.q_5^{v}$	$r_6^{iv}+\theta_6^{iv}.r_5^{v}$	$z_6^{iv}+\theta_6^{iv}.z_5^{v}$
$e_7^{iv}+\theta_7^{iv}.e_5^{v}$	$f_7^{iv}+\theta_7^{iv}.f_5^{v}$	$g_7^{iv}+\theta_7^{iv}.g_5^{v}$	$h_7^{iv}+\theta_7^{iv}.h_5^{v}$	$l_7^{iv}+\theta_7^{iv}.l_5^{v}$	$m_7^{iv}+\theta_7^{iv}.m_5^{v}$	$n_7^{iv}+\theta_7^{iv}.n_5^{v}$	$q_7^{iv}+\theta_7^{iv}.q_5^{v}$	$r_7^{iv}+\theta_7^{iv}.r_5^{v}$	$z_7^{iv}+\theta_7^{iv}.z_5^{v}$
$e_8^{iv}+\theta_8^{iv}.e_5^{v}$	$f_8^{iv}+\theta_8^{iv}.f_5^{v}$	$g_8^{iv}+\theta_8^{iv}.g_5^{v}$	$h_8^{iv}+\theta_8^{iv}.h_5^{v}$	$l_8^{iv}+\theta_8^{iv}.l_5^{v}$	$m_8^{iv}+\theta_8^{iv}.m_5^{v}$	$n_8^{iv}+\theta_8^{iv}.n_5^{v}$	$q_8^{iv}+\theta_8^{iv}.q_5^{v}$	$r_8^{iv}+\theta_8^{iv}.r_5^{v}$	$z_8^{iv}+\theta_8^{iv}.z_5^{v}$
$e_9^{iv}+\theta_9^{iv}.e_5^{v}$	$f_9^{iv}+\theta_9^{iv}.f_5^{v}$	$g_9^{iv}+\theta_9^{iv}.g_5^{v}$	$h_9^{iv}+\theta_9^{iv}.h_5^{v}$	$l_9^{iv}+\theta_9^{iv}.l_5^{v}$	$m_9^{iv}+\theta_9^{iv}.m_5^{v}$	$n_9^{iv}+\theta_9^{iv}.n_5^{v}$	$q_9^{iv}+\theta_9^{iv}.q_5^{v}$	$r_9^{iv}+\theta_9^{iv}.r_5^{v}$	$z_9^{iv}+\theta_9^{iv}.z_5^{v}$
$-e_6^{v}\div\phi_6^{v}$	$-f_6^{v}\div\phi_6^{v}$	$-g_6^{v}\div\phi_6^{v}$	$-h_6^{v}\div\phi_6^{v}$	$-l_6^{v}\div\phi_6^{v}$	$-m_6^{v}\div\phi_6^{v}$	$-n_6^{v}\div\phi_6^{v}$	$-q_6^{v}\div\phi_6^{v}$	$-r_6^{v}\div\phi_6^{v}$	$-z_6^{v}\div\phi_6^{v}$
$e_7^{v}+\phi_7^{v}.e_6^{vi}$	$f_7^{v}+\phi_7^{v}.f_6^{vi}$	$g_7^{v}+\phi_7^{v}.g_6^{vi}$	$h_7^{v}+\phi_7^{v}.h_6^{vi}$	$l_7^{v}+\phi_7^{v}.l_6^{vi}$	$m_7^{v}+\phi_7^{v}.m_6^{vi}$	$n_7^{v}+\phi_7^{v}.n_6^{vi}$	$q_7^{v}+\phi_7^{v}.q_6^{vi}$	$r_7^{v}+\phi_7^{v}.r_6^{vi}$	$z_7^{v}+\phi_7^{v}.z_6^{vi}$
$e_8^{v}+\phi_8^{v}.e_6^{vi}$	$f_8^{v}+\phi_8^{v}.f_6^{vi}$	$g_8^{v}+\phi_8^{v}.g_6^{vi}$	$h_8^{v}+\phi_8^{v}.h_6^{vi}$	$l_8^{v}+\phi_8^{v}.l_6^{vi}$	$m_8^{v}+\phi_8^{v}.m_6^{vi}$	$n_8^{v}+\phi_8^{v}.n_6^{vi}$	$q_8^{v}+\phi_8^{v}.q_6^{vi}$	$r_8^{v}+\phi_8^{v}.r_6^{vi}$	$z_8^{v}+\phi_8^{v}.z_6^{vi}$
$e_9^{v}+\phi_9^{v}.e_6^{vi}$	$f_9^{v}+\phi_9^{v}.f_6^{vi}$	$g_9^{v}+\phi_9^{v}.g_6^{vi}$	$h_9^{v}+\phi_9^{v}.h_6^{vi}$	$l_9^{v}+\phi_9^{v}.l_6^{vi}$	$m_9^{v}+\phi_9^{v}.m_6^{vi}$	$n_9^{v}+\phi_9^{v}.n_6^{vi}$	$q_9^{v}+\phi_9^{v}.q_6^{vi}$	$r_9^{v}+\phi_9^{v}.r_6^{vi}$	$z_9^{v}+\phi_9^{v}.z_6^{vi}$
$-e_7^{vi}\div\omega_7^{vi}$	$-f_7^{vi}\div\omega_7^{vi}$	$-g_7^{vi}\div\omega_7^{vi}$	$-h_7^{vi}\div\omega_7^{vi}$	$-l_7^{vi}\div\omega_7^{vi}$	$-m_7^{vi}\div\omega_7^{vi}$	$-n_7^{vi}\div\omega_7^{vi}$	$-q_7^{vi}\div\omega_7^{vi}$	$-r_7^{vi}\div\omega_7^{vi}$	$-z_7^{vi}\div\omega_7^{vi}$
$e_8^{vi}+\omega_8^{vi}.e_7^{vii}$	$f_8^{vi}+\omega_8^{vi}.f_7^{vii}$	$g_8^{vi}+\omega_8^{vi}.g_7^{vii}$	$h_8^{vi}+\omega_8^{vi}.h_7^{vii}$	$l_8^{vi}+\omega_8^{vi}.l_7^{vii}$	$m_8^{vi}+\omega_8^{vi}.m_7^{vii}$	$n_8^{vi}+\omega_8^{vi}.n_7^{vii}$	$q_8^{vi}+\omega_8^{vi}.q_7^{vii}$	$r_8^{vi}+\omega_8^{vi}.r_7^{vii}$	$z_8^{vi}+\omega_8^{vi}.z_7^{vii}$
$e_9^{vi}+\omega_9^{vi}.e_7^{vii}$	$f_9^{vi}+\omega_9^{vi}.f_7^{vii}$	$g_9^{vi}+\omega_9^{vi}.g_7^{vii}$	$h_9^{vi}+\omega_9^{vi}.h_7^{vii}$	$l_9^{vi}+\omega_9^{vi}.l_7^{vii}$	$m_9^{vi}+\omega_9^{vi}.m_7^{vii}$	$n_9^{vi}+\omega_9^{vi}.n_7^{vii}$	$q_9^{vi}+\omega_9^{vi}.q_7^{vii}$	$r_9^{vi}+\omega_9^{vi}.r_7^{vii}$	$z_9^{vi}+\omega_9^{vi}.z_7^{vii}$
$-e_8^{vii}\div\varepsilon_8^{vii}$	$-f_8^{vii}\div\varepsilon_8^{vii}$	$-g_8^{vii}\div\varepsilon_8^{vii}$	$-h_8^{vii}\div\varepsilon_8^{vii}$	$-l_8^{vii}\div\varepsilon_8^{vii}$	$-m_8^{vii}\div\varepsilon_8^{vii}$	$-n_8^{vii}\div\varepsilon_8^{vii}$	$-q_8^{vii}\div\varepsilon_8^{vii}$	$-r_8^{vii}\div\varepsilon_8^{vii}$	$-z_8^{vii}\div\varepsilon_8^{vii}$
$e_9^{vii}+\varepsilon_9^{vii}.e_8^{viii}$	$f_9^{vii}+\varepsilon_9^{vii}.f_8^{viii}$	$g_9^{vii}+\varepsilon_9^{vii}.g_8^{viii}$	$h_9^{vii}+\varepsilon_9^{vii}.h_8^{viii}$	$l_9^{vii}+\varepsilon_9^{vii}.l_8^{viii}$	$m_9^{vii}+\varepsilon_9^{vii}.m_8^{viii}$	$n_9^{vii}+\varepsilon_9^{vii}.n_8^{viii}$	$q_9^{vii}+\varepsilon_9^{vii}.q_8^{viii}$	$r_9^{vii}+\varepsilon_9^{vii}.r_8^{viii}$	$z_9^{vii}+\varepsilon_9^{vii}.z_8^{viii}$
$-e_9^{viii}\div\lambda_9^{viii}$	$-f_9^{viii}\div\lambda_9^{viii}$	$-g_9^{viii}\div\lambda_9^{viii}$	$-h_9^{viii}\div\lambda_9^{viii}$	$-l_9^{viii}\div\lambda_9^{viii}$	$-m_9^{viii}\div\lambda_9^{viii}$	$-n_9^{viii}\div\lambda_9^{viii}$	$-q_9^{viii}\div\lambda_9^{viii}$	$-r_9^{viii}\div\lambda_9^{viii}$	$-z_9^{viii}\div\lambda_9^{viii}$

PLATE 12.

set down as calculated. Such a table with the values of all these coefficients is given in Plates 11 and 12. These plates are applicable to the solution for all stories except, of course, the coefficients in the first nine equations *A–a–*1, *A–b–*1, etc., values of which for the first and succeeding stories are discussed later. These calculations are of the very simplest kind and, with a Thacher rule, may be rapidly made.

In Plate 12 the term ω is expanded into its constituent terms, *i. e.*, into first story unknowns and a numerical term z. After calculating and tabulating the quantities as shown in Plates 11 and 12 it is seen that the bottom line gives V_0^C in terms of the terms comprised in ω only, *i. e.*, in first story unknowns and a numerical term. Now, by substituting this value of V_0^C in the value of V_0^B given on Plate 10, this quantity may also be found in terms of first story unknowns only. Then, using this value of V_0^B and the above value of V_0^C, substitute in the expression for S_0^C in Plate 10; continue thus until all the 0 story unknowns are given in terms of first story unknowns and the quantities z. The values of the coefficients of the first story unknowns and the numerical quantities, involved in these values of the 0 story unknowns, are given on Plate 13. Thus for the value of M_0^D, the coefficient of S_1^A is given in the column headed M_0^D and in the horizontal row marked on the left S_1^A. All the coefficients for any unknown are given in the vertical column headed by this unknown. The tables on Plate 13 have also been extended so as to give the coefficients for the values of C_1^A, C_1^B, C_1^C, C_1^D, Δ_0^a, Δ_0^b, Δ_0^d, C_2^A, C_2^B, C_2^C, C_2^D, Δ_1^a, Δ_1^b, and Δ_1^d, in terms of first story unknowns only. It is important to note the new symbol here introduced; for example $m^{\Delta-b}$ *with no subscript* means the value of the coefficient of S_1^B in the value of Δ_0^b when expressed in terms of first story unknowns only; also l^{V-B} represents the value of the coefficient of S_1^A in the expression for V_0^B. But quantities with subscripts as $o_1^{\Delta-b}$ represent the coefficients as already computed in plates 4, 5, 6, 7, and 8. These new coefficients may be properly associated by remembering that the letters are as in the original symbolic equations: thus e is a coefficient of M_1^A, l of S_1^A, m of s_1^B, etc.; and the exponents, as $\Delta - b$ for instance, refer to the quantity for the 0 story in the expression for the value of which the new



	V_0^C	V_0^B	S_0^C	S_0^B	S_0^A	M_0^D	M_0^C
M_1^A	e_9^{ix}	$e_8^{viii}+\lambda_8^{viii}.e^{V-C}$	$e_7^{vii}+\varepsilon_7^{vii}.e^{V-B}+\lambda_7^{vii}.e^{V-C}$	$e_6^{vi}+\psi_6^{vi}.e^{S-C}+\varepsilon_6^{vi}.e^{V-B}+\lambda_6^{vi}.e^{V-C}$	$e_5^{v}+\phi_5^{v}.e^{S-B}+\psi_5^{v}.e^{S-C}+\varepsilon_5^{v}.e^{V-B}+\lambda_5^{v}.e^{V-C}$	$e_4^{iv}+\theta_4^{iv}.e^{S-A}+\phi_4^{iv}.e^{S-B}+\psi_4^{iv}.e^{S-C}+\varepsilon_4^{iv}.e^{V-B}+\lambda_4^{iv}.e^{V-C}$	$e_3'''+\delta_3'''.e^{M-D}+\theta_3'''.e^{S-A}+\phi_3'''.e^{S-B}+\psi_3'''.e^{S-C}+\varepsilon_3'''.e^{V-B}+\lambda_3'''.e^{V-C}$
M_1^B	f_9^{ix}	$f_8^{viii}+\lambda_8^{viii}.f^{V-C}$	$f_7^{vii}+\varepsilon_7^{vii}.f^{V-B}+\lambda_7^{vii}.f^{V-C}$	$f_6^{vi}+\psi_6^{vi}.f^{S-C}+\varepsilon_6^{vi}.f^{V-B}+\lambda_6^{vi}.f^{V-C}$	$f_5^{v}+\phi_5^{v}.f^{S-B}+\psi_5^{v}.f^{S-C}+\varepsilon_5^{v}.f^{V-B}+\lambda_5^{v}.f^{V-C}$	$f_4^{iv}+\theta_4^{iv}.f^{S-A}+\phi_4^{iv}.f^{S-B}+\psi_4^{iv}.f^{S-C}+\varepsilon_4^{iv}.f^{V-B}+\lambda_4^{iv}.f^{V-C}$	$f_3'''+\delta_3'''.f^{M-D}+\theta_3'''.f^{S-A}+\phi_3'''.f^{S-B}+\psi_3'''.f^{S-C}+\varepsilon_3'''.f^{V-B}+\lambda_3'''.f^{V-C}$
M_1^C	g_9^{ix}	$g_8^{viii}+\lambda_8^{viii}.g^{V-C}$	$g_7^{vii}+\varepsilon_7^{vii}.g^{V-B}+\lambda_7^{vii}.g^{V-C}$	$g_6^{vi}+\psi_6^{vi}.g^{S-C}+\varepsilon_6^{vi}.g^{V-B}+\lambda_6^{vi}.g^{V-C}$	$g_5^{v}+\phi_5^{v}.g^{S-B}+\psi_5^{v}.g^{S-C}+\varepsilon_5^{v}.g^{V-B}+\lambda_5^{v}.g^{V-C}$	$g_4^{iv}+\theta_4^{iv}.g^{S-A}+\phi_4^{iv}.g^{S-B}+\psi_4^{iv}.g^{S-C}+\varepsilon_4^{iv}.g^{V-B}+\lambda_4^{iv}.g^{V-C}$	$g_3'''+\delta_3'''.g^{M-D}+\theta_3'''.g^{S-A}+\phi_3'''.g^{S-B}+\psi_3'''.g^{S-C}+\varepsilon_3'''.g^{V-B}+\lambda_3'''.g^{V-C}$
M_1^D	h_9^{ix}	$h_8^{viii}+\lambda_8^{viii}.h^{V-C}$	$h_7^{vii}+\varepsilon_7^{vii}.h^{V-B}+\lambda_7^{vii}.h^{V-C}$	$h_6^{vi}+\psi_6^{vi}.h^{S-C}+\varepsilon_6^{vi}.h^{V-B}+\lambda_6^{vi}.h^{V-C}$	$h_5^{v}+\phi_5^{v}.h^{S-B}+\psi_5^{v}.h^{S-C}+\varepsilon_5^{v}.h^{V-B}+\lambda_5^{v}.h^{V-C}$	$h_4^{iv}+\theta_4^{iv}.h^{S-A}+\phi_4^{iv}.h^{S-B}+\psi_4^{iv}.h^{S-C}+\varepsilon_4^{iv}.h^{V-B}+\lambda_4^{iv}.h^{V-C}$	$h_3'''+\delta_3'''.h^{M-D}+\theta_3'''.h^{S-A}+\phi_3'''.h^{S-B}+\psi_3'''.h^{S-C}+\varepsilon_3'''.h^{V-B}+\lambda_3'''.h^{V-C}$
S_1^A	l_9^{ix}	$l_8^{viii}+\lambda_8^{viii}.l^{V-C}$	$l_7^{vii}+\varepsilon_7^{vii}.l^{V-B}+\lambda_7^{vii}.l^{V-C}$	$l_6^{vi}+\psi_6^{vi}.l^{S-C}+\varepsilon_6^{vi}.l^{V-B}+\lambda_6^{vi}.l^{V-C}$	$l_5^{v}+\phi_5^{v}.l^{S-B}+\psi_5^{v}.l^{S-C}+\varepsilon_5^{v}.l^{V-B}+\lambda_5^{v}.l^{V-C}$	$l_4^{iv}+\theta_4^{iv}.l^{S-A}+\phi_4^{iv}.l^{S-B}+\psi_4^{iv}.l^{S-C}+\varepsilon_4^{iv}.l^{V-B}+\lambda_4^{iv}.l^{V-C}$	$l_3'''+\delta_3'''.l^{M-D}+\theta_3'''.l^{S-A}+\phi_3'''.l^{S-B}+\psi_3'''.l^{S-C}+\varepsilon_3'''.l^{V-B}+\lambda_3'''.l^{V-C}$
S_1^B	m_9^{ix}	$m_8^{viii}+\lambda_8^{viii}.m^{V-C}$	$m_7^{vii}+\varepsilon_7^{vii}.m^{V-B}+\lambda_7^{vii}.m^{V-C}$	$m_6^{vi}+\psi_6^{vi}.m^{S-C}+\varepsilon_6^{vi}.m^{V-B}+\lambda_6^{vi}.m^{V-C}$	$m_5^{v}+\phi_5^{v}.m^{S-B}+\psi_5^{v}.m^{S-C}+\varepsilon_5^{v}.m^{V-B}+\lambda_5^{v}.m^{V-C}$	$m_4^{iv}+\theta_4^{iv}.m^{S-A}+\phi_4^{iv}.m^{S-B}+\psi_4^{iv}.m^{S-C}+\varepsilon_4^{iv}.m^{V-B}+\lambda_4^{iv}.m^{V-C}$	$m_3'''+\delta_3'''.m^{M-D}+\theta_3'''.m^{S-A}+\phi_3'''.m^{S-B}+\psi_3'''.m^{S-C}+\varepsilon_3'''.m^{V-B}+\lambda_3'''.m^{V-C}$
S_1^C	n_9^{ix}	$n_8^{viii}+\lambda_8^{viii}.n^{V-C}$	$n_7^{vii}+\varepsilon_7^{vii}.n^{V-B}+\lambda_7^{vii}.n^{V-C}$	$n_6^{vi}+\psi_6^{vi}.n^{S-C}+\varepsilon_6^{vi}.n^{V-B}+\lambda_6^{vi}.n^{V-C}$	$n_5^{v}+\phi_5^{v}.n^{S-B}+\psi_5^{v}.n^{S-C}+\varepsilon_5^{v}.n^{V-B}+\lambda_5^{v}.n^{V-C}$	$n_4^{iv}+\theta_4^{iv}.n^{S-A}+\phi_4^{iv}.n^{S-B}+\psi_4^{iv}.n^{S-C}+\varepsilon_4^{iv}.n^{V-B}+\lambda_4^{iv}.n^{V-C}$	$n_3'''+\delta_3'''.n^{M-D}+\theta_3'''.n^{S-A}+\phi_3'''.n^{S-B}+\psi_3'''.n^{S-C}+\varepsilon_3'''.n^{V-B}+\lambda_3'''.n^{V-C}$
V_1^B	q_9^{ix}	$q_8^{viii}+\lambda_8^{viii}.q^{V-C}$	$q_7^{vii}+\varepsilon_7^{vii}.q^{V-B}+\lambda_7^{vii}.q^{V-C}$	$q_6^{vi}+\psi_6^{vi}.q^{S-C}+\varepsilon_6^{vi}.q^{V-B}+\lambda_6^{vi}.q^{V-C}$	$q_5^{v}+\phi_5^{v}.q^{S-B}+\psi_5^{v}.q^{S-C}+\varepsilon_5^{v}.q^{V-B}+\lambda_5^{v}.q^{V-C}$	$q_4^{iv}+\theta_4^{iv}.q^{S-A}+\phi_4^{iv}.q^{S-B}+\psi_4^{iv}.q^{S-C}+\varepsilon_4^{iv}.q^{V-B}+\lambda_4^{iv}.q^{V-C}$	$q_3'''+\delta_3'''.q^{M-D}+\theta_3'''.q^{S-A}+\phi_3'''.q^{S-B}+\psi_3'''.q^{S-C}+\varepsilon_3'''.q^{V-B}+\lambda_3'''.q^{V-C}$
V_1^C	r_9^{ix}	$r_8^{viii}+\lambda_8^{viii}.r^{V-C}$	$r_7^{vii}+\varepsilon_7^{vii}.r^{V-B}+\lambda_7^{vii}.r^{V-C}$	$r_6^{vi}+\psi_6^{vi}.r^{S-C}+\varepsilon_6^{vi}.r^{V-B}+\lambda_6^{vi}.r^{V-C}$	$r_5^{v}+\phi_5^{v}.r^{S-B}+\psi_5^{v}.r^{S-C}+\varepsilon_5^{v}.r^{V-B}+\lambda_5^{v}.r^{V-C}$	$r_4^{iv}+\theta_4^{iv}.r^{S-A}+\phi_4^{iv}.r^{S-B}+\psi_4^{iv}.r^{S-C}+\varepsilon_4^{iv}.r^{V-B}+\lambda_4^{iv}.r^{V-C}$	$r_3'''+\delta_3'''.r^{M-D}+\theta_3'''.r^{S-A}+\phi_3'''.r^{S-B}+\psi_3'''.r^{S-C}+\varepsilon_3'''.r^{V-B}+\lambda_3'''.r^{V-C}$
3	3_9^{ix}	$3_8^{viii}+\lambda_8^{viii}.3^{V-C}$	$3_7^{vii}+\varepsilon_7^{vii}.3^{V-B}+\lambda_7^{vii}.3^{V-C}$	$3_6^{vi}+\psi_6^{vi}.3^{S-C}+\varepsilon_6^{vi}.3^{V-B}+\lambda_6^{vi}.3^{V-C}$	$3_5^{v}+\phi_5^{v}.3^{S-B}+\psi_5^{v}.3^{S-C}+\varepsilon_5^{v}.3^{V-B}+\lambda_5^{v}.3^{V-C}$	$3_4^{iv}+\theta_4^{iv}.3^{S-A}+\phi_4^{iv}.3^{S-B}+\psi_4^{iv}.3^{S-C}+\varepsilon_4^{iv}.3^{V-B}+\lambda_4^{iv}.3^{V-C}$	$3_3'''+\delta_3'''.3^{M-D}+\theta_3'''.3^{S-A}+\phi_3'''.3^{S-B}+\psi_3'''.3^{S-C}+\varepsilon_3'''.3^{V-B}+\lambda_3'''.3^{V-C}$

	M_0^B	M_0^A	C_1^A	C_1^B	C_1^C
M_1^A	$e_2''+\gamma_2''.e^{M-C}+\delta_2''.e^{M-D}+\theta_2''.e^{S-A}+\phi_2''.e^{S-B}+\psi_2''.e^{S-C}+\varepsilon_2''.e^{V-B}+\lambda_2''.e^{V-C}$	$e_1'+\beta_1'.e^{M-B}+\gamma_1'.e^{M-C}+\delta_1'.e^{M-D}+\theta_1'.e^{S-A}+\phi_1'.e^{S-B}+\psi_1'.e^{S-C}+\varepsilon_1'.e^{V-B}+\lambda_1'.e^{V-C}$	$a_0^{C-A}.e^{M-A}+i_0^{C-A}.e^{S-A}$	$b_0^{C-B}.e^{M-B}+j_0^{C-B}.e^{S-B}$	$c_0^{C-C}.e^{M-C}+k_0^{C-C}.e^{S-C}$
M_1^B	$f_2''+\gamma_2''.f^{M-C}+\delta_2''.f^{M-D}+\theta_2''.f^{S-A}+\phi_2''.f^{S-B}+\psi_2''.f^{S-C}+\varepsilon_2''.f^{V-B}+\lambda_2''.f^{V-C}$	$f_1'+\beta_1'.f^{M-B}+\gamma_1'.f^{M-C}+\delta_1'.f^{M-D}+\theta_1'.f^{S-A}+\phi_1'.f^{S-B}+\psi_1'.f^{S-C}+\varepsilon_1'.f^{V-B}+\lambda_1'.f^{V-C}$	$a_0^{C-A}.f^{M-A}+i_0^{C-A}.f^{S-A}$	$b_0^{C-B}.f^{M-B}+j_0^{C-B}.f^{S-B}$	$c_0^{C-C}.f^{M-C}+k_0^{C-C}.f^{S-C}$
M_1^C	$g_2''+\gamma_2''.g^{M-C}+\delta_2''.g^{M-D}+\theta_2''.g^{S-A}+\phi_2''.g^{S-B}+\psi_2''.g^{S-C}+\varepsilon_2''.g^{V-B}+\lambda_2''.g^{V-C}$	$g_1'+\beta_1'.g^{M-B}+\gamma_1'.g^{M-C}+\delta_1'.g^{M-D}+\theta_1'.g^{S-A}+\phi_1'.g^{S-B}+\psi_1'.g^{S-C}+\varepsilon_1'.g^{V-B}+\lambda_1'.g^{V-C}$	$a_0^{C-A}.g^{M-A}+i_0^{C-A}.g^{S-A}$	$b_0^{C-B}.g^{M-B}+j_0^{C-B}.g^{S-B}$	$c_0^{C-C}.g^{M-C}+k_0^{C-C}.g^{S-C}$
M_1^D	$h_2''+\gamma_2''.h^{M-C}+\delta_2''.h^{M-D}+\theta_2''.h^{S-A}+\phi_2''.h^{S-B}+\psi_2''.h^{S-C}+\varepsilon_2''.h^{V-B}+\lambda_2''.h^{V-C}$	$h_1'+\beta_1'.h^{M-B}+\gamma_1'.h^{M-C}+\delta_1'.h^{M-D}+\theta_1'.h^{S-A}+\phi_1'.h^{S-B}+\psi_1'.h^{S-C}+\varepsilon_1'.h^{V-B}+\lambda_1'.h^{V-C}$	$a_0^{C-A}.h^{M-A}+i_0^{C-A}.h^{S-A}$	$b_0^{C-B}.h^{M-B}+j_0^{C-B}.h^{S-B}$	$c_0^{C-C}.h^{M-C}+k_0^{C-C}.h^{S-C}$
S_1^A	$l_2''+\gamma_2''.l^{M-C}+\delta_2''.l^{M-D}+\theta_2''.l^{S-A}+\phi_2''.l^{S-B}+\psi_2''.l^{S-C}+\varepsilon_2''.l^{V-B}+\lambda_2''.l^{V-C}$	$l_1'+\beta_1'.l^{M-B}+\gamma_1'.l^{M-C}+\delta_1'.l^{M-D}+\theta_1'.l^{S-A}+\phi_1'.l^{S-B}+\psi_1'.l^{S-C}+\varepsilon_1'.l^{V-B}+\lambda_1'.l^{V-C}$	$a_0^{C-A}.l^{M-A}+i_0^{C-A}.l^{S-A}$	$b_0^{C-B}.l^{M-B}+j_0^{C-B}.l^{S-B}$	$c_0^{C-C}.l^{M-C}+k_0^{C-C}.l^{S-C}$
S_1^B	$m_2''+\gamma_2''.m^{M-C}+\delta_2''.m^{M-D}+\theta_2''.m^{S-A}+\phi_2''.m^{S-B}+\psi_2''.m^{S-C}+\varepsilon_2''.m^{V-B}+\lambda_2''.m^{V-C}$	$m_1'+\beta_1'.m^{M-B}+\gamma_1'.m^{M-C}+\delta_1'.m^{M-D}+\theta_1'.m^{S-A}+\phi_1'.m^{S-B}+\psi_1'.m^{S-C}+\varepsilon_1'.m^{V-B}+\lambda_1'.m^{V-C}$	$a_0^{C-A}.m^{M-A}+i_0^{C-A}.m^{S-A}$	$b_0^{C-B}.m^{M-B}+j_0^{C-B}.m^{S-B}$	$c_0^{C-C}.m^{M-C}+k_0^{C-C}.m^{S-C}$
S_1^C	$n_2''+\gamma_2''.n^{M-C}+\delta_2''.n^{M-D}+\theta_2''.n^{S-A}+\phi_2''.n^{S-B}+\psi_2''.n^{S-C}+\varepsilon_2''.n^{V-B}+\lambda_2''.n^{V-C}$	$n_1'+\beta_1'.n^{M-B}+\gamma_1'.n^{M-C}+\delta_1'.n^{M-D}+\theta_1'.n^{S-A}+\phi_1'.n^{S-B}+\psi_1'.n^{S-C}+\varepsilon_1'.n^{V-B}+\lambda_1'.n^{V-C}$	$a_0^{C-A}.n^{M-A}+i_0^{C-A}.n^{S-A}$	$b_0^{C-B}.n^{M-B}+j_0^{C-B}.n^{S-B}$	$c_0^{C-C}.n^{M-C}+k_0^{C-C}.n^{S-C}$
V_1^B	$q_2''+\gamma_2''.q^{M-C}+\delta_2''.q^{M-D}+\theta_2''.q^{S-A}+\phi_2''.q^{S-B}+\psi_2''.q^{S-C}+\varepsilon_2''.q^{V-B}+\lambda_2''.q^{V-C}$	$q_1'+\beta_1'.q^{M-B}+\gamma_1'.q^{M-C}+\delta_1'.q^{M-D}+\theta_1'.q^{S-A}+\phi_1'.q^{S-B}+\psi_1'.q^{S-C}+\varepsilon_1'.q^{V-B}+\lambda_1'.q^{V-C}$	$a_0^{C-A}.q^{M-A}+i_0^{C-A}.q^{S-A}$	$b_0^{C-B}.q^{M-B}+j_0^{C-B}.q^{S-B}$	$c_0^{C-C}.q^{M-C}+k_0^{C-C}.q^{S-C}$
V_1^C	$r_2''+\gamma_2''.r^{M-C}+\delta_2''.r^{M-D}+\theta_2''.r^{S-A}+\phi_2''.r^{S-B}+\psi_2''.r^{S-C}+\varepsilon_2''.r^{V-B}+\lambda_2''.r^{V-C}$	$r_1'+\beta_1'.r^{M-B}+\gamma_1'.r^{M-C}+\delta_1'.r^{M-D}+\theta_1'.r^{S-A}+\phi_1'.r^{S-B}+\psi_1'.r^{S-C}+\varepsilon_1'.r^{V-B}+\lambda_1'.r^{V-C}$	$a_0^{C-A}.r^{M-A}+i_0^{C-A}.r^{S-A}$	$b_0^{C-B}.r^{M-B}+j_0^{C-B}.r^{S-B}$	$c_0^{C-C}.r^{M-C}+k_0^{C-C}.r^{S-C}$
3	$3_2''+\gamma_2''.3^{M-C}+\delta_2''.3^{M-D}+\theta_2''.3^{S-A}+\phi_2''.3^{S-B}+\psi_2''.3^{S-C}+\varepsilon_2''.3^{V-B}+\lambda_2''.3^{V-C}$	$3_1'+\beta_1'.3^{M-B}+\gamma_1'.3^{M-C}+\delta_1'.3^{M-D}+\theta_1'.3^{S-A}+\phi_1'.3^{S-B}+\psi_1'.3^{S-C}+\varepsilon_1'.3^{V-B}+\lambda_1'.3^{V-C}$	$a_0^{C-A}.3^{M-A}+i_0^{C-A}.3^{S-A}$	$b_0^{C-B}.3^{M-B}+j_0^{C-B}.3^{S-B}$	$c_0^{C-C}.3^{M-C}+k_0^{C-C}.3^{S-C}$

	C_1^D	Δ_0^a	Δ_0^b	Δ_0^d
M_1^A	$d_0^{C-D}.e^{M-D}+i_0^{C-D}.e^{S-A}+j_0^{C-D}.e^{S-B}+k_0^{C-D}.e^{S-C}$	$a_0^{\Delta-a}.e^{M-A}+b_0^{\Delta-a}.e^{M-B}+c_0^{\Delta-a}.e^{M-C}+d_0^{\Delta-a}.e^{M-D}+o_0^{\Delta-a}.e^{V-B}+p_0^{\Delta-a}.e^{V-C}$	$o_0^{\Delta-b}.e^{V-B}+p_0^{\Delta-b}.e^{V-C}$	$a_0^{\Delta-d}.e^{M-A}+b_0^{\Delta-d}.e^{M-B}+c_0^{\Delta-d}.e^{M-C}+d_0^{\Delta-d}.e^{M-D}+o_0^{\Delta-d}.e^{V-B}+p_0^{\Delta-d}.e^{V-C}$
M_1^B	$d_0^{C-D}.f^{M-D}+i_0^{C-D}.f^{S-A}+j_0^{C-D}.f^{S-B}+k_0^{C-D}.f^{S-C}$	$a_0^{\Delta-a}.f^{M-A}+b_0^{\Delta-a}.f^{M-B}+c_0^{\Delta-a}.f^{M-C}+d_0^{\Delta-a}.f^{M-D}+o_0^{\Delta-a}.f^{V-B}+p_0^{\Delta-a}.f^{V-C}$	$o_0^{\Delta-b}.f^{V-B}+p_0^{\Delta-b}.f^{V-C}$	$a_0^{\Delta-d}.f^{M-A}+b_0^{\Delta-d}.f^{M-B}+c_0^{\Delta-d}.f^{M-C}+d_0^{\Delta-d}.f^{M-D}+o_0^{\Delta-d}.f^{V-B}+p_0^{\Delta-d}.f^{V-C}$
M_1^C	$d_0^{C-D}.g^{M-D}+i_0^{C-D}.g^{S-A}+j_0^{C-D}.g^{S-B}+k_0^{C-D}.g^{S-C}$	$a_0^{\Delta-a}.g^{M-A}+b_0^{\Delta-a}.g^{M-B}+c_0^{\Delta-a}.g^{M-C}+d_0^{\Delta-a}.g^{M-D}+o_0^{\Delta-a}.g^{V-B}+p_0^{\Delta-a}.g^{V-C}$	$o_0^{\Delta-b}.g^{V-B}+p_0^{\Delta-b}.g^{V-C}$	$a_0^{\Delta-d}.g^{M-A}+b_0^{\Delta-d}.g^{M-B}+c_0^{\Delta-d}.g^{M-C}+d_0^{\Delta-d}.g^{M-D}+o_0^{\Delta-d}.g^{V-B}+p_0^{\Delta-d}.g^{V-C}$
M_1^D	$d_0^{C-D}.h^{M-D}+i_0^{C-D}.h^{S-A}+j_0^{C-D}.h^{S-B}+k_0^{C-D}.h^{S-C}$	$a_0^{\Delta-a}.h^{M-A}+b_0^{\Delta-a}.h^{M-B}+c_0^{\Delta-a}.h^{M-C}+d_0^{\Delta-a}.h^{M-D}+o_0^{\Delta-a}.h^{V-B}+p_0^{\Delta-a}.h^{V-C}$	$o_0^{\Delta-b}.h^{V-B}+p_0^{\Delta-b}.h^{V-C}$	$a_0^{\Delta-d}.h^{M-A}+b_0^{\Delta-d}.h^{M-B}+c_0^{\Delta-d}.h^{M-C}+d_0^{\Delta-d}.h^{M-D}+o_0^{\Delta-d}.h^{V-B}+p_0^{\Delta-d}.h^{V-C}$
S_1^A	$d_0^{C-D}.l^{M-D}+i_0^{C-D}.l^{S-A}+j_0^{C-D}.l^{S-B}+k_0^{C-D}.l^{S-C}$	$a_0^{\Delta-a}.l^{M-A}+b_0^{\Delta-a}.l^{M-B}+c_0^{\Delta-a}.l^{M-C}+d_0^{\Delta-a}.l^{M-D}+o_0^{\Delta-a}.l^{V-B}+p_0^{\Delta-a}.l^{V-C}$	$o_0^{\Delta-b}.l^{V-B}+p_0^{\Delta-b}.l^{V-C}$	$a_0^{\Delta-d}.l^{M-A}+b_0^{\Delta-d}.l^{M-B}+c_0^{\Delta-d}.l^{M-C}+d_0^{\Delta-d}.l^{M-D}+o_0^{\Delta-d}.l^{V-B}+p_0^{\Delta-d}.l^{V-C}$
S_1^B	$d_0^{C-D}.m^{M-D}+i_0^{C-D}.m^{S-A}+j_0^{C-D}.m^{S-B}+k_0^{C-D}.m^{S-C}$	$a_0^{\Delta-a}.m^{M-A}+b_0^{\Delta-a}.m^{M-B}+c_0^{\Delta-a}.m^{M-C}+d_0^{\Delta-a}.m^{M-D}+o_0^{\Delta-a}.m^{V-B}+p_0^{\Delta-a}.m^{V-C}$	$o_0^{\Delta-b}.m^{V-B}+p_0^{\Delta-b}.m^{V-C}$	$a_0^{\Delta-d}.m^{M-A}+b_0^{\Delta-d}.m^{M-B}+c_0^{\Delta-d}.m^{M-C}+d_0^{\Delta-d}.m^{M-D}+o_0^{\Delta-d}.m^{V-B}+p_0^{\Delta-d}.m^{V-C}$
S_1^C	$d_0^{C-D}.n^{M-D}+i_0^{C-D}.n^{S-A}+j_0^{C-D}.n^{S-B}+k_0^{C-D}.n^{S-C}$	$a_0^{\Delta-a}.n^{M-A}+b_0^{\Delta-a}.n^{M-B}+c_0^{\Delta-a}.n^{M-C}+d_0^{\Delta-a}.n^{M-D}+o_0^{\Delta-a}.n^{V-B}+p_0^{\Delta-a}.n^{V-C}$	$o_0^{\Delta-b}.n^{V-B}+p_0^{\Delta-b}.n^{V-C}$	$a_0^{\Delta-d}.n^{M-A}+b_0^{\Delta-d}.n^{M-B}+c_0^{\Delta-d}.n^{M-C}+d_0^{\Delta-d}.n^{M-D}+o_0^{\Delta-d}.n^{V-B}+p_0^{\Delta-d}.n^{V-C}$
V_1^B	$d_0^{C-D}.q^{M-D}+i_0^{C-D}.q^{S-A}+j_0^{C-D}.q^{S-B}+k_0^{C-D}.q^{S-C}$	$a_0^{\Delta-a}.q^{M-A}+b_0^{\Delta-a}.q^{M-B}+c_0^{\Delta-a}.q^{M-C}+d_0^{\Delta-a}.q^{M-D}+o_0^{\Delta-a}.q^{V-B}+p_0^{\Delta-a}.q^{V-C}$	$o_0^{\Delta-b}.q^{V-B}+p_0^{\Delta-b}.q^{V-C}$	$a_0^{\Delta-d}.q^{M-A}+b_0^{\Delta-d}.q^{M-B}+c_0^{\Delta-d}.q^{M-C}+d_0^{\Delta-d}.q^{M-D}+o_0^{\Delta-d}.q^{V-B}+p_0^{\Delta-d}.q^{V-C}$
V_1^C	$d_0^{C-D}.r^{M-D}+i_0^{C-D}.r^{S-A}+j_0^{C-D}.r^{S-B}+k_0^{C-D}.r^{S-C}$	$a_0^{\Delta-a}.r^{M-A}+b_0^{\Delta-a}.r^{M-B}+c_0^{\Delta-a}.r^{M-C}+d_0^{\Delta-a}.r^{M-D}+o_0^{\Delta-a}.r^{V-B}+p_0^{\Delta-a}.r^{V-C}$	$o_0^{\Delta-b}.r^{V-B}+p_0^{\Delta-b}.r^{V-C}$	$a_0^{\Delta-d}.r^{M-A}+b_0^{\Delta-d}.r^{M-B}+c_0^{\Delta-d}.r^{M-C}+d_0^{\Delta-d}.r^{M-D}+o_0^{\Delta-d}.r^{V-B}+p_0^{\Delta-d}.r^{V-C}$
3	$3_0^{C-D}+d_0^{C-D}.3^{M-D}+i_0^{C-D}.3^{S-A}+j_0^{C-D}.3^{S-B}+k_0^{C-D}.3^{S-C}$	$3_0^{\Delta-a}+a_0^{\Delta-a}.3^{M-A}+b_0^{\Delta-a}.3^{M-B}+c_0^{\Delta-a}.3^{M-C}+d_0^{\Delta-a}.3^{M-D}+o_0^{\Delta-a}.3^{V-B}+p_0^{\Delta-a}.3^{V-C}$	$o_0^{\Delta-b}.3^{V-B}+p_0^{\Delta-b}.3^{V-C}$	$3_0^{\Delta-d}+a_0^{\Delta-d}.3^{M-A}+b_0^{\Delta-d}.3^{M-B}+c_0^{\Delta-d}.3^{M-C}+d_0^{\Delta-d}.3^{M-D}+o_0^{\Delta-d}.3^{V-B}+p_0^{\Delta-d}.3^{V-C}$

	C_2^A	C_2^B	C_2^C	C_2^D	Δ_1^a	Δ_1^b	Δ_1^d
M_1^A	$a_1^{C-A}+e^{C-A}$	e^{C-B}	e^{C-C}	e^{C-D}	$a_1^{\Delta-a}+e^{\Delta-a}$	$e^{\Delta-b}$	$a_1^{\Delta-d}+e^{\Delta-d}$
M_1^B	f^{C-A}	$b_1^{C-B}+f^{C-B}$	f^{C-C}	f^{C-D}	$b_1^{\Delta-a}+f^{\Delta-a}$	$f^{\Delta-b}$	$b_1^{\Delta-d}+f^{\Delta-d}$
M_1^C	g^{C-A}	g^{C-B}	$c_1^{C-C}+g^{C-C}$	g^{C-D}	$c_1^{\Delta-a}+g^{\Delta-a}$	$g^{\Delta-b}$	$c_1^{\Delta-d}+g^{\Delta-d}$
M_1^D	h^{C-A}	h^{C-B}	h^{C-C}	$d_1^{C-D}+h^{C-D}$	$d_1^{\Delta-a}+h^{\Delta-a}$	$h^{\Delta-b}$	$d_1^{\Delta-d}+h^{\Delta-d}$
S_1^A	$i_1^{C-A}+l^{C-A}$	l^{C-B}	l^{C-C}	$i_1^{C-D}+l^{C-D}$	$l^{\Delta-a}$	$l^{\Delta-b}$	$l^{\Delta-d}$
S_1^B	m^{C-A}	$j_1^{C-B}+m^{C-B}$	m^{C-C}	$j_1^{C-D}+m^{C-D}$	$m^{\Delta-a}$	$m^{\Delta-b}$	$m^{\Delta-d}$
S_1^C	n^{C-A}	n^{C-B}	$k_1^{C-C}+n^{C-C}$	$k_1^{C-D}+n^{C-D}$	$n^{\Delta-a}$	$n^{\Delta-b}$	$n^{\Delta-d}$
V_1^B	q^{C-A}	q^{C-B}	q^{C-C}	q^{C-D}	$o_1^{\Delta-a}+q^{\Delta-a}$	$o_1^{\Delta-b}+q^{\Delta-b}$	$o_1^{\Delta-d}+q^{\Delta-d}$
V_1^C	r^{C-A}	r^{C-B}	r^{C-C}	r^{C-D}	$p_1^{\Delta-a}+r^{\Delta-a}$	$p_1^{\Delta-b}+r^{\Delta-b}$	$p_1^{\Delta-d}+r^{\Delta-d}$
3	3^{C-A}	3^{C-B}	3^{C-C}	$3_1^{C-D}+3^{C-D}$	$3_1^{\Delta-a}+3^{\Delta-a}$	$3^{\Delta-b}$	$3_1^{\Delta-d}+3^{\Delta-d}$

TABLES GIVING VALUES OF QUANTITIES IN
TERMS OF THE FIRST STORY UNKNOWNS

Note. For other stories use the same table, changing only the subscripts of the unknowns and of the coefficients of the Symbolic Equations to agree with the story with exceptions as noted in Text.

PLATE 13.

coefficient is found. Calculating and tabulating the quantities as shown on Plate 13 finishes the solution for the 0 story.

Now, in starting the solution for the first story to express first, story unknowns in terms of second story unknowns the values of α, β, γ, δ, θ, φ, ψ, ϵ, λ, and z for the nine equations A–a–1, A–b–1, etc., must be found before starting the tables of Plates 11 and 12. The rest of the coefficients involved in the term ω, *i. e.*, the coefficients of second story unknowns, will be as shown on Plate 12 except that the subscripts will be 2 instead of 1. Now, referring to the symbolic equations A_1^a, A_1^b, A_1^d, B_1^a, B_1^b, B_1^d, C_1^a, C_1^b, and C_1^d it will be seen that the other quantities α, β, γ, etc., would also be as given in Plates 11 and 12 with only a change of subscript were it not for the additional terms $s_1^{A-a}C_1^A + t_1^{A-a}C_1^B + P_1^{A-a}S_0^A$ in equation A_1^a, $t_1^{A-b}C_1^B + u_1^{A-b}C_1^C + P_1^{A-b}S_0^A + Q_1^{A-b}S_0^B$ in equation A_1^b, $u_1^{A-d}C_1^C + v_1^{A-d}C_1^D + P_1^{A-d}S_0^A + Q_1^{A-d}S_0^B + R_1^{A-d}S_0^C$ in equation A_1^d, $s_1^{B-a}C_1^A + t_1^{B-a}C_1^B$ in equation E_1^a, $t_1^{B-b}C_1^B + u_1^{B-b}C_1^C$ in equation B_1^b, $u_1^{B-d}C_1^C + v_1^{B-d}C_1^D$ in equation E_1^d, $u_1^{C-a}\Delta_0^a + t_1^{C-a}C_1^B$ in equation C_1^a, $x_1^{C-b}\Delta_0^b + u_1^{C-b}C_1^C$ in equation C_1^b, and $y_1^{C-d}\Delta_0^d + v_1^{C-d}C_1^D$ in equation C_1^d.

Now, on Plate 13 is given the values of all these terms C_1^A, C_1^B, C_1^C, C_1^D, Δ_0^a, Δ_0^b, and Δ_0^d in terms of first story unknowns only. Therefore the coefficients α, β, γ, etc., in equations A–a–1, A–b–1, etc., may be made up as were those for C_2^A, C_2^B, etc., as given on Plate 13: thus α_1 would be $a_1^{A-a} + s_1^{A-a}e^{C-A} + t_1^{A-a}e^{C-B} + P_1^{A-a}e^{s-A}$; α_4 would be $a_1^{B-a} + s_1^{B-a}h^{C-A} + t_1^{B-a}h^{C-B}$, etc. In this manner the coefficients for the nine equations A–a–1, A–b–1, etc., may be easily computed and then the tables on Plates 11, 12, and 13 may be computed up as far as the value of C_1^A on Plate 13 (would be C_2^A for first story). The values of C_2^A, C_2^B, C_2^C, C_2^D, Δ_1^a, Δ_1^b, and Δ_1^d must be known in terms of second story unknowns before the coefficients α, β, γ, etc., can be computed for starting the solution for the second story; just as the values of C_1^A, C_1^B, C_1^C, C_1^D, Δ_0^a, Δ_0^b, and Δ_0^d were needed for preparing for the first story solution. But, on Plate 13 of the 0 story solution values of these quantities were computed in terms of first story unknowns, every one of these nine unknowns being involved. Therefore, for Plate 13, for the first and all succeeding stories each of the column of values from C_1^A to Δ_0^d inclusive (would be C_2^A to Δ_1^d inclusive for first story solution) would contain nine terms for each coefficient (ten for the numerical term z). The method of writing the values of these coefficients is illustrated for C_2^A thus: Let the coefficients of M_1^A, M_1^B, M_1^C, M_1^D, etc., under C_2^A of Plate 13 for the 0 story be now known as e^{C-A}, f^{C-A}, g^{C-A}, h^{C-A}, etc., and let the coefficients of M_2^A, M_2^B, M_2^C, etc., in the values of M_1^A, M_1^B, etc., in terms of second story unknowns only, for Plate 13 for the first story solution be known, as they were before for the 0 story solution, as e^{M-A}, f^{M-A}, g^{M-A}, e^{M-B}, f^{M-B}, g^{M-B}, etc. Then the coefficient of M_2^A in the values of C_2^A will be $e^{C-A}e^{M-A} + f^{C-A}e^{M-B} + g^{C-A}e^{M-C} + h^{C-A}e^{M-D} + l^{C-A}e^{S-A} + m^{C-A} e^{S-B} + n^{C-A}e^{S-C} + g^{C-A}e^{V-B} + r^{C-A}e^{V-C}$.

In this manner the values of the coefficients of all the second story unknowns may be written for the values of the quantities C_2^A to Δ_1^d inclusive. Now the tables of values of C_3^A to Δ_2^d inclusive may be filled out; they will be as given on Plate 13 for C_2^A to Δ_1^d inclusive excepting for the change in the subscripts of the symbolic coefficients there given. Plate 13 as thus constructed for the first story solution will then be standard for all succeeding stories.

As no example of this general structure was calculated, due to a lack of time, the additional plates required for the revised plate 13 and for the revised coefficients α, β, γ, etc., as given above, for first and succeeding story solutions have been omitted. They may be readily constructed as described above by any one desiring to investigate such a structure. Now the work for each story is merely a repetition of that for the first story, and when the top story is reached it is evident that ω (Plate 12) will then consist of only the column z and the result of the tabulated calculations of Plate 13 for the top story gives numerical values of all top story unknowns. Substitution of these values thus found in connection with the tabulated coefficients of Plate 13 for the next lower story, gives the values of the unknowns for that story. This process is then continued until the 0 story is reached, when all unknowns will have become known. In the calculations of plates 11 and 12 a very nice check on the calculations may be kept by using Gauss's method as given in Chapter X of "Method of Least Squares," by Merriman.

Special Case of a Building of Only Two Columns to a Transverse Section.—The symbolic equations and values of the coefficients in

SYMBOLIC EQUATIONS FOR A TWO COLUMN CROSS SECTION.

Values of C-A.

$$C_0^A = 0$$

$$C_1^A = a_0^{C-A} \cdot M_0^A + i_0^{C-A} \cdot S_0^A.$$

$$C_2^A = a_1^{C-A} \cdot M_1^A + i_1^{C-A} \cdot S_1^A + C_1^A.$$

$$C_3^A = a_2^{C-A} \cdot M_2^A + i_2^{C-A} \cdot S_2^A + C_2^A.$$

Values of δ-A.

$$\delta_0^A = a_0^{\delta-A} \cdot M_0^A + i_0^{\delta-A} \cdot S_0^A.$$

$$\delta_1^A = a_1^{\delta-A} \cdot M_1^A + i_1^{\delta-A} \cdot S_1^A + s_1^{\delta-A} \cdot C_1^A.$$

$$\delta_2^A = a_2^{\delta-A} \cdot M_2^A + i_2^{\delta-A} \cdot S_2^A + s_2^{\delta-A} \cdot C_2^A.$$

Values of H-a.

$$H_0^a = l_0^{H-a} \cdot S_1^A + i_0^{H-a} \cdot S_0^A + z_0^{H-a}.$$

$$H_1^a = l_1^{H-a} \cdot S_2^A + i_1^{H-a} \cdot S_1^A + z_1^{H-a}.$$

$$H_2^a = l_2^{H-a} \cdot S_3^A + i_2^{H-a} \cdot S_2^A + z_2^{H-a}.$$

Values of V-A.

$$V_0^A = a_0^{V-A} \cdot M_0^A + b_0^{V-A} \cdot M_0^B + z_0^{V-A}.$$

$$V_1^A = a_1^{V-A} \cdot M_1^A + b_1^{V-A} \cdot M_1^B + z_1^{V-A}.$$

$$V_2^A = a_2^{V-A} \cdot M_2^A + b_2^{V-A} \cdot M_2^B + z_2^{V-A}.$$

Values of α-A.

$$\alpha_0^A = a_0^{\alpha-A} \cdot M_0^A + b_0^{\alpha-A} \cdot M_0^B + z_0^{\alpha-A}.$$

$$\alpha_1^A = a_1^{\alpha-A} \cdot M_1^A + b_1^{\alpha-A} \cdot M_1^B + z_1^{\alpha-A}.$$

$$\alpha_2^A = a_2^{\alpha-A} \cdot M_2^A + b_2^{\alpha-A} \cdot M_2^B + z_2^{\alpha-A}.$$

Values of Δ-a.

$$\Delta_0^a = a_0^{\Delta-a} \cdot M_0^A + b_0^{\Delta-a} \cdot M_0^B + z_0^{\Delta-a}.$$

$$\Delta_1^a = a_1^{\Delta-a} \cdot M_1^A + b_1^{\Delta-a} \cdot M_1^B + z_1^{\Delta-a} + \Delta_0^a.$$

$$\Delta_2^a = a_2^{\Delta-a} \cdot M_2^A + b_2^{\Delta-a} \cdot M_2^B + z_2^{\Delta-a} + \Delta_1^a.$$

Values of R-a.

$$R_0^a = a_0^{R-a} \cdot M_0^A + b_0^{R-a} \cdot M_0^B + e_0^{R-a} \cdot M_1^A + f_0^{R-a} \cdot M_1^B + z_0^{R-a}.$$

$$R_1^a = a_1^{R-a} \cdot M_1^A + b_1^{R-a} \cdot M_1^B + e_1^{R-a} \cdot M_2^A + f_1^{R-a} \cdot M_2^B + z_1^{R-a}.$$

$$R_2^a = a_2^{R-a} \cdot M_2^A + b_2^{R-a} \cdot M_2^B + e_2^{R-a} \cdot M_3^A + f_2^{R-a} \cdot M_3^B + z_2^{R-a}.$$

Values of C-B.

$$C_0^B = 0$$

$$C_1^B = b_0^{C-B} \cdot M_0^B + i_0^{C-B} \cdot S_0^A + z_0^{C-B}.$$

$$C_2^B = b_1^{C-B} \cdot M_1^B + i_1^{C-B} \cdot S_1^A + z_1^{C-B} + C_1^B.$$

$$C_3^B = b_2^{C-B} \cdot M_2^B + i_2^{C-B} \cdot S_2^A + z_2^{C-B} + C_2^B.$$

Values of δ-B.

$$\delta_0^B = b_0^{\delta-B} \cdot M_0^B + i_0^{\delta-B} \cdot S_0^A + z_0^{\delta-B}.$$

$$\delta_1^B = b_1^{\delta-B} \cdot M_1^B + i_1^{\delta-B} \cdot S_1^A + z_1^{\delta-B} + t_1^{\delta-B} \cdot C_1^B.$$

$$\delta_2^B = b_2^{\delta-B} \cdot M_2^B + i_2^{\delta-B} \cdot S_2^A + z_2^{\delta-B} + t_2^{\delta-B} \cdot C_2^B.$$

Values of λ-a.

$$\lambda_0^a = l_0^{\lambda-a} \cdot S_1^A + i_0^{\lambda-a} \cdot S_0^A + z_0^{\lambda-a}.$$

$$\lambda_1^a = l_1^{\lambda-a} \cdot S_2^A + i_1^{\lambda-a} \cdot S_1^A + z_1^{\lambda-a}.$$

$$\lambda_2^a = l_2^{\lambda-a} \cdot S_3^A + i_2^{\lambda-a} \cdot S_2^A + z_2^{\lambda-a}.$$

Values of V-B.

$$V_0^B = a_0^{V-B} \cdot M_0^A + b_0^{V-B} \cdot M_0^B + z_0^{V-B}.$$

$$V_1^B = a_1^{V-B} \cdot M_1^A + b_1^{V-B} \cdot M_1^B + z_1^{V-B}.$$

$$V_2^B = a_2^{V-B} \cdot M_2^A + b_2^{V-B} \cdot M_2^B + z_2^{V-B}$$

Values of α-B.

$$\alpha_0^B = a_0^{\alpha-B} \cdot M_0^A + b_0^{\alpha-B} \cdot M_0^B + z_0^{\alpha-B}.$$

$$\alpha_1^B = a_1^{\alpha-B} \cdot M_1^A + b_1^{\alpha-B} \cdot M_1^B + z_1^{\alpha-B}.$$

$$\alpha_2^B = a_2^{\alpha-B} \cdot M_2^A + b_2^{\alpha-B} \cdot M_2^B + z_2^{\alpha-B}.$$

Values of Q-a.

$$Q_0^a = i_0^{Q-a} \cdot S_0^A + l_0^{Q-a} \cdot S_1^A + a_0^{Q-a} \cdot M_0^A + e_0^{Q-a} \cdot M_1^A + z_0^{Q-a}$$

$$Q_1^a = i_1^{Q-a} \cdot S_1^A + l_1^{Q-a} \cdot S_2^A + a_1^{Q-a} \cdot M_1^A + e_1^{Q-a} \cdot M_2^A + z_1^{Q-a}.$$

$$Q_2^a = i_2^{Q-a} \cdot S_2^A + l_2^{Q-a} \cdot S_3^A + a_2^{Q-a} \cdot M_2^A + e_2^{Q-a} \cdot M_3^A + z_2^{Q-a}.$$

Equations A-a.

$$a_0^{A-a} \cdot M_0^A + b_0^{A-a} \cdot M_0^B + i_0^{A-a} \cdot S_0^A + l_0^{A-a} \cdot S_1^A + z_0^{A-a} = 0$$

$$a_1^{A-a} \cdot M_1^A + b_1^{A-a} \cdot M_1^B + i_1^{A-a} \cdot S_1^A + l_1^{A-a} \cdot S_2^A + z_1^{A-a} + s_1^{A-a} \cdot C_1^A + t_1^{A-a} \cdot C_1^B + P_1^{A-a} \cdot S_0^A = 0.$$

$$a_2^{A-a} \cdot M_2^A + b_2^{A-a} \cdot M_2^B + i_2^{A-a} \cdot S_2^A + l_2^{A-a} \cdot S_3^A + z_2^{A-a} + s_2^{A-a} \cdot C_2^A + t_2^{A-a} \cdot C_2^B + P_2^{A-a} \cdot S_1^A = 0.$$

Equations B-a.

$$a_0^{B-a} \cdot M_0^A + b_0^{B-a} \cdot M_0^B + e_0^{B-a} \cdot M_1^A + f_0^{B-a} \cdot M_1^B + i_0^{B-a} \cdot S_0^A + l_0^{B-a} \cdot S_1^A + z_0^{B-a} = 0.$$

$$a_1^{B-a} \cdot M_1^A + b_1^{B-a} \cdot M_1^B + e_1^{B-a} \cdot M_2^A + f_1^{B-a} \cdot M_2^B + i_1^{B-a} \cdot S_1^A + l_1^{B-a} \cdot S_2^A + z_1^{B-a} + s_1^{B-a} \cdot C_1^A + t_1^{B-a} \cdot C_1^B = 0$$

$$a_2^{B-a} \cdot M_2^A + b_2^{B-a} \cdot M_2^B + e_2^{B-a} \cdot M_3^A + f_2^{B-a} \cdot M_3^B + i_2^{B-a} \cdot S_2^A + l_2^{B-a} \cdot S_3^A + z_2^{B-a} + s_2^{B-a} \cdot C_2^A + t_2^{B-a} \cdot C_2^B = 0$$

Equations C-a.

$$a_0^{C-a} \cdot M_0^A + b_0^{C-a} \cdot M_0^B + e_0^{C-a} \cdot M_1^A + f_0^{C-A} \cdot M_1^B + i_0^{C-a} \cdot S_0^A + l_0^{C-a} \cdot S_1^A + z_0^{C-a} = 0.$$

$$a_1^{C-a} \cdot M_1^A + b_1^{C-a} \cdot M_1^B + e_1^{C-a} \cdot M_2^A + f_1^{C-A} \cdot M_2^B + i_1^{C-a} \cdot S_1^A + l_1^{C-a} \cdot S_2^A + z_1^{C-a} + t_1^{C-a} \cdot C_1^B + w_1^{C-a} \cdot \Delta_0^a = 0$$

$$a_2^{C-a} \cdot M_2^A + b_2^{C-a} \cdot M_2^B + e_2^{C-a} \cdot M_3^A + f_2^{C-a} \cdot M_3^B + i_2^{C-a} \cdot S_2^A + l_2^{C-a} \cdot S_3^A + z_2^{C-a} + t_2^{C-a} \cdot C_2^B + w_2^{C-a} \cdot \Delta_1^a = 0$$

PLATE 14.

VALUES OF THE COEFFICIENTS IN THE SYMBOLIC EQUATIONS FOR A TWO COLUMN CROSS SECTION.

	a_n	b_n	e_n	f_n	i_n	l_n	s_n	t_n	w_n	P_n	z_n
C-A	$C_n \div I_n^A$	0	0	0	$-C_n^2 \div 2I_n^A$	0	1	0	0	0	0
C-B	0	$C_n \div I_n^B$	0	0	$C_n^2 \div 2I_n^B$	0	0	1	0	0	$-C_n^2 \Sigma F_n \div 2I_n^B$
δ-A	$C_n^2 \div 2I_n^A$	0	0	0	$-C_n^3 \div 6I_n^A$	0	C_n	0	0	0	0
δ-B	0	$C_n^2 \div 2I_n^B$	0	0	$C_n^3 \div 6I_n^B$	0	0	C_n	0	0	$-C_n^3 \Sigma F_n \div 6I_n^B$
H-a	0	0	0	0	-1	1	0	0	0	0	F_n^A
λ-a	0	0	0	0	$-b \div A_n^a$	$b \div A_n^a$	0	0	0	0	$b.z_n^{H-a} \div A_n^a$
V-A	$1 \div b$	$1 \div b$	0	0	0	0	0	0	0	0	$-X_n \div b$
V-B	$-1 \div b$	$-1 \div b$	0	0	0	0	0	0	0	0	$Y_n + X_n \div b$
α-A	$(C_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)a_n^{V-A} \div A_n^A$	$(C_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)b_n^{V-A} \div A_n^A$	0	0	0	0	0	0	0	0	$(C_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)z_n^{V-A} \div A_n^A$
α-B	$(C_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)a_n^{V-B} \div A_n^B$	$(C_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)b_n^{V-B} \div A_n^B$	0	0	0	0	0	0	0	0	$(C_n + \frac{1}{2}d_{n-1} + \frac{1}{2}d_n)z_n^{V-B} \div A_n^B$
Δ-a	$-a_n^{\alpha-A} + a_n^{\alpha-B}$	$-b_n^{\alpha-A} + b_n^{\alpha-B}$	0	0	0	0	0	0	1	0	$-z_n^{\alpha-A} + z_n^{\alpha-B}$
Q-a	$-1 \div I_n^a$	0	$1 \div I_n^a$	0	$(C_n + \frac{1}{2}d_n) \div I_n^a$	$d_n \div 2I_n^a$	0	0	0	0	$-P_n^A.e_n^A \div I_n^a$
R-a	$a_n^{V-A} \div I_n^a$	$b_n^{V-A} \div I_n^a$	$-a_{n+1}^{V-A} \div I_n^a$	$-b_{n+1}^{V-A} \div I_n^a$	0	0	0	0	0	0	$(z_n^{V-A} - z_{n+1}^{V-A} - P_n^A) \div I_n^a$
A-a	$a_n^{\delta-A} + \frac{1}{2}d_n.a_n^{C-A}$	$-b_n^{\delta-B} - \frac{1}{2}d_n.b_n^{C-B}$	0	0	$i_n^{\delta-A} - i_n^{\delta-B} + \frac{1}{2}d_n(i_n^{C-A} - i_n^{C-B}) - i_n^{\lambda-a} + l_{n-1}^{\lambda-a}$	$-l_n^{\lambda-a}$	$s_n^{\delta-A} + \frac{1}{2}d_n.s_n^{C-A} + \frac{1}{2}d_{n-1}$	$-t_n^{\delta-B} - \frac{1}{2}d_n.t_n^{C-B} - \frac{1}{2}d_{n-1}$	0	$-b \div A_{n-1}^a$	$-z_n^{\lambda-a} + z_{n-1}^{\lambda-a} - z_n^{\delta-B} - \frac{1}{2}d_n.z_n^{C-B}$
B-a	$(a_n^{Q-a} + b.a_n^{R-a})b - \frac{1}{2}b^2 a_n^{R-a} - a_n^{C-A}$	$\frac{1}{2}b^2 b_n^{R-a} + b_n^{C-B}$	$(e_n^{Q-a} + b.e_n^{R-a})b - \frac{1}{2}b^2 e_n^{R-a}$	$\frac{1}{2}b^2 f_n^{R-a}$	$b.i_n^{Q-a} - i_n^{C-A} + i_n^{C-B}$	$b.l_n^{Q-a}$	$-s_n^{C-A}$	t_n^{C-B}	0	0	$(z_n^{Q-a} + b.z_n^{R-a})b - \frac{1}{2}b^2 z_n^{R-a} - b^3 w_n \div 6.I_n^a + z_n^{C-B}$
C-a	$(a_n^{Q-a} + b.a_n^{R-a})\frac{1}{2}b^2 - \frac{1}{6}b^3 a_n^{R-a} - a_n^{\Delta-a}$	$\frac{1}{3}b^3 b_n^{R-a} + b.b_n^{C-B} - b_n^{\Delta-a}$	$(e_n^{Q-a} + b.e_n^{R-a})\frac{1}{2}b^2 - \frac{1}{6}b^3 e_n^{R-a}$	$\frac{1}{3}b^3 f_n^{R-a}$	$\frac{1}{2}b^2 i_n^{Q-a} + b.i_n^{C-B}$	$\frac{1}{2}b^2 l_n^{Q-a}$	0	$b.t_n^{C-B}$	-1	0	$(z_n^{Q-a} + b.z_n^{R-a})\frac{1}{2}b^2 - \frac{1}{6}b^3 z_n^{R-a} - b^4 w_n \div 8I_n^a + b.z_n^{C-B} - z_n^{\Delta-a}$

The following are Rewritten Equations in which values of H^a have been substituted for values of S to agree with the Solution used in Chapters VI-VII-VIII & IX. Values of Coefficients are as given in the Above Table excepting only those whose values are given in the Table to the Right

$$C_{n+1}^A = a_n^{C-A}.M_n^A + i_n^{C-A}(S_0^B - H_0^a - H_1^a \cdots - H_{n-1}^a) + C_n^A + z_n^{C-A}$$

$$C_{n+1}^B = b_n^{C-B}.M_n^B + i_n^{C-B}(S_0^B - H_0^a - H_1^a \cdots - H_{n-1}^a) + C_n^B + z_n^{C-B}$$

$$\Delta_n^a = a_n^{\Delta-a}M_n^A + b_n^{\Delta-a}M_n^B + z_n^{\Delta-a} + \Delta_{n-1}^a$$

$$a_n^{A-a}.M_n^A + b_n^{A-a}.M_n^B + i_n^{A-a}(S_0^B - H_0^a - H_1^a \cdots - H_{n-1}^a) + l_n^{A-a}.H_n^a + z_n^{A-a} + s_n^{A-a}C_n^A + t_n^{A-a}C_n^B + P_n^{A-a}H_{n-1}^a = 0$$

$$a_n^{B-a}.M_n^A + b_n^{B-a}.M_n^B + e_n^{B-a}.M_{n+1}^A + f_n^{B-a}.M_{n+1}^B + i_n^{B-a}(S_0^B - H_0^a - H_1^a \cdots - H_{n-1}^a) + l_n^{B-a}H_n^a + z_n^{B-a} + s_n^{B-a}C_n^A + t_n^{B-a}C_n^B = 0$$

$$a_n^{C-a}.M_n^A + b_n^{C-a}.M_n^B + e_n^{C-a}.M_{n+1}^A + f_n^{C-a}.M_{n+1}^B + i_n^{C-a}(S_0^B - H_0^a - H_1^a \cdots - H_{n-1}^a) + l_n^{C-a}.H_n^a + z_n^{C-a} + w_n^{C-a}\Delta_{n-1}^a + t_n^{C-a}C_n^B = 0$$

Values of New Coefficients in Equations as Rewritten

	i_n	P_n	z_n
C-A	$-l_n^{C-A}$	0	$i_n^{C-A}(\Sigma F - F_0^A - F_1^A \cdots - F_{n-1}^A)$
C-B	$-l_n^{C-B}$	0	$z_n^{C-B} + i_n^{C-B}(\Sigma F - F_0^A - F_1^A \cdots - F_{n-1}^A)$
Δ-a	0	0	$z_n^{\Delta-a}$
A-a	$-(i_n^{A-a} + P_n^{A-a} + l_n^{A-a})$	$-P_n^{A-a}$	$z_n^{A-a} + (i_n^{A-a} + P_n^{A-a} + l_n^{A-a})(\Sigma F - F_0^A - F_1^A \cdots - F_{n-1}^A) - l_n^{A-a}.F_n^A + P_n^{A-a}F_{n-1}^A$
B-a	$-(i_n^{B-a} + l_n^{B-a})$	0	$z_n^{B-a} + (i_n^{B-a} + l_n^{B-a})(\Sigma F - F_0^A - F_1^A \cdots - F_{n-1}^A) - l_n^{B-a}.F_n^A$
C-a	$-(i_n^{C-a} + l_n^{C-a})$	0	$z_n^{C-a} + (i_n^{C-a} + l_n^{C-a})(\Sigma F - F_0^A - F_1^A \cdots - F_{n-1}^A) - l_n^{C-a}.F_n^A$

PLATE 15.

these equations for this simple structure are readily written from those given for the general case. These equations are given on Plate 14 and the values of the coefficients are given on Plate 15. In applying this method in later chapters the quantities H^a and S_0^B were considered as unknowns in place of the quantities S^A in the formulae of Plate 14. This was due to the fact that these applications were made before the general case and formulae were attempted. The transformed equations necessary for the solution, as required by this modification are given in the lower left-hand corner of Plate 15 and the values of those coefficients affected by the transformation are given in the lower right-hand corner of the same plate. The transformation was effected by writing

$$S_n^A = [(\Sigma F - F_0^A - F_1^A - \cdots - F_{n-1}^A) - (S_0^B - H_0 - H_1 \\ - \cdots - H_{n-1})],$$

except in the term PS_{n-1}^A of equations A, in which was substituted

$$S_{n-1}^A = S_n^A - (H_{n-1}^A - F_{n-1}^A).$$

Now the equations may be further simplified as follows:

First. In equations A–a, B–a, and C–a, the last letter "a" of the exponents of the coefficients may be dropped.

Second. Dividing all coefficients in equations A–a by l, these equations may, for the nth story, be written

$$H_n = c_n^A M_n^A + l_n^A M_n^B + i_n^A(S_0^B H_0 - H_1 - \cdots - H_{n-1}) \\ + z_n^A + s_n^A C_n^A + t_n^A C_n^B + P_n^A H_{n-1}$$

in which the coefficients are, of course, different from the values already given, due to the operation just performed.

Third. Dividing all the coefficients of equations B–a by $b/4I_n^a$, and all those of equations C–a by $b^2/6I_n^a$ and then subtracting equations C–a from equations B–a, there results, for the nth story, the equation

$$M_{n+1}^A = a_n^B M_n^A + l_n^B M_n^B + i_n^B(S_0^B - H_0 - H_1 - \cdots - H_{n-1}) \\ + l_n^B H_n + z_n^B + s_n^B C_n^A + t_n^B C_n^B + w_n^B \Delta_{n-1},$$

in which the values of the coefficients are determined by the operation.

Fourth. Dividing all the coefficients of equations $B - a$ by $b/2I_n^a$, and all those of equations $C - a$ by $b^2/6I_n^a$ and then subtracting equations $B - a$ from equations $C - a$ there results, for the nth story, the equation

$$M_{n+1}^B = a_n^C M_n^A + b_n^C M_n^B + i_n^C (S_0^B - H_0 - H_1 - \cdots - H_{n-1}) \\ + l_n^C H_n + z_n^C + s_n^C C_n^A + t_n^A C_n^B + w_n^C \Delta_{n-1},$$

in which the values of the coefficients are determined by the operation.

When the equations $A - a$, $B - a$ and $C - a$, and the coefficients occurring in these equations, are referred to in the following solution, the reference is to these revised equations and their coefficients.

The method used in solving these equations is now given on Plate 16, which, it is believed, explains itself. The Greek letter and other coefficients used on this plate have entirely different meanings from any they have had where previously used. The values of these coefficients are given on Plate 17. Plates 16 and 17 are for the first three stories only. For all succeeding stories the solution and coefficients may be written exactly the same as for the second story except for the subscripts, which must agree with the story for which the coefficients are written. When the last story of the building is reached the coefficients in the lines marked 9–10–11–13–14 and 15 may be omitted entirely and the two central columns of those in lines marked 6, 7, 8, and 12 may be omitted also; since the terms omitted are for quantities whose values are either already given in the previous story or they are terms or coefficients of terms which do not exist for the last story.

The solution for any structure, then, consists mainly of calculating the coefficients of Plate 17. For this purpose calculation sheets are ruled up in a form corresponding to Plate 17 and in each blank space the calculations for the coefficient found in the corresponding space of Plate 17 are inserted, these calculations being made by slide rule. Plate 17 must be kept constantly in view during calculations.

SOLUTION FOR A TWO COLUMN CROSS SECTION.

From equation A_0^A there is obtained the expression	$S_0^B = \frac{1}{c_0} H_0 - \frac{a_0}{c_0} M_0^A - \frac{b_0}{c_0} M_0^B - \frac{d_0}{c_0}$.
Substituting this value of S_0^B in equation B_0^A there results	$M_0^A + A_0 M_0^B = B_0 M_1^A + C_0 H_0 + D_0$
Substituting this value of S_0^B in equation C_0^A there results	$M_0^A + A_0' M_0^B = B_0' M_1^B + C_0' H_0 + D_0'$.
Solving these two resulting equations there is obtained	$M_0^A = \alpha_0 H_0 + \beta_0 M_1^A + \gamma_0 M_1^B + \delta_0$.
	$M_0^B = \alpha_0' H_0 + \beta_0' M_1^A + \gamma_0' M_1^B + \delta_0'$.
Substituting these values of M_0^A & M_0^B in above value of S_0^B	$S_0^B = \alpha_0'' H_0 + \beta_0'' M_1^A + \gamma_0'' M_1^B + \delta_0''$
Subtracting H_0 from both sides of this equation	$S_0^B - H_0 = (\alpha_0'' - 1) H_0 + \beta_0'' M_1^A + \gamma_0'' M_1^B + \delta_0''$.
Substituting above values of S_0^B, M_0^A & M_0^B in expression for C_1^A.	$C_1^A = \alpha_0''' H_0 + \beta_0''' M_1^A + \gamma_0''' M_1^B + \delta_0'''$.
" " " " " " " " " " C_1^B.	$C_1^B = \alpha_0^{iv} H_0 + \beta_0^{iv} M_1^A + \gamma_0^{iv} M_1^B + \delta_0^{iv}$
" " " " " " " " " " Δ_0^A.	$\Delta_0^A = \alpha_0^{v} H_0 + \beta_0^{v} M_1^A + \gamma_0^{v} M_1^B + \delta_0^{v}$.
Substituting values of $S_0^B - H_0$ & C_1^A in expression for C_2^A.	$C_2^A = \theta_0 H_0 + \phi_0 M_1^A + \psi_0 M_1^B + \omega_0$.
" " " " " C_1^B " " " C_2^B.	$C_2^B = \theta_0' H_0 + \phi_0' M_1^A + \psi_0' M_1^B + \omega_0'$.
" " " Δ_0^A " " " Δ_1^A.	$\Delta_1^A = \theta_0'' H_0 + \phi_0'' M_1^A + \psi_0'' M_1^B + \omega_0''$.
" " " $S_0^B - H_0$, C_1^A & C_1^B in equation A_1^A there results	$H_0 = \frac{1}{c_1} H_1 - \frac{a_1}{c_1} M_1^A - \frac{b_1}{c_1} M_1^B - \frac{d_1}{c_1}$
" " " " " " & Δ_0^A " " B_1^A " "	$M_1^A + A_1 M_1^B = B_1 M_2^A + C_1 H_1 + D_1$.
" " " " " " " " " " C_1^A " "	$M_1^A + A_1' M_1^B = B_1' M_2^B + C_1' H_1 + D_1'$
Solving these two resulting equations there is obtained	$M_1^A = \alpha_1 H_1 + \beta_1 M_2^A + \gamma_1 M_2^B + \delta_1$
	$M_1^B = \alpha_1' H_1 + \beta_1' M_2^A + \gamma_1' M_2^B + \delta_1'$
Substituting these values of M_1^A & M_1^B in above value of H_0	$H_0 = \alpha_1'' H_1 + \beta_1'' M_2^A + \gamma_1'' M_2^B + \delta_1''$
" " " " M_1^A, M_1^B & H_0 " " " " C_2^A	$C_2^A = \alpha_1''' H_1 + \beta_1''' M_2^A + \gamma_1''' M_2^B + \delta_1'''$
" " " " " " " " " " " C_2^B	$C_2^B = \alpha_1^{iv} H_1 + \beta_1^{iv} M_2^A + \gamma_1^{iv} M_2^B + \delta_1^{iv}$
" " " " " " " " " " " Δ_1^A	$\Delta_1^A = \alpha_1^{v} H_1 + \beta_1^{v} M_2^A + \gamma_1^{v} M_2^B + \delta_1^{v}$
" " " " " " " " " " " $S_0^B - H_0$	$S_0^B - H_0 = \alpha_1^{vi} H_1 + \beta_1^{vi} M_2^A + \gamma_1^{vi} M_2^B + \delta_1^{vi}$
Subtracting H_1 from both sides of this equation	$S_0^B - H_0 - H_1 = (\alpha_1^{vi} - 1) H_1 + \beta_1^{vi} M_2^A + \gamma_1^{vi} M_2^B + \delta_1^{vi}$
Substituting above values of $S_0^B - H_0 - H_1$ & C_2^A in expression for C_3^A.	$C_3^A = \theta_1 H_1 + \phi_1 M_2^A + \psi_1 M_2^B + \omega_1$.
" " " " " C_2^B " " " C_3^B.	$C_3^B = \theta_1' H_1 + \phi_1' M_2^A + \psi_1' M_2^B + \omega_1'$.
" " " " Δ_1^A " " " Δ_2^A.	$\Delta_2^A = \theta_1'' H_1 + \phi_1'' M_2^A + \psi_1'' M_2^B + \omega_1''$
" " " $S_0^B - H_0 - H_1$, C_2^A & C_2^B in equation A_2^A gives	$H_1 = \frac{1}{c_2} H_2 - \frac{a_2}{c_2} M_2^A - \frac{b_2}{c_2} M_2^B - \frac{d_2}{c_2}$
" " " " " " & Δ_1^A " " B_2^A "	$M_2^A + A_2 M_2^B = B_2 M_3^A + C_2 H_2 + D_2$.
" " " " " " " " " " C_2^A "	$M_2^A + A_2' M_2^B = B_2' M_3^B + C_2' H_2 + D_2'$.
Solving these two resulting equations there is obtained	$M_2^A = \alpha_2 H_2 + \beta_2 M_3^A + \gamma_2 M_3^B + \delta_2$.
	$M_2^B = \alpha_2' H_2 + \beta_2' M_3^A + \gamma_2' M_3^B + \delta_2'$.
Substituting these values of M_2^A & M_2^B in above value of H_1	$H_1 = \alpha_2'' H_2 + \beta_2'' M_3^A + \gamma_2'' M_3^B + \delta_2''$.
" " " " M_2^A, M_2^B, & H_1 " " " " C_3^A	$C_3^A = \alpha_2''' H_2 + \beta_2''' M_3^A + \gamma_2''' M_3^B + \delta_2'''$
" " " " " " " " " " " C_3^B	$C_3^B = \alpha_2^{iv} H_2 + \beta_2^{iv} M_3^A + \gamma_2^{iv} M_3^B + \delta_2^{iv}$
" " " " " " " " " " " Δ_2^A	$\Delta_2^A = \alpha_2^{v} H_2 + \beta_2^{v} M_3^A + \gamma_2^{v} M_3^B + \delta_2^{v}$
" " " " " " " " " " " $S_0^B - H_0 - H_1$	$S_0^B - H_0 - H_1 = \alpha_2^{vi} H_2 + \beta_2^{vi} M_3^A + \gamma_2^{vi} M_3^B + \delta_2^{vi}$.
Subtracting H_2 from both sides of this equation	$S_0^B - H_0 - H_1 - H_2 = (\alpha_2^{vi} - 1) H_2 + \beta_2^{vi} M_3^A + \gamma_2^{vi} M_3^B + \delta_2^{vi}$.
Substituting above values of $S_0^B - H_0 - H_1 - H_2$ & C_3^A in expression for C_4^A	$C_4^A = \theta_2 H_2 + \phi_2 M_3^A + \psi_2 M_3^B + \omega_2$
" " " " " C_3^B " " " C_4^B	$C_4^B = \theta_2' H_2 + \phi_2' M_3^A + \psi_2' M_3^B + \omega_2'$
" " " " Δ_2^A " " " Δ_3^A	$\Delta_3^A = \theta_2'' H_2 + \phi_2'' M_3^A + \psi_2'' M_3^B + \omega_2''$

PLATE 16.

	TABULATED VALUES OF THE COEFFICIENTS IN THE SOLUTION FOR THE 0 STORY				
	$c_0 = i_0^A$	$a_0 = a_0^A$	$b_0 = b_0^A$	$d_0 = z_0^A$	
	$A_0 = \frac{i_0^A b_0^B - i_0^B b_0^A}{i_0^A a_0^B - i_0^B a_0^A}$	$B_0 = \frac{i_0^A}{i_0^A a_0^B - i_0^B a_0^A}$	$C_0 = \frac{-i_0^A l_0^B - i_0^B}{i_0^A a_0^B - i_0^B a_0^A}$	$D_0 = \frac{-i_0^A z_0^B + i_0^B z_0^A}{i_0^A a_0^B - i_0^B a_0^A}$	
	$A_0' = \frac{i_0^A b_0^C - i_0^C b_0^A}{i_0^A a_0^C - i_0^C a_0^A}$	$B_0' = \frac{i_0^A}{i_0^A a_0^C - i_0^C a_0^A}$	$C_0' = \frac{-i_0^A l_0^C - i_0^C}{i_0^A a_0^C - i_0^C a_0^A}$	$D_0' = \frac{-i_0^A z_0^C + i_0^C z_0^A}{i_0^A a_0^C - i_0^C a_0^A}$	
	$\alpha_0 = \frac{A_0 C_0' - A_0' C_0}{A_0 - A_0'}$	$\beta_0 = -\frac{A_0' . B_0}{A_0 - A_0'}$	$\gamma_0 = \frac{A_0 . B_0'}{A_0 - A_0'}$	$\delta_0 = \frac{A_0 D_0' - A_0' D_0}{A_0 - A_0'}$	
	$\alpha_0' = \frac{C_0 - C_0'}{A_0 - A_0'}$	$\beta_0' = \frac{B_0}{A_0 - A_0'}$	$\gamma_0' = -\frac{B_0'}{A_0 - A_0'}$	$\delta_0' = \frac{D_0 - D_0'}{A_0 - A_0'}$	
	$\alpha_0'' = \frac{1}{c_0} - \frac{a_0 . \alpha_0}{c_0} - \frac{b_0 . \alpha_0'}{c_0}$	$\beta_0'' = -\frac{a_0 . \beta_0}{c_0} - \frac{b_0 . \beta_0'}{c_0}$	$\gamma_0'' = -\frac{a_0 . \gamma_0}{c_0} - \frac{b_0 . \gamma_0'}{c_0}$	$\delta_0'' = -\frac{d_0}{c_0} - \frac{a_0 . d_0}{c_0} - \frac{b_0 . d_0'}{c_0}$	
	$\alpha_0''' = a_0^{C-A} . \alpha_0 + i_0^{C-A} . \alpha_0''$	$\beta_0''' = a_0^{C-A} \beta_0 + i_0^{C-A} \beta_0''$	$\gamma_0''' = a_0^{C-A} . \gamma_0 + i_0^{C-A} \gamma_0''$	$\delta_0''' = z_0^{C-A} + a_0^{C-A} . \delta_0 + i_0^{C-A} . \delta_0''$	
	$\alpha_0^{IV} = b_0^{C-B} . \alpha_0' + i_0^{C-B} \alpha_0''$	$\beta_0^{IV} = b_0^{C-B} . \beta_0' + i_0^{C-B} \beta_0''$	$\gamma_0^{IV} = b_0^{C-B} \gamma_0' + i_0^{C-B} . \gamma_0''$	$\delta_0^{IV} = z_0^{C-B} + b_0^{C-B} . \delta_0' + i_0^{C-B} . \delta_0''$	
	$\alpha_0^{V} = a_0^{\Delta-a} . \alpha_0 + b_0^{\Delta-a} \alpha_0'$	$\beta_0^{V} = a_0^{\Delta-a} . \beta_0 + b_0^{\Delta-a} \beta_0'$	$\gamma_0^{V} = a_0^{\Delta-a} . \gamma_0 + b_0^{\Delta-a} . \gamma_0'$	$\delta_0^{V} = z_0^{\Delta-A} + a_0^{\Delta-a} . \delta_0 + b_0^{\Delta-A} \delta_0'$	
	$\theta_0 = \alpha_0''' + i_1^{C-A} . (\alpha_0'' - 1)$	$\phi_0 = a_1^{C-A} + \beta_0''' + i_1^{C-A} . \beta_0''$	$\psi_0 = \gamma_0''' + i_1^{C-A} \gamma_0''$	$\omega_0 = z_1^{C-A} + \delta_0''' + i_1^{C-A} . \delta_0''$	
	$\theta_0' = \alpha_0^{IV} + i_1^{C-B} . (\alpha_0'' - 1)$	$\phi_0' = \beta_0^{IV} + i_1^{C-B} . \beta_0''$	$\psi_0' = \gamma_0^{IV} + i_1^{C-B} \gamma_0'' + b_1^{C-B}$	$\omega_0' = z_1^{C-B} + \delta_0^{IV} + i_1^{C-B} \delta_0''$	
	$\theta_0'' = \alpha_0^{V}$	$\phi_0'' = a_1^{\Delta-a} + \beta_0^{V}$	$\psi_0'' = b_1^{\Delta-a} + \gamma_0^{V}$	$\omega_0'' = z_1^{\Delta-A} + \delta_0^{V}$	
	TABULATED VALUES OF THE COEFFICIENTS IN THE SOLUTION FOR THE 1ST STORY				
	$c_1 = i_1^A(\alpha_0'' - 1) + s_1^A \alpha_0''' + t_1^A \alpha_0^{IV} + p_1^A$	$a_1 = a_1^A + i_1^A \beta_0'' + s_1^A \beta_0''' + t_1^A \beta_0^{IV}$	$b_1 = b_1^A + i_1^A \gamma_0'' + s_1^A \gamma_0''' + t_1^A \gamma_0^{IV}$	$d_1 = z_1^A + i_1^A \delta_0'' + s_1^A \delta_0''' + t_1^A \delta_0^{IV}$	
	$i_1 = i_1^B(\alpha_0'' - 1) + s_1^B \alpha_0''' + t_1^B \alpha_0^{IV} + w_1^B \alpha_0^{V}$	$g_1 = a_1^B + i_1^B \beta_0'' + s_1^B \beta_0''' + t_1^B \beta_0^{IV} + w_1^B \beta_0^{V}$	$h_1 = b_1^B + i_1^B \gamma_0'' + s_1^B \gamma_0''' + t_1^B \gamma_0^{IV} + w_1^B \gamma_0^{V}$	$k_1 = -z_1^B - i_1^B \delta_0'' - s_1^B \delta_0''' - t_1^B \delta_0^{IV} - w_1^B \delta_0^{V}$	
	$p_1 = i_1^C(\alpha_0'' - 1) + s_1^C \alpha_0''' + t_1^C \alpha_0^{IV} + w_1^C \alpha_0^{V}$	$n_1 = a_1^C + i_1^C \beta_0'' + s_1^C \beta_0''' + t_1^C \beta_0^{IV} + w_1^C \beta_0^{V}$	$o_1 = b_1^C + i_1^C \gamma_0'' + s_1^C \gamma_0''' + t_1^C \gamma_0^{IV} + w_1^C \gamma_0^{V}$	$q_1 = -z_1^C - i_1^C \delta_0'' - s_1^C \delta_0''' - t_1^C \delta_0^{IV} - w_1^C \delta_0^{V}$	
	$A_1 = \frac{c_1 . h_1 - i_1 . b_1}{c_1 . g_1 - i_1 . a_1}$	$B_1 = \frac{c_1}{c_1 . g_1 - i_1 . a_1}$	$C_1 = \frac{-c_1 . l_1^B - i_1}{c_1 . g_1 - i_1 . a_1}$	$D_1 = \frac{c_1 . k_1 + i_1 . d_1}{c_1 . g_1 - i_1 . a_1}$	
	$A_1' = \frac{c_1 . o_1 - p_1 . b_1}{c_1 . n_1 - p_1 . a_1}$	$B_1' = \frac{c_1}{c_1 . n_1 - p_1 . a_1}$	$C_1' = \frac{-c_1 . l_1^C - p_1}{c_1 . n_1 - p_1 . a_1}$	$D_1' = \frac{c_1 . q_1 + p_1 . d_1}{c_1 . n_1 - p_1 . a_1}$	
	$\alpha_1 = \frac{A_1 . C_1' - A_1' C_1}{A_1 - A_1'}$	$\beta_1 = -\frac{A_1' . B_1}{A_1 - A_1'}$	$\gamma_1 = \frac{A_1 . B_1'}{A_1 - A_1'}$	$\delta_1 = \frac{A_1 . D_1' - A_1' . D_1}{A_1 - A_1'}$	
	$\alpha_1' = \frac{C_1 - C_1'}{A_1 - A_1'}$	$\beta_1' = \frac{B_1}{A_1 - A_1'}$	$\gamma_1' = -\frac{B_1'}{A_1 - A_1'}$	$\delta_1' = \frac{D_1 - D_1'}{A_1 - A_1'}$	
	$\alpha_1'' = \frac{1}{c_1} - \frac{a_1 . \alpha_1}{c_1} - \frac{b_1 . \alpha_1'}{c_1}$	$\beta_1'' = -\frac{a_1 . \beta_1}{c_1} - \frac{b_1 . \beta_1'}{c_1}$	$\gamma_1'' = -\frac{a_1 . \gamma_1}{c_1} - \frac{b_1 . \gamma_1'}{c_1}$	$\delta_1'' = -\frac{d_1}{c_1} - \frac{a_1 . \delta_1}{c_1} - \frac{b_1 . \delta_1'}{c_1}$	
	$\alpha_1''' = \theta_0 . \alpha_1'' + \phi_0 . \alpha_1 + \psi_0 . \alpha_1'$	$\beta_1''' = \theta_0 . \beta_1'' + \phi_0 . \beta_1 + \psi_0 . \beta_1'$	$\gamma_1''' = \theta_0 . \gamma_1'' + \phi_0 . \gamma_1 + \psi_0 . \gamma_1'$	$\delta_1''' = \omega_0 + \theta_0 . \delta_1'' + \phi_0 . \delta_1 + \psi_0 . \delta_1'$	
	$\alpha_1^{IV} = \theta_0' . \alpha_1'' + \phi_0' . \alpha_1 + \psi_0' . \alpha_1'$	$\beta_1^{IV} = \theta_0' \beta_1'' + \phi_0' . \beta_1 + \psi_0' . \beta_1'$	$\gamma_1^{IV} = \theta_0' \gamma_1'' + \phi_0' \gamma_1 + \psi_0' \gamma_1'$	$\delta_1^{IV} = \omega_0' + \theta_0' \delta_1'' + \phi_0' . \delta_1 + \psi_0' \delta_1'$	
	$\alpha_1^{V} = \theta_0'' \alpha_1'' + \phi_0'' \alpha_1 + \psi_0'' \alpha_1'$	$\beta_1^{V} = \theta_0'' \beta_1'' + \phi_0'' \beta_1 + \psi_0'' \beta_1'$	$\gamma_1^{V} = \theta_0'' \gamma_1'' + \phi_0'' \gamma_1 + \psi_0'' \gamma_1'$	$\delta_1^{V} = \omega_0'' + \theta_0'' \delta_1'' + \phi_0'' \delta_1 + \psi_0'' \delta_1'$	
	$\alpha_1^{VI} = (\alpha_0'' - 1) \alpha_1'' + \beta_0'' \alpha_1 + \gamma_0'' \alpha_1'$	$\beta_1^{VI} = (\alpha_0'' - 1) \beta_1'' + \beta_0'' \beta_1 + \gamma_0'' \beta_1'$	$\gamma_1^{VI} = (\alpha_0'' - 1) \gamma_1'' + \beta_0'' \gamma_1 + \gamma_0'' \gamma_1'$	$\delta_1^{VI} = \delta_0'' + (\alpha_0'' - 1) \delta_1'' + \beta_0'' \delta_1 + \gamma_0'' \delta_1'$	
	$\theta_1 = \alpha_1''' + i_2^{C-A} (\alpha_1^{VI} - 1)$	$\phi_1 = a_2^{C-A} + \beta_1''' + i_2^{C-A} \beta_1^{VI}$	$\psi_1 = \gamma_1''' + i_2^{C-A} \gamma_1^{VI}$	$\omega_1 = z_2^{C-A} + \delta_1''' + i_2^{C-A} \delta_1^{VI}$	
	$\theta_1' = \alpha_1^{IV} + i_2^{C-B} (\alpha_1^{VI} - 1)$	$\phi_1' = \beta_1^{IV} + i_2^{C-B} \beta_1^{VI}$	$\psi_1' = b_2^{C-B} + \gamma_1^{IV} + i_2^{C-B} \gamma_1^{VI}$	$\omega_1' = z_2^{C-B} + \delta_1^{IV} + i_2^{C-B} \delta_1^{VI}$	
	$\theta_1'' = \alpha_1^{V}$	$\phi_1'' = a_2^{\Delta-A} + \beta_1^{V}$	$\psi_1'' = b_2^{\Delta-a} + \gamma_1^{V}$	$\omega_1'' = z_2^{\Delta-A} + \delta_1^{V}$	
	TABULATED VALUES OF THE COEFFICIENTS IN THE SOLUTION FOR THE 2ND STORY				
1	$c_2 = i_2^A(\alpha_1^{VI} - 1) + s_2^A \alpha_1''' + t_2^A \alpha_1^{IV} + p_2^A$	$a_2 = a_2^A + i_2^A \beta_1^{VI} + s_2^A \beta_1''' + t_2^A \beta_1^{IV}$	$b_2 = b_2^A + i_2^A \gamma_1^{VI} + s_2^A \gamma_1''' + t_2^A \gamma_1^{IV}$	$d_2 = z_2^A + i_2^A \delta_1^{VI} + s_2^A \delta_1''' + t_2^A \delta_1^{IV}$	1
2	$i_2 = i_2^B(\alpha_1^{VI} - 1) + s_2^B \alpha_1''' + t_2^B \alpha_1^{IV} + w_2^B \alpha_1^{V}$	$g_2 = a_2^B + i_2^B \beta_1^{VI} + s_2^B \beta_1''' + t_2^B \beta_1^{IV} + w_2^B \beta_1^{V}$	$h_2 = b_2^B + i_2^B \gamma_1^{VI} + s_2^B \gamma_1''' + t_2^B \gamma_1^{IV} + w_2^B \gamma_1^{V}$	$k_2 = -z_2^B - i_2^B \delta_1^{VI} - s_2^B \delta_1''' - t_2^B \delta_1^{IV} - w_2^B \delta_1^{V}$	2
3	$p_2 = i_2^C(\alpha_1^{VI} - 1) + s_2^C \alpha_1''' + t_2^C \alpha_1^{IV} + w_2^C \alpha_1^{V}$	$n_2 = a_2^C + i_2^C \beta_1^{VI} + s_2^C \beta_1''' + t_2^C \beta_1^{IV} + w_2^C \beta_1^{V}$	$o_2 = b_2^C + i_2^C \gamma_1^{VI} + s_2^C \gamma_1''' + t_2^C \gamma_1^{IV} + w_2^C \gamma_1^{V}$	$q_2 = -z_2^C - i_2^C \delta_1^{VI} - s_2^C \delta_1''' - t_2^C \delta_1^{IV} - w_2^C \delta_1^{V}$	3
4	$A_2 = \frac{c_2 h_2 - i_2 b_2}{c_2 g_2 - i_2 . a_2}$	$B_2 = \frac{c_2}{c_2 g_2 - i_2 . a_2}$	$C_2 = \frac{-c_2 l_2^B - i_2}{c_2 g_2 - i_2 . a_2}$	$D_2 = \frac{c_2 k_2 + i_2 d_2}{c_2 . g_2 - i_2 . a_2}$	4
5	$A_2' = \frac{c_2 . o_2 - p_2 . b_2}{c_2 . n_2 - p_2 . a_2}$	$B_2' = \frac{c_2}{c_2 n_2 - p_2 a_2}$	$C_2' = \frac{-c_2 l_2^C - p_2}{c_2 n_2 - p_2 a_2}$	$D_2' = \frac{c_2 . q_2 - p_2 . d_2}{c_2 . n_2 - p_2 . a_2}$	5
6	$\alpha_2 = \frac{A_2 C_2' - A_2' . C_2}{A_2 - A_2'}$	$\beta_2 = -\frac{A_2' . B_2}{A_2 - A_2'}$	$\gamma_2 = \frac{A_2 . B_2'}{A_2 - A_2'}$	$\delta_2 = \frac{A_2 D_2' - A_2' . D_2}{A_2 - A_2'}$	6
7	$\alpha_2' = \frac{C_2 - C_2'}{A_2 - A_2'}$	$\beta_2' = \frac{B_2}{A_2 - A_2'}$	$\gamma_2' = -\frac{B_2'}{A_2 - A_2'}$	$\delta_2' = \frac{D_2 - D_2'}{A_2 - A_2'}$	7
8	$\alpha_2'' = \frac{1}{c_2} - \frac{a_2 . \alpha_2}{c_2} - \frac{b_2 . \alpha_2'}{c_2}$	$\beta_2'' = -\frac{a_2 \beta_2}{c_2} - \frac{b_2 \beta_2'}{c_2}$	$\gamma_2'' = -\frac{a_2 . \gamma_2}{c_2} - \frac{b_2 . \gamma_2'}{c_2}$	$\delta_2'' = -\frac{d_2}{c_2} - \frac{a_2 d_2}{c_2} - \frac{b_2 . \delta_2'}{c_2}$	8
9	$\alpha_2''' = \theta_1 . \alpha_2'' + \phi_1 . \alpha_2 + \psi_1 . \alpha_2'$	$\beta_2''' = \theta_1 . \beta_2'' + \phi_1 . \beta_2 + \psi_1 . \beta_2'$	$\gamma_2''' = \theta_1 . \gamma_2'' + \phi_1 . \gamma_2 + \psi_1 . \gamma_2'$	$\delta_2''' = \omega_1 + \theta_1 . \delta_2'' + \phi_1 . \delta_2 + \psi_1 . \delta_2'$	9
10	$\alpha_2^{IV} = \theta_1' \alpha_2'' + \phi_1' \alpha_2 + \psi_1' \alpha_2'$	$\beta_2^{IV} = \theta_1' . \beta_2'' + \phi_1' . \beta_2 + \psi_1' . \beta_2'$	$\gamma_2^{IV} = \theta_1' \gamma_2'' + \phi_1' \gamma_2 + \psi_1' . \gamma_2'$	$\delta_2^{IV} = \omega_1' + \theta_1' \delta_2'' + \phi_1' \delta_2 + \psi_1' \delta_2'$	10
11	$\alpha_2^{V} = \theta_1'' \alpha_2'' + \phi_1'' \alpha_2 + \psi_1'' \alpha_2'$	$\beta_2^{V} = \theta_1'' \beta_2'' + \phi_1'' \beta_2 + \psi_1'' . \beta_2'$	$\gamma_2^{V} = \theta_1'' \gamma_2'' + \phi_1'' \gamma_2 + \psi_1'' \gamma_2'$	$\delta_2^{V} = \omega_1'' + \theta_1'' \delta_2'' + \phi_1'' \delta_2 + \psi_1'' \delta_2'$	11
12	$\alpha_2^{VI} = (\alpha_1^{VI} - 1) \alpha_2'' + \beta_1^{VI} \alpha_2 + \gamma_1^{VI} \alpha_2'$	$\beta_2^{VI} = (\alpha_1^{VI} - 1) \beta_2'' + \beta_1^{VI} \beta_2 + \gamma_1^{VI} \beta_2'$	$\gamma_2^{VI} = (\alpha_1^{VI} - 1) \gamma_2'' + \beta_1^{VI} \gamma_2 + \gamma_1^{VI} \gamma_2'$	$\delta_2^{VI} = \delta_1^{VI} + (\alpha_1^{VI} - 1) \delta_2'' + \beta_1^{VI} \delta_2 + \gamma_1^{VI} \delta_2'$	12
13	$\theta_2 = \alpha_2''' + i_3^{C-A} (\alpha_2^{VI} - 1)$	$\phi_2 = a_3^{C-A} + \beta_2''' + i_3^{C-A} \beta_2^{VI}$	$\psi_2 = \gamma_2''' + i_3^{C-A} \gamma_2^{VI}$	$\omega_2 = z_3^{C-A} + \delta_2''' + i_3^{C-A} \delta_2^{VI}$	13
14	$\theta_2' = \alpha_2^{IV} + i_3^{C-B} (\alpha_2^{VI} - 1)$	$\phi_2' = \beta_2^{IV} + i_3^{C-B} \beta_2^{VI}$	$\psi_2' = b_3^{C-B} + \gamma_2^{IV} + i_3^{C-B} \gamma_2^{VI}$	$\omega_2' = z_3^{C-B} + \delta_2^{IV} + i_3^{C-B} \delta_2^{VI}$	14
15	$\theta_2'' = \alpha_2^{V}$	$\phi_2'' = a_3^{\Delta-a} + \beta_2^{V}$	$\psi_2'' = b_3^{\Delta-a} + \gamma_2^{V}$	$\omega_2'' = z_3^{\Delta-a} + \delta_2^{V}$	15

PLATE 17.

Writing out the equation represented by the coefficients in line 12 of the last story gives

$$S_0^B - H_0 - H_1 - \cdots - H_{n-1} = \alpha_n^{\text{vi}} H_n + \beta_n^{\text{vi}} M_{n+1}^A + \gamma_n^{\text{vi}} M_{n+1}^B + \delta_n^{\text{vi}},$$

where the nth is the last story; but there are no terms M_{n+1}^A and M_{n+1}^B, hence the coefficients β_n^{vi} and γ_n^{vi} may be omitted as mentioned above and the equation may be written

$$S_0^B - H_0 - H_1 - \cdots - H_{n-1} = \alpha_n^{\text{vi}} H_n + \delta_n^{\text{vi}}.$$

Now, from the principle that the sum of the horizontal forces acting on a section must equal zero, a vertical section being taken so as to cut through all the floor girders of the structure and, the portion of the building on the right of this section being considered,

$$S_0^B - H_0 - H_1 - \cdots - H_{n+1} = H_n - \Sigma F^B.$$

Hence, making this substitution in the equation above

$$H_n(1 - \alpha_n^{\text{vi}}) = \delta_n^{\text{vi}} + \Sigma F^B,$$

which gives the value of H_n. Now, starting at the bottom of the tabular computations and using this value of H_n, find, in the last story, the values of M_n^A and M_n^B. Then, using these values of H_n, M_n^A and M_n^B, in the next to the last story deduce the values of all quantities whose coefficients have been tabulated for this story. Again, using the values thus found for H_{n-1} M_{n-1}^A, and M_{n-1}^B evaluate similarly the unknowns of the story below. This operation may be repeated until the unknowns for all stories have been evaluated.

Checks.—The following checks may now be applied.

First. After all values of H and S_0^B are computed see if the condition given above, *i. e.*,

$$S_0^B - H_0 - H_1 - \cdots - H_{n-1} = H_n - \Sigma F^B,$$

is satisfied. This condition may be satisfied and yet all results may be very far from the true solution.

Second. The conditions forming, when expressed symbolically, equations A are simple geometrical ones and must be fulfilled for *every story.*

This is indeed a very delicate check and, when satisfied for *all* stories, assures that the calculations for all quantities are free from error, with the possible exception of the quantity Δ, the coefficients for which are so small.

Third. When there are a large number of stories it will be very advisable to use a check calculation in connection with Plate 17, somewhat similar to that used by Gauss in solution of normal equations. This will enable the calculator to check his work at each story as it progresses.

The check on the values of H is given first because, for a few stories only, it is naturally the first check available. The second is the real check of them all. If desired, the values of C^A, and C^B may be evaluated from their original expressions using the values of H, M^A, and M^B found in the solution. Likewise any other condition of the problem may be tested, but if the second check is fully satisfied these extra checks are unnecessary. It is well to note that all the calculations necessary may be quite accurately made on a Thacher slide rule, for the most part without any scratch figures not shown in the blanks. Calculations for various loadings differ only in the fourth column of Plate 17.

CHAPTER IV.

A STUDY OF THE EFFECT OF ECCENTRIC LOADING ON COLUMNS.

This little chapter is introduced in order, first, to present the results arrived at by the late Professor A. H. Heller in "Stresses in Structures" in what is considered a more comprehensive manner, and, second, to give a simple application of the general systematic method of solving the stresses in a structure composed of several stories. No new conclusions are pointed out here and the reader is referred to Chapter VII, IX, and XVI for original calculations and conclusions.

The equations of the elastic line for any section of the column shown in Fig. 7, *a*, will, in general, be the same as those given on page 39 of Chapter III. (Compare Fig. 6, *b*.) These equations are

$$E\frac{d^2y}{dx^2} = \frac{M}{I} - \frac{S}{I}x; \quad E\frac{dy}{dx} = \frac{M}{I}x - \frac{S}{2I}x^2 + K;$$

$$Ey = \frac{M}{2I}x^2 - \frac{S}{6I}x^3 + Kx + K'.$$

The column taken as an illustration for this chapter is a five story one, Fig. 7, *a*, with eccentric loads on the same side of the column at every floor. The column is considered as of constant moment of inertia from top to bottom. This column may be regarded as one of two symmetrical columns of a two column cross section symmetrically loaded, the two columns being joined by shallow girders which are considered as pin connected. The arrows, Fig. 7, *a*, indicate the assumed direction of the axial stresses in the girders; any stress coming out negative in the solution must act in a direction opposite to that shown. Assuming that the girders do not change in length under stress, a vertical line through the foot of the column will pass through all the intersections of girders with the column, *i. e.*, $\delta = 0$, for each column section. Also assume the column to be held vertically at the anchorage. Consider the column section for each story as having a separate origin

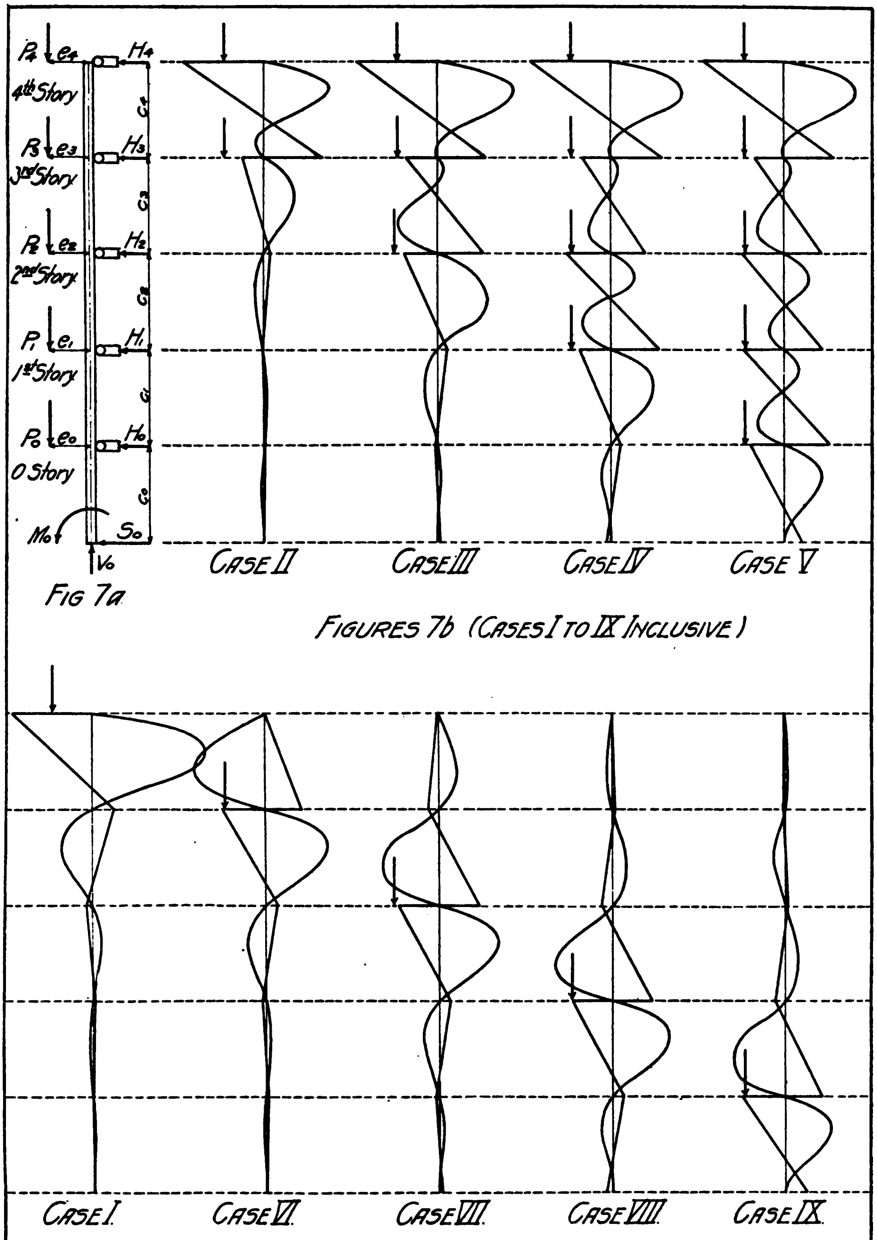


FIG 7a

FIGURES 7b (CASES I TO IX INCLUSIVE)

NOTE—Bending Moments and Deformations Measured Positively to the Right of coordinates at the foot of the column section. Then, as in Chapter III, $K = C$, and $K' = 0$ and the column slopes and deflections may be written as follows:

E_x Column Slopes.

$$C_0 = 0,$$

$$C_1 = \frac{M_0 c_0}{I_0} - \frac{S_0 c_0^2}{2I_0},$$

$$C_2 = \frac{M_1 c_1}{I_1} - \frac{S_1 c_1^2}{2I_1} + C_1,$$

$$C_3 = \frac{M_2 c_2}{I_2} - \frac{S_2 c_2^2}{2I_2} + C_2.$$

E_x Column Deflections.

$$\delta_0 = \frac{M_0 c_0^2}{2I_0} - \frac{S_0 c_0^3}{6I_0} = 0,$$

$$\delta_1 = \frac{M_1 c_1^2}{2I_1} - \frac{S_1 c_1^3}{6I_1} + C_1 c_1 = 0,$$

$$\delta_2 = \frac{M_2 c_2^2}{2I_2} - \frac{S_2 c_2^3}{6I_2} + C_2 c_2 = 0.$$

The column bending moment existing just above the top of the nth story is M_{n+1} which must equal $- S_n c_n + M_n + P_n e_n$. These equations may be written for all values of n as follows:

$$\begin{aligned} M_n - S_n c_n &= M_{n+1} - P_n e_n, \\ M_0 - S_0 c_0 &= M_1 - P_0 e_0, \\ M_1 - S_1 c_1 &= M_2 - P_1 e_1, \\ M_2 - S_2 c_2 &= M_3 - P_2 e_2. \end{aligned}$$

Now, rewriting the three sets of equations above symbolically using coefficients corresponding to Chapter III as nearly as may be,

Equations C.

$$\begin{aligned} C_0 &= 0, \\ C_1 &= a_0^C M_0 + i_0^C S_0, \\ C_2 &= a_1^C M_1 + i_1^C S_1 + C_1, \\ C_3 &= a_2^C M_2 + i_2^C S_2 + C_2. \end{aligned}$$

Equations δ.

$$\begin{aligned} a_0^\delta M_0 + i_0^\delta S_0 &= 0, \\ a_1^\delta M_1 + i_1^\delta S_1 + s_1^\delta C_1 &= 0, \\ a_2^\delta M_2 + i_2^\delta S_2 + s_2^\delta C_2 &= 0. \end{aligned}$$

Equations M.

$$\begin{aligned} M_0 + i_0^M S_0 &= M_1 + z_0^M, \\ M_1 + i_1^M S_1 &= M_2 + z_1^M, \\ M_2 + i_2^M S_2 &= M_3 + z_2^M. \end{aligned}$$

The values of the coefficients in the above equations are:

Equation.	a_n.	i_n,	s_n.	z_n.
C	$C_n \div I_n$	$-C_n^2 \div 2I_n$	1.0	0
δ	$C_n^2 \div 2I_n$	$-C_n^3 \div 6I_n$	C_n	0
M	0	$-C_n$	0	$-P_n e_n$

These equations are now solved as follows (cf. Chapter III, Plate 16):

From equation δ_0 $S_0 = \alpha_0 M_0 + \beta_0$.
Substituting this value of S_0 in equation C_1 $C_1 = \alpha_0' M_0 + \beta_0'$.
Substituting this value of S_0 in equation M_0 $M_0 = \alpha_1 M_1 + \beta_1$.
Substituting this value of M_0 in above value of S_0 $S_0 = \alpha_1' M_1 + \beta_1'$.
Substituting this value of M_0 in above value of C_1 $C_1 = \alpha_1'' M_1 + \beta_1''$.
Substituting this value of C_1 in equation δ_1 $S_1 = \alpha_1''' M_1 + \beta_1'''$.
Substituting this value of S_1 and C_1 in equation C_2 $C_2 = \alpha_1^{\text{iv}} M_1 + \beta_1^{\text{iv}}$.
Substituting this value of S_1 in equation M_1 $M_1 = \alpha_2 M_2 + \beta_2$.
Substituting this value of M_1 in above value of S_1 $S_1 = \alpha_2' M_2 + \beta_2'$.
Substituting this value of M_1 in above value of C_2 $C_2 = \alpha_2'' M_2 + \beta_2''$.
Substituting this value of C_2 in equation δ_2 $S_2 = \alpha_2''' M_2 + \beta_2'''$.
Substituting this value of S_2 and C_2 in equation C_3 $C_3 = \alpha_2^{\text{iv}} M_2 + \beta_2^{\text{iv}}$.

The solution may thus be continued as above for all succeeding stories. In this solution the Greek letter coefficients have the following values.

VALUES OF GREEK LETTER COEFFICIENTS.

Zero Story.

S_0	$\alpha_0 = -a_0^\delta \div i_0^\delta$	$\beta_0 = 0$
C_1	$\alpha_0' = a_0^c + i_0^c \alpha_0$	$\beta_0' = 0$

First Story.

M_0	$\alpha_1 = 1 \div (1 + i_0^M \alpha_0)$	$\beta_1 = z_0^M \div (1 + i_0^M \alpha_0)$
S_0	$\alpha_1' = \alpha_0 \alpha_1$	$\beta_1' = \alpha_0 \beta_1$
C_1	$\alpha_1'' = \alpha_0' \alpha_1$	$\beta_1'' = \alpha_0' \beta_1$
S_1	$\alpha_1''' = (-a_1^\delta - s_1^\delta \alpha_1'') \div i_1^\delta$	$\beta_1''' = -s_1^\delta \beta_1'' \div i_1^\delta$
C_2	$\alpha_1^{\text{iv}} = a_1^c + i_1^c \alpha_1''' + \alpha_1''$	$\beta_1^{\text{iv}} = i_1^c \beta_1''' + \beta_1''$

Second Story

M_1	$\alpha_2 = 1 \div (1 + i_1^M \alpha_1''')$	$\beta_2 = (z_1^M - i_1^M \beta_1''') \div (1 + i_1^M \alpha_1''')$
S_1	$\alpha_2' = \alpha_1''' \alpha_2$	$\beta_2' = \beta_1''' + \alpha_1''' \beta_2$
C_2	$\alpha_2'' = \alpha_1^{\text{iv}} \alpha_2$	$\beta_2'' = \beta_1^{\text{iv}} + \alpha_1^{\text{iv}} \beta_2$
S_2	$\alpha_2''' = (-a_2^\delta - s_2^\delta \alpha_2'') \div i_2^\delta$	$\beta_2''' = -s_2^\delta \beta_2'' \div i_2^\delta$
C_3	$\alpha_2^{\text{iv}} = a_2^c + i_2^c \alpha_2''' + \alpha_2''$	$\beta_2^{\text{iv}} = i_2^c \beta_2''' + \beta_2''$

The values of these coefficients for all other stories will be the same as for the second story if proper values are assigned to the subscripts in place of those given.

Application of Theory.—The following example will be taken to illustrate the application of this method of solution, as well as to furnish data for discussion.

Assume the column five stories in height, all story heights, c, equal to 100 in., and the moment of inertia of the column section constant for all sections and equal to 1,000$''^4$. Also assume, the eccentric loads P to be 10,000 lb each and their eccentricities, e, all the same and equal to 10 in. The following table gives the values of the symbolic coefficients a, i, s, and z.

Coefficient.	Story.				
	0	1	2	3	4
a^c	0.1	0.1	0.1	0.1	0.1
a^δ	5.0	5.0	5.0	5.0	5.0
a^M	0	0	0	0	0
i^c	−5.0	−5.0	−5.0	−5.0	−5.0
i^δ	−166.67	−166.67	−166.67	−166.67	−166.67
i^M	−100.00	−100.00	−100.00	−100.00	−100.00
s^c	1.0	1.0	1.0	1.0	1.0
s^δ	100.00	100.00	100.00	100.00	100.00
s^M	0	0	0	0	0
z^c	0	0	0	0	0
z^δ	0	0	0	0	0
z^M	−100,000	−100,000	−100,000	−100,000	−100,000

Of course, for those stories where loads are left off z will be 0.

The solution is now made for the following cases:

Case I.—Load P_4 only.

Case II.—Loads P_4 and P_3 only.

Case III.—Loads P_4, P_3, and P_2 only.

Case IV.—Loads P_4, P_3, P_2, and P_1 only.

Case V.—Loads P_4, P_3, P_2, P_1, and P_0.

The solutions for all five cases are given in the following table which, in form, corresponds to the table of Greek letter coefficients. The first and second column under each case gives values of α and β, while the third column gives values of the quantities S, M, and C, as shown in the corresponding lines at the left of the table. The third column, in each case, is calculated from the bottom up because S_4 will be the first quantity found and will be determined by

	α, All Cases.	Case I.		Case II.		Case III.		Case IV.		Case V.	
		β, Case I.	Results.	β, Case II.	Results.	β, Case III.	Results.	β, Case IV.	Results.	β, Case V.	Results.
S_0	α_0 +0.03	β_0 0	+ 8.3	β_0 0	− 8.3	β_0 0	+ 49.6	β_0 0	− 165.9	β_0 0	+ 637.5
C_1	α_0' −0.05	β_0' 0	− 13.9	β_0' 0	+ 13.8	β_0' 0	− 82.8	β_0' 0	+ 276.5	β_0' 0	−1,062.5
M_0	α_1 −0.50	β_1 0	+ 277.5	β_1 0	− 275.0	β_1 0	+1,655	β_1 0	−5,530	β_1 +50,000	+21,250
S_0	α_1' −0.015	β_1' 0	+ 8.3	β_1' 0	− 8.3	β_1' 0	+ 49.7	β_1' 0	− 165.7	β_1' + 1,500	+ 639
C_1	α_1'' +0.025	β_1'' 0	− 13.9	β_1'' 0	+ 13.8	β_1'' 0	− 82.7	β_1'' 0	+ 276	β_1'' − 2,500	− 1,063
S_1	α_1''' +0.045	β_1''' 0	− 25.0	β_1''' 0	+ 24.8	β_1''' 0	− 149.0	β_1''' 0	+ 497.5	β_1''' − 1,500	+ 1,085
C_2	α_1^{iv} −0.100	β_1^{iv} 0	+ 55.5	β_1^{iv} 0	− 55.0	β_1^{iv} 0	+ 331.0	β_1^{iv} 0	−1,105.0	β_1^{iv} + 5,000	− 745
M_1	α_2 −0.285	β_2 0	− 55.5	β_2 0	+ 550	β_2 0	−3,310	β_2 +28,550	+11,050	β_2 +71,400	+57,450
S_1	α_2' −0.0128	β_2' 0	− 25.0	β_2' 0	+ 24.8	β_2' 0	− 149.0	β_2' + 1,285	+ 498	β_2' + 1,715	+ 1,086
C_2	α_2'' +0.0285	β_2'' 0	+ 55.5	β_2'' 0	− 55.0	β_2'' 0	+ 331.0	β_2'' − 2,855	− 1,105	β_2'' − 2,140	− 745
S_2	α_2''' +0.0471	β_2''' 0	+ 91.8	β_2''' 0	− 90.9	β_2''' 0	+ 548.0	β_2''' − 1,713	+ 1,177	β_2''' − 1,284	+ 1,026
C_3	α_2^{iv} −0.1070	β_2^{iv} 0	− 208	β_2^{iv} 0	+ 207	β_2^{iv} 0	−1,242	β_2^{iv} + 5,710	− 850	β_2^{iv} + 4,280	− 960
M_2	α_3 −0.270	β_3 0	+1,945	β_3 0	−1,930	β_3 +27,000	+11,600	β_3 +73,100	+61,280	β_3 +61,600	+48,870
S_2	α_3' −0.0127	β_3' 0	+ 91.4	β_3' 0	− 90.9	β_3' + 1,272	+ 548	β_3' + 1,702	+ 1,145	β_3' + 1,616	+ 1,017
C_3	α_3'' +0.0289	β_3'' 0	− 208	β_3'' 0	+ 207	β_3'' − 2,885	− 1,235	β_3'' − 2,120	− 855	β_3'' − 2,320	− 960
S_3	α_3''' +0.0473	β_3''' 0	− 340	β_3''' 0	+ 338	β_3''' − 1,731	+ 969	β_3''' − 1,272	+ 798	β_3''' − 1,392	+ 838
C_4	α_3^{iv} −0.1076	β_3^{iv} 0	+ 773	β_3^{iv} 0	− 769	β_3^{iv} + 5,770	− 360	β_3^{iv} + 4,240	− 470	β_3^{iv} + 4,640	− 430
M_3	α_4 −0.268	β_4 0	−7,200	β_4 +26,800	+7,150	β_4 +73,200	+57,000	β_4 +61,000	+43,820	β_4 +64,000	+47,100
S_3	α_4' −0.0127	β_4' 0	− 340	β_4' + 1,266	+ 338	β_4' + 1,724	+ 956	β_4' + 1,618	+ 807	β_4' + 1,638	+ 840
C_4	α_4'' +0.0288	β_4'' 0	+ 774	β_4'' − 2,880	− 765	β_4'' − 2,100	− 358	β_4'' − 2,320	− 478	β_4'' − 2,250	− 435
S_4	α_4''' +0.0473	β_4''' 0	+1,267	β_4''' − 1,728	+1,737	β_4''' − 1,260	+ 1,600	β_4''' − 1,392	+ 1,638	β_4''' − 1,350	+ 1,630
C_5	α_4^{iv} −0.1076	β_4^{iv} 0	−2,885	β_4^{iv} + 5,760	−2,140	β_4^{iv} + 4,200	− 2,300	β_4^{iv} + 4,640	− 2,250	β_4^{iv} + 4,500	− 2,280
M_4	α_5 0	β_5 +26,800	+26,800	β_5 +73,300	+ 73,300	β_5 +60,500	+60,500	β_5 +64,000	+64,000	β_5 +63,000	+63,000
		M_5'	−99,900	M_5'	−100,400	M_5'	−99,500	M_5'	−99,800	M_5'	−100,000

$S_4 = \beta'_5$, since, in the equation $S_4 = \alpha'_5 M_5 + \beta'_5$, $M_5 = 0$ as there is no restraining flexural influence at the top of the column. The column α will be the same for all five cases since α is independent of the type of loading; this column is therefore only given once. As will be noted frequent checks are obtained in the last column in each case. The calculations are as follows.

M'_5 represents the bending moment just *below* the load P_4.

Now, by combining the results above four other important cases are secured as follows:

Case VI.—Load P_0 only—by subtracting Case IV from Case V.
Case VII.—Load P_1 only—by subtracting Case III from Case IV.
Case VIII.—Load P_2 only—by subtracting Case II from Case III.
Case IX.—Load P_3 only—by subtracting Case I from Case II.

Figs. 7, *b*, show the bending moment diagrams and the deflection curves (values of Ey) for all nine cases, to scale.

Scale for bending moments 1 in. = 240,000 in. lbs.
Scale for Deflections (Ey) 1 in. = 80,000 in.

The values of M, S, H, and C for all cases are given in the following table. Figs. 7, *b*, clearly show the fact that the effect of the eccentric load is very small at points two or more stories away from the load.

PRESENT PRACTICE IN DESIGNING FOR ECCENTRIC LOADS.

Class A Buildings.—Referring to Fig. 5, *a*, of Chapter II, suppose the structure there shown to have diagonal bracing and it is desired to design the two story column section marked "*AA*." Evidently the lower or second story portion of the column will receive the heaviest load and the axial stress in this portion will consist of the dead, live, and wind loads from above plus the column weight and fireproofing for the second story.

The eccentric load P_n^A would be considered as producing, at the point marked "x" on the extreme outer fibre, a unit compressive stress equal to $P_n^A e_n^A v/I$ and the effect of all other eccentric loads, above or below, on the design of this column section is regarded as nil.

Class B Buildings.—As was shown in Chapter II, if the girders are considered rigid (and this is the assumption made in designing

	Case I.				Case II.				Case III.			
	M	*S*	*H*	*C*	*M*	*S*	*H*	*C*	*M*	*S*	*H*	*C*
0	+ 277.5	+ 8.3	− 33.3	− 13.9	− 275	− 8.3	+ 33.1	+ 13.8	+ 1,655	+ 49.7	− 198.7	− 82.7
1	− 555	− 25.0	+ 116.4	+ 55.5	+ 550	+ 24.8	− 115.7	− 55.0	− 3,310	− 149.0	+ 697.0	+ 331.0
2	+ 1,945	+ 91.6	− 431.4	− 208.0	− 1,930	− 90.9	+ 428.9	+ 207.0	+11,600	+ 548.0	+ 408.0	−1,238.0
3	− 7,200	− 340.0	+1,607.0	+ 774.0	+ 7,150	+ 338.0	+1,399.0	− 767.0	+57,000	+ 956.0	+ 644.0	− 359.0
4	+26,800	+1,267.0	−1,267.0	−2,885.0	+73,300	+1,737.0	−1,737.0	−2,140.0	+60,500	+1,600.0	−1,600.0	−2,300.0
	Case IV.				Case V.				Case VI.			
0	− 5,530	− 165.9	+ 663.7	+ 276.0	+21,250	+ 639.0	+ 447.0	−1,063.0	− 552.5	− 16.6	+ 66.4	+ 27.7
1	+11,050	+ 497.8	+ 647.0	−1,105.0	+57,450	+1,086.0	− 69.0	− 745.0	+ 1,105.0	+ 49.8	− 232.1	− 110.5
2	+61,280	+1,145.0	− 338.0	− 852.0	+48,870	+1,017.0	− 177.0	− 960.0	− 3,875.0	− 182.3	+ 860.3	+ 415.0
3	+43,820	+ 807.0	+ 831.0	− 474.0	+47,100	+ 840.0	+ 790.0	− 432.0	+14,350.0	+ 678.0	− 208.0	−1,541.0
4	+64,000	+1,638.0	−1,638.0	−2,250.0	+63,000	+1,630.0	−1,630.0	−2,280.0	+46,500.0	+ 470.0	− 470.0	+ 745.0
	Case VII.				Case VIII.				Case IX.			
0	+ 1,930	+ 58.0	− 231.8	− 96.5	− 1,185	− 215.4	+ 862.4	+ 358.7	+26,780	+ 804.7	− 216.7	−1,339.0
1	− 3,860	− 173.8	+ 812.7	+ 386.0	+14,360	+ 647.0	− 50.0	−1,436.0	+46,400	+ 588.0	− 716.0	+ 360.0
2	+13,530	+ 638.9	− 20.9	−1,445.0	+49,680	+ 597.0	− 746.0	+ 386.0	−12,410	− 128.0	+ 161.0	− 108.0
3	+49,850	+ 618.0	− 755.0	+ 408.0	−13,180	− 149.0	+ 187.0	− 115.0	+ 3,280	+ 33.0	− 41.0	+ 42.0
4	−12,800	− 137.0	+ 137.0	− 160.0	+ 3,500	+ 38.0	− 38.0	+ 50.0	− 1,000	− 8.0	+ 8.0	− 30.0

practice) the column can receive no bending moment from eccentric loading, no matter how large the load or its eccentricity, *i. e.*, the column is designed for axial stress and bending due to wind only.

CONCLUSIONS POINTED OUT BY PROFESSOR HELLER AND VERIFIED IN THIS CHAPTER.

First. The maximum moment for eccentric loads on a column will always occur for a load at the roof line and there equals Pe (considering all loads and eccentricities as equal).

Second. For equal loads with equal eccentricities at *all* floors, the bending moment approaches the constant value $\frac{1}{2}Pe$ for floors about three stories or more from either top or bottom of the building.

Third. For a single load at any point, the bending moment in the column passes from a value of about $\frac{1}{2}Pe$ just above the load, through zero, to a value of approximately $-\frac{1}{2}Pe$ just below the load.

Professor Heller's figures are based on a column hinged at the anchorage. His results agree very well with the results here given, based on a column fixed vertically at the anchorage.

CHAPTER V.

A STUDY OF THE EFFECT OF HORIZONTAL AND VERTICAL LOADS ON A SINGLE PLATE GIRDER PORTAL. BY THREE METHODS.

In this chapter two portals of extremely different proportions are investigated. One of these is a high portal with comparatively large columns and a very light shallow girder, the other is a low portal with light columns and a deep rigid girder. These two extremes were chosen in order to compare the theory based on rigid girders with that based on an entirely elastic structure. The stresses and distortions are first calculated for each portal on the assumption of rigid girders; then these quantities are calculated by the methods given in Chapter III. Finally each structure is treated as an *arch* of variable moment of inertia and the stresses calculated by Howe's summation method. (See "A Treatise on Arches," by M. A. Howe.) The two portals will be referred to as P_1 and P_2, P_1 being the tall one. The properties assumed for each portal are given as follows:

	P_1	P_2		P_1	P_2
c_0	200	100	$A_0{}^B$	20	15
d_0	20	50	$A_0{}^a$	20	25
b	200	200	F_0	10,000	10,000
$I_0{}^A$	1,000	500	W_0	300	300
$I_0{}^B$	1,000	500	ΣF	10,000	10,000
$I_0{}^a$	1,200	5,000	V^A	7.00	7.00
$A_0{}^A$	20	15	V^B	7.00	7.00

STRESSES AND DEFORMATION — RIGID GIRDERS.

Portal P_1.—For Floor Load w_0 Only.—The total floor load will be $w_0 b = 300$ ℔ × 200 in. = 60,000 ℔. Referring to Chapter II, by symmetry

$$h^A = -h^B = 100 \text{ in.}, \quad X = 60{,}000 \times 0 = 0; \quad Y = 60{,}000.$$

$$s = 0, \quad p = \frac{60{,}000}{20 + 20} = 1{,}500 \text{ ℔ per sq. in.},$$

$$V_0^A = V_0^B = 1{,}500 \times 20 = 30{,}000 \text{ ℔}.$$

$$S_0^A = S_0^B = 0$$

and hence $H_0 = 0$.

$$M_0^A = M_0^B = 0,$$

hence the column receives no bending.

Girder stresses,

top flange,

$$\text{Max. compression} = 30{,}000 \times \frac{100}{20} - \frac{300 \times \overline{100^2}}{40} = 75{,}000 \text{ lb} \quad (x = 100 \text{ in.}),$$

$$\text{Max. tension} = 0 \quad (x = 0 \text{ in. and } x = 200 \text{ in.}).$$

bottom flange,

$$\text{Max. tension} = 75{,}000 \text{ lb} \quad (x = 100 \text{ in.}),$$

$$\text{Max. compression} = 0 \quad (x = 0 \text{ in. and } x = 200 \text{ in.}).$$

The girder does not deform (by assumption). The columns do not change in length (by assumption).

For Wind Load F_0 Only.—$h^A = -h^B = 100$ in. as above;

$$X = 10{,}000 \times 110 = 1{,}100{,}000 \text{ in. lb}; \quad Y = 0.$$

$$s = +\frac{1{,}100{,}000}{20 \times \overline{100^2} + 20 \times \overline{100^2}} = +2.75 \text{ lb per sq. in.}$$

$$p = 0; \quad s^A = -2.75 \times 100 = -275 \text{ lb per sq. in.} = -s^B.$$

$$V_0^A = -V_0^B = -275 \times 20 = -5{,}500 \text{ lb};$$

$$S_0^A = S_0^B = \frac{1{,}000}{2{,}000} \times 10{,}000 = +5{,}000 \text{ lb};$$

hence

$$H_0 = S_0^B = +5{,}000 \text{ lb};$$

$$M_0^A = M_0^B = 5{,}000 \times \frac{200}{2} = +500{,}000 \text{ in. lb}.$$

Max. column compressive fibre stress

$$= f^B = \frac{500{,}000 \times 7.00}{1{,}000} + 275 = 3{,}500 + 275 = 3{,}775 \text{ lb per sq. in.}$$

Girder stresses,

top flange,

$$\text{Max. compression} \frac{5{,}000 \times 200}{40} + 5{,}000 = 30{,}000 \text{ lb} \quad (x = 0),$$

$$\text{Max. tension } \frac{5{,}500 \times 200}{20} - 30{,}000 = 25{,}000 \text{ lb} \quad (x = 200 \text{ in.}).$$

bottom flange,

$$\text{Max. tension } \frac{5{,}000 \times 240}{40} - 5{,}000 = 25{,}000 \text{ lb} \qquad (x = 0),$$

$$\text{Max. compression } \frac{5{,}500 \times 200}{20} - 25{,}000 = 30{,}000 \text{ lb}$$

$$(x = 200 \text{ in.}).$$

The girder does not deform.

For the columns

$$\delta_0^A = \delta_0^B = + \frac{5{,}000 \times \overline{200}^3}{12{,}000} = + 3{,}333{,}333.$$

Portal P_2—For Floor Load w_0 Only.—All calculations for portal P_1 hold also for this portal excepting only those for girder stresses. These girder stresses will be 20/50 times those given for P_1, since this is the ratio of the two girder depths. The girder stresses are then

For top flange, max. compression $= 0.4 \times 75{,}000 = 30{,}000$ lb. Max tension = 0.

For bottom flange, max. tension $= 0.4 \times 75{,}000 = 30{,}000$ lb. Max. compression = 0.

For Wind Load F_0 Only.—

$$h^A = - h^B = 100 \text{ in.}; \quad X = 10{,}000 \times 75 = + 750{,}000 \text{ in. lb};$$

$$Y = 0.$$

$$s = + \frac{750{,}000}{15 \times \overline{100}^2 + 15 \times \overline{100}^2} = + 2.50; \quad p = 0.$$

$$s^A = - 2.50 \times 100 = - 250 \text{ lb per sq. in.} = - s^B.$$

$$V_0^A = - V_0^B = - 250 \times 15 = - 3{,}750 \text{ lb};$$

$$S_0^A = S_0^B = \frac{500}{1{,}000} \times 10{,}000 = + 5{,}000 \text{ lb};$$

hence

$$H_0 = S_0^B = + 5{,}000 \text{ lb}.$$

$$M_0^A = M_0^B = + 5{,}000 \times 50 = + 250{,}000 \text{ in. lb}.$$

Max. column compressive fibre stress

$$= f^B = \frac{250{,}000 \times 7.00}{500} + 250 = 3{,}500 + 250 = 3{,}750 \text{ lb per sq. in.}$$

Girder stresses,

top flange,

$$\text{Max. compression, } \frac{5{,}000 \times 100}{100} + 5{,}000 = +\ 10{,}000 \text{ lb} \quad (x = 0),$$

$$\text{Max. tension, } \frac{3{,}750 \times 200}{50} - 10{,}000 = +\ 5{,}000 \text{ lb} \quad (x = 200).$$

bottom flange,

$$\text{Max. tension, } \frac{5{,}000 \times 200}{100} - 5{,}000 = +\ 5{,}000 \text{ lb} \quad (x = 0),$$

$$\text{Max. compression, } \frac{3{,}750 \times 200}{50} - 5{,}000 = +\ 10{,}000 \text{ lb} \quad (x = 200).$$

The girder does not deform. For the column,

$$\delta_0^A = \delta_0^B = + \frac{5{,}000 \times \overline{100}^3}{6{,}000} = +\ 833{,}333.$$

Stresses and Deformations—Entire Structure Elastic.

Portal P_1 (both loadings calculated separately but in the same table).—Referring to Plate 15, the values of the symbolic coefficients become as shown on page 91.

The simplified equations are:

$$\begin{aligned} H_0 &= a_0^A M_0^A + b_0^A M_0^B + i_0^A S_0^B + z_0^A, \\ 0 &= a_0^B M_0^A + b_0^B M_0^B + i_0^B S_0^B + l_0^B H_0 + z_0^B, \\ 0 &= a_0^C M_0^A + b_0^C M_0^B + i_0^C S_0^B + l_0^C H_0 + z_0^C. \end{aligned}$$

Note that, although $S_1^A = 0$, the coefficient "l_0" was carried in the solution. Now, referring to Plate 17, the solution for the unknowns C_1^A, C_1^B, Δ_0^a, S_0^A and S_0^B is made by filling out calculations corresponding to the values given in the blanks of this plate, using the coefficients given in the above table in so doing. For coefficients of C_1^A, C_1^B, and Δ_0^a use those given in the upper part of the above table excepting where other values of the coefficients are

SYMBOLIC COEFFICIENTS — PORTAL P_1.

	a_0	b_0	e_0	f_0	i_0	l_0	s_0	t_0	w_0	P_0	z_0	
											For F_0.	For w_0.
C–A	+ 0.2	0.0	0	0	− 20.0	0	0	0	0	0	0.0	0
C–B	0.0	+ 0.2	0	0	+ 20.0	0	0	0	0	0	− 200,000.0	0
δ–A	+20.0	0.0	0	0	−1,333.3	0	0	0	0	0	0.0	0
δ–B	0.0	+20.0	0	0	+1,333.3	0	0	0	0	0	−13,333,333.3	0
H–a	0.0	0.0	0	0	− 1.0	+ 1.0	0	0	0	0	+ 10,000	0
λ–a	0.0	0.0	0	0	− 10.0	+ 10.0	0	0	0	0	+ 100,000	0
V–A	+ 0.005	+ 0.005	0	0	0	0	0	0	0	0	− 10,500	+ 30,000
V–B	− 0.005	− 0.005	0	0	0	0	0	0	0	0	+ 10,500	+ 30,000
α–A	+ 0.0525	+ 0.0525	0	0	0	0	0	0	0	0	− 110,250	+ 315,000
α–B	− 0.0525	− 0.0525	0	0	0	0	0	0	0	0	+ 110,250	+ 315,000
Δ–a	− 0.1050	− 0.1050	0	0	0	0	0	0	0	0	+ 220,500	0
Q–a	− 0.000,833	0.0	0	0	+ 0.175	+ 0.00833	0	0	0	0	0	0
R–a	+ 0.000,004,167	+ 0.000,004,167	0	0	0	0	0	0	0	0	− 8.75	+ 25.0
A–a	+22.0	−22.0	0	0	−3,056.7	− 10.0	0	0	0	0	+15,233,333.3	0
B–a	− 0.2833	+ 0.2833	0	0	+ 75.0	+ 1.6667	0	0	0	0	− 375,000	+ 166,667
C–a	− 5.4505	+51.21	0	0	+7,500.0	+166.667	0	0	0	0	−63,553,833.3	+16,666,667
Values of coefficients affected by transformed equations—see Plate 15.												
C–A					+ 20.0					0	− 200,000	0
C–B					− 20.0					0	0	0
Δ–a					0.0					0	+ 220,500	0
A–a					+3,066.7					0	−15,333,333	0
B–a					− 76.667					0	+ 375,000	+ 166,667
C–a					−7,666.7					0	+11,446,167	+16,666,667
Final coefficients after simplifying equations *A*, *B*, and *C*—see text, Chap. III.												
A–a	+ 2.2000	− 2.2000			+ 306.67						− 1,533,333	0
B–a	+ 5.8189	+ 2.4189			+ 460.00	− 10.0					− 6,939,690	− 1,000,000
C–a	+ 2.4189	+ 5.8189			− 460.00	+ 10.0					− 2,439,690	+ 1,000,000

SOLUTION OF EQUATIONS.

			For F_0	For W_0
A_0 +0.960 +1,784 +1.312 −1,012 den. = +772 +2.272	B_0 0	C_0 +3.975 −0.596 +3.379	D_0 +2,758,000 − 914,000 +1,844,000	D_0 +397,500 0 +397,500
A_0' +1.018 $A_0 - A_0' = +1.832$ −0.578 + 741.5 +1012.0 den. = +1,753 +0.440	B_0' 0	C_0' −1.752 +0.262 −1.490	D_0' +427,500 +402,500 +830,000	D_0' −175,200 0 −175,200
α_0 −1.847 −0.811 −2.658	B_0 0	γ_0 0	δ_0 +1,029,000 − 443,000 +586,000	δ_0 −217,300 − 95,500 −312,800
α_0' $\alpha_0 + \alpha_0' = -0.003$ +2.655	B_0' 0	γ_0' 0	δ_0' $\delta_0 + \delta_0' = +1{,}139{,}500$ +553,500	δ_0' $\delta_0 + \delta_0' = -300$ +312,500
α_0'' +0.00326 +0.01905 +0.01904 +0.04135	B_0'' 0	γ_0'' 0	δ_0'' +4,990 −4,200 +3,967 +4,757	δ_0'' 0 +2,245 +2,240 +4,485
α_0''' −0.5316 +0.8270 +0.2954	β_0''' 0	γ_0''' 0	δ_0''' −200,000 +117,200 + 95,140 +12,340	δ_0''' 0 −62,560 +89,700 +27,140
α_0^{iv} +0.531 −0.827 −0.296	β_0^{iv} 0	γ^{iv} 0	δ_0^{iv} 0 +110,700 − 95,140 +15,560	δ_0^{iv} 0 +62,500 −89,700 −27,200
α_0^{v} +0.000315	β_0^{v} 0	γ_0^{v} 0	δ_0^{v} +220,500 −119,600 +100,900	δ_0^{v} 0 +31.5 +31.5

shown in the middle part of the table under "Coefficients for Transformed Equations," when these should be used. For the coefficients in Equations A, B, and C, use the "Final Coefficients" as above given. The tabular calculations corresponding to Plate 17 are as shown in the table on page 92.

Now, referring to the "Solution" on Plate 16 and using the coefficients in the above and preceding table, the values of the unknowns are deduced for each loading as follows:

For Wind Load F_0 Only.

$$S_0^B = +\,0.04135 H_0 + 4{,}757.$$

But

$$H_0 = S_0^B,$$

hence

$$0.9586 S_0^B = +\,4{,}757 \quad \text{or} \quad S_0^B = +\,4{,}965 \text{ lb} = H_0,$$

$$M_0^A = -\,2.658\, H_0 + 586{,}000 = +\,572{,}800 \text{ in. lb},$$

$$M_0^B = +\,2.655\, H_0 + 553{,}500 = +\,566{,}700 \text{ in. lb},$$

$$C_1^A = +\,0.2954\, H_0 + 12{,}340 = +\,13{,}807,$$

$$C_1^B = -\,0.296\, H_0 + 15{,}560 = +\,14{,}090,$$

$$\Delta_0^a = +\,0.0003\, H_0 + 100{,}900 = +\,100{,}900,$$

$$S_0^A = \Sigma F - S_0^B = 10{,}000 - 4{,}965 = +\,5{,}035,$$

$$X_0^A = +\frac{572{,}800}{5{,}035} = +\,113.8 \text{ in.},$$

$$X_0^B = +\frac{566{,}700}{4{,}965} = +\,114.1 \text{ in.},$$

$$\delta_0^A = +\,20.0\, M_0^A - 1{,}333.3\, S_0^A = \qquad +4{,}742{,}700 \text{ in.},$$

$$\delta_0^B = +\,20.0\, M_0^B + 1{,}333.3\, S_0^A - 13{,}333{,}333 = +4{,}714{,}000 \text{ in.},$$

$$\lambda_0^a = -\,10.0\, S_0^A + 100{,}000 = \qquad +\ \ 49{,}650 \text{ in.},$$

$$V_0^A = +\,0.005\, M_0^A + 0.005\, M_0^B - 10{,}500 = -\,4{,}802 \text{ lb} = -\,V_0^B,$$

$$\alpha_0^A = +\,0.0525\, M_0^A + 0.0525\, M_0^B - 110{,}250 \text{ ft.}$$
$$= -\,50{,}427 \text{ in.} = -\,\alpha_0^B.$$

Girder Stresses.

Top flange:

$$\text{Max. comp.} = \tfrac{200}{20} S_0^A - \tfrac{1}{20} M_0^A + \tfrac{1}{2} F_0 = +\,26{,}710 \text{ lb} \quad (x = 0 \text{ in.}),$$

$$\text{Max. tens.} = \tfrac{200}{20} S_0^B - \tfrac{1}{20} M_0^B = +\,21{,}315 \text{ lb} \qquad (x = 200 \text{ in.}).$$

Bottom flange:

$$\text{Max. tens.} = \tfrac{220}{20} S_0^A - \tfrac{1}{20} M_0^A - \tfrac{1}{2} F_0 = +21{,}745 \text{ lb} \quad (x = 0),$$
$$\text{Max. comp.} = \tfrac{220}{20} S_0^B - \tfrac{1}{20} M_0^B = +26{,}280 \text{ lb} \quad (x = 200 \text{ in.}).$$

Max. Compressive Column Fibre Stress.

$$\text{For Col. } A = \frac{572{,}800 \times 7.00}{1{,}000} - \frac{4{,}802}{20} = 4{,}010 - 240$$
$$= +3{,}770 \text{ lb per sq. in.}$$

This occurs at the inside bottom fibre.

$$\text{For Col. } B = \frac{566{,}700 \times 7.00}{1{,}000} + \frac{4{,}802}{20} = 3{,}967 + 240$$
$$= +4{,}207 \text{ lb per sq. in.}$$

This occurs at the outside bottom fibre.

Check.

Using the geometrical check of Equation A,

$$\left.\begin{aligned} \delta_0^A + \frac{d_0}{2} C_1^A - \lambda_0^a &= +4{,}831{,}120 \\ \delta_0^B + \frac{d_0}{2} C_1^B &= +4{,}854{,}900 \end{aligned}\right\} \text{checks.}$$

For Floor Load w_0 Only.

$$S_0^B = +0.04135\, H_0 + 4{,}485.$$

But

$$H_0 = S_0^B,$$

hence

$$0.9586\, S_0^B = +4{,}485 \quad \text{or} \quad S_0^B = +4{,}680 \text{ lb} = H_0,$$

$$\left.\begin{aligned} M_0^A &= -2.658\, H_0 - 312{,}800 = -325{,}200 \\ M_0^B &= +2.655\, H_0 + 312{,}500 = +324{,}900 \end{aligned}\right\} \begin{aligned} &\text{Average} \\ &= 325{,}050 \text{ in. lb,} \end{aligned}$$

$$\left.\begin{aligned} C_1^A &= +0.2954\, H_0 + 27{,}140 = +28{,}524 \\ C_1^B &= -0.296\, H_0 - 27{,}200 = -28{,}584 \end{aligned}\right\} \text{Average} = 28{,}554,$$

$$\Delta_0^a = +0.0003\, H_0 + 31.5 = +33. \quad \text{Should be } 0,$$

$$S_0^A = -S_0^B = -4{,}680 \text{ lb},$$

$$X_0^A = X_0^B = \frac{325{,}050}{4{,}680} = +69.4 \text{ in},$$

$$\begin{aligned}
\delta_0^A &= +\,20.0\,M_0^A - 1{,}333.3\,S_0^A = -\,261{,}000 \text{ in.},\\
\delta_0^B &= +\,20.0\,M_0^B + 1{,}333.3\,S_0^A = +\,261{,}000 \text{ in.},\\
\lambda_0^a &= -\,10.0\,S_0^A = +\,46{,}800 \text{ in.},\\
V_0^A &= +\,0.005\,M_0^A + 0.005\,M_0^B + 30{,}000 = +\,30{,}000 \text{ ℔} = +\,V_0^B,\\
\alpha_0^A &= +\,0.0525\,M_0^A + 0.0525\,M_0^B + 315{,}000\\
&\qquad\qquad = +\,315{,}000 \text{ in.} = +\,\alpha_0^B.
\end{aligned}$$

Girder Stresses.

Top flange:

$$\text{Max. comp.} = \frac{200}{20}S_0^A - \frac{1}{20}M_0^A + \frac{100}{20}V_0^A - \frac{w_0\overline{100}^2}{2\times 20}$$

$$= +\,44{,}450 \text{ ℔} \quad (x = 100 \text{ in.}),$$

$$\text{Max. tens.} = -\tfrac{200}{20}S_0^A + \tfrac{1}{20}M_0^A = +\,30{,}550 \text{ ℔}$$

$$(x = 0 \text{ in.}, \epsilon x = 200 \text{ in.}).$$

Bottom flange:

$$\text{Max. tens.} = \frac{220}{20}S_0^A - \frac{1}{20}M_0^A + \frac{100}{20}V_0^A - \frac{w_0\overline{100}^2}{2\times 20}$$

$$= +\,39{,}770 \text{ ℔} \quad (x = 100 \text{ in.}),$$

$$\text{Max. comp.} = -\tfrac{220}{20}S_0^A + \tfrac{1}{20}M_0^A = +\,35{,}230 \text{ ℔}$$

$$(x = 0 \text{ in.}, \epsilon x = 200 \text{ in.}).$$

Max. Compressive Column Fibre Stress.

$$\text{For Col. } A = \frac{(200\,S_0^A - M_0^A)7.00}{1{,}000} + \frac{30{,}000}{20}$$

$$= +\,4{,}277 + 1{,}500 = +\,5{,}777 \text{ ℔ per sq. in.}$$

This occurs at the inside top fibre.

For Col. *B*,

Same as for Column *A*.

Check.

Using the geometrical check of Equation *A*,

$$\left.\begin{aligned}
\delta_0^A + \frac{d_0}{2}C_1^A - \lambda_0^a &= -\,22{,}260\\
\delta_0^B + \frac{d_0}{2}C_1^B \qquad &= -\,24{,}540
\end{aligned}\right\}$$

Considering the large quantities involved and the slide rule work (18 in. Mannheim) this checks.

Portal P_2 (both loadings calculated separately but in the same table).—Referring to Plate 15, the values of the symbolic coefficients become:

SYMBOLIC COEFFICIENTS—PORTAL P_2.

	a_0	b_0	e_0	f_0	i_0	l_0	s_0	t_0	w_0	P_0	z_0	
											For F_0.	For w_0.
C–A	+ 0.2	0.0	0	0	− 10.0	0.0	0	0	0	0	0.0	0.0
C–B	0.0	+ 0.2	0	0	+ 10.0	0.0	0	0	0	0	− 100,000.0	0.0
δ–A	+10.0	0.0	0	0	− 333.33	0.0	0	0	0	0	0.0	0.0
δ–B	0.0	+10.0	0	0	+ 333.33	0.0	0	0	0	0	− 3,333,333.0	0.0
H–a	0.0	0.0	0	0	− 1.00	+ 1.0	0	0	0	0	+ 10,000.0	0.0
λ–a	0.0	0.0	0	0	− 8.00	+ 8.0	0	0	0	0	+ 80,000.0	0.0
V–A	+ 0.005	+ 0.005	0	0	0.0	0.0	0	0	0	0	− 6,250.0	+ 30,000.0
V–B	− 0.005	− 0.005	0	0	0.0	0.0	0	0	0	0	+ 6,250.0	+ 30,000.0
α–A	+ 0.04167	+ 0.04167	0	0	0.0	0.0	0	0	0	0	− 52,083.0	+ 250,000.0
α–B	− 0.04167	− 0.04167	0	0	0.0	0.0	0	0	0	0	+ 52,083.0	+ 250,000.0
Δ–a	− 0.0833	− 0.0833	0	0	0.0	0.0	0	0	0	0	+ 104,167.0	0.0
Q–a	− 0.000200	0.0	0	0	+ 0.025	+ 0.005	0	0	0	0	0.0	0.0
R–a	+ 0.0000010	+ 0.0000010	0	0	0.0	0.0	0	0	0	0	− 1.25	+ 6.0
A–a	+15.000	−15.000	0	0	−1,158.67	− 8.0	0	0	0	0	+ 5,753,333	0.0
B–a	− 0.22	+ 0.22	0	0	+ 25.0	+ 1.0	0	0	0	0	− 125,000	+ 40,000.0
C–a	− 1.25	+42.75	0	0	+2,500.0	+100.0	0	0	0	0	−23,437,500	+4,000,000.0
Values of coefficients affected by transformed equations, see Plate 15.												
C–A					+ 10.0					0	− 100,000	0.0
C–B					− 10.0					0	0	0.0
Δ–a					0.0					0	+ 104,167	0.0
A–a					+1,166.67					0	− 5,753,333	0.0
B–a					− 26.0					0	+ 125,000	+ 40,000.0
C–a					−2,600					0	+ 1,562,500	+ 400,000.0
Final coefficients—after simplifying equations *A*, *B*, and *C*, see text, Chap. III.												
A–a	+ 1.875	− 1.875			+ 145.83						− 729,167	0
B–a	+21.0625	+10.0625			+ 650.00	− 25.0					−11,328,000	−1,000,000
C–a	+10.0625	+21.0625			− 650.00	+ 25.0					− 5,078,125	+1,000,000

SOLUTION OF EQUATIONS.

A_0 +3,074 − −1,218 den = +1,856	+0.7915 +0.6565 +1.4480	B_0	0	C_0	+1.965 −0.350 +1.615	D_0	+890,000 −255,300 +634,700	D_0	+78,600 0 +78,600
A_0' $A_0-A_0'=$ +0.757 +1,468 +1,218 den = +2,686	+1.144 −0.453 +0.691	B_0'	0	C_0'	−1.357 +0.242 −1.115	D_0'	+276,000 +176,300 +452,300	D_0'	−54,280 0 −54,280
α_0	−2.133 −1.473 −3.606	β_0	0	γ_0	0	δ_0	+865,500 −579,000 +286,500	δ_0	−104,000 − 71,700 −175,700
α_0'	+2.132 +1.472 +3.604	β_0'	0	γ_0'	0	δ_0'	+839,000 −597,500 +241,500	δ_0'	+103,900 + 71,750 +175,650
α_0''	+0.00686 +0.0464 +0.0464 +0.0997	β_0''	0	γ_0''	0	δ_0''	+5,000 −3,685 +3,105 +4,420	δ_0''	0 +2,259 +2,259 +4,518
α_0'''	−0.7212 +0.997 +0.276	β_0'''	0	γ_0'''	0	δ_0'''	−100,000 + 57,300 + 44,200 +1,500	δ_0'''	0 −35,140 +45,180 +10,040
α_0^{iv}	+0.721 −0.997 −0.276	β_0^{iv}	0	γ_0^{iv}	0	δ_0^{iv}	0 +48,300 −44,200 +4,100	δ_0^{iv}	0 +35,140 −45,180 −10,040
α_0^{v}	+0.3005 −0.3005 0	β_0^{v}		γ_0^{v}	0	δ_0^{v}	+104,167 − 23,875 − 20,125 +60,167	δ_0^{v}	0 −14,642 +14,642 0

Now, proceeding as described for the Portal P_1, the tabular calculations corresponding to Plate 17 are as given in the table on page 97.

Now, referring to the "solution" on Plate 16 and using the coefficients in the above and preceding tables, the values of the unknowns are deduced for each loading as follows:

For Wind Load F_0 Only.

$$S_0^B = +0.0997\,H_0 + 4{,}420.$$

But

$$H_0 = S_0^B;$$

hence

$$0.9003\,S_0^B = +4{,}420 \quad \text{or} \quad S_0^B = +4{,}915\text{ lb} = H_0,$$

$$M_0^A = -3.606\,H_0 + 286{,}500 = +268{,}780\text{ in. lb},$$

$$M_0^B = +3.604\,H_0 + 241{,}500 = +259{,}220\text{ in. lb},$$

$$C_1^A = +0.276\,H_0 + 1{,}500 = +2{,}860,$$

$$C_1^B = -0.276\,H_0 + 4{,}100 = +2{,}740,$$

$$\Delta_0^a = 0\,H_0 + 60{,}167 = +60{,}167,$$

$$S_0^A = \Sigma F - S_0^B = 10{,}000 - 4{,}915 = +5{,}085\text{ lb},$$

$$X_0^A = \frac{268{,}780}{5{,}085} = +52.85\text{ in.},$$

$$X_0^B = \frac{259{,}220}{4{,}915} = +52.75\text{ in.},$$

$$\delta_0^A = +10.0\,M_0^A - 333.33\,S_0^A = +992{,}800\text{ in.},$$

$$\delta_0^B = +10.0\,M_0^B + 333.33\,S_0^A - 3{,}333{,}333 = +953{,}900\text{ in.},$$

$$\lambda_0^A = -8.0\,S_0^A + 80{,}000 = +39{,}320\text{ in.},$$

$$V_0^A = +0.005\,M_0^A + 0.005\,M_0^B - 6{,}250 = -3{,}610\text{ lb} = -V_0^B,$$

$$\alpha_0^A = +0.0417\,M_0^A + 0.0417\,M_0^B - 52{,}083 = -30{,}065\text{ in.} = -\alpha_0^B,$$

Girder Stresses.

Top flange:

Max. comp. $= \frac{100}{50}S_0^A - \frac{1}{50}M_0^A + \frac{1}{2}F_0 = +9{,}794$ lb ($x = 0$ in.)

Max. tens. $= \frac{100}{50}S_0^B - \frac{1}{50}M_0^B = +4{,}646$ lb ($x = 200$ in.).

Bottom flange:

Max. tens. $= \frac{150}{50}S_0^A - \frac{1}{50}M_0^A - \frac{1}{2}F_0 = +4{,}879$ lb ($x = 0$ in.),

Max. comp. $= \frac{150}{50}S_0^B - \frac{1}{50}M_0^B = +9{,}561$ lb ($x = 200$ in.).

Max. Compressive Column Fibre Stress.

For Col. $A = \dfrac{268{,}780 \times 7.00}{500} - \dfrac{3{,}610}{15} = 3{,}763 - 241$

$= +\, 3{,}522$ ℔ per sq. in.

This occurs at the inside bottom fibre.

For Col. $B = \dfrac{259{,}220 \times 7.00}{500} + \dfrac{3{,}610}{15} = 3{,}629 + 241$

$= +\, 3{,}870$ ℔ per sq. in.

This occurs on the outside bottom fibre.

Checks.

Using the geometrical check of Equation A,

$$\left.\begin{aligned} \delta_0^A + \frac{d_0}{2} C_1^A - \lambda_0^a &= +\, 1{,}025{,}000 \text{ in.} \\ \delta_0^B + \frac{d_0}{2} C_1^B &= +\, 1{,}022{,}400 \text{ in.} \end{aligned}\right\} \text{checks.}$$

For Floor Load w_0 Only.

$$S_0^B = +\, 0.0997\, H_0 + 4{,}518.$$

But

$$H_0 = S_0^B;$$

hence

$0.9003\, S_0^B = +\, 4{,}518$ or $S_0^B = +\, 5{,}020$ ℔ $= H_0$,

$M_0^A = -\, 3.606\, H_0 - 175{,}700 = -\, 193{,}800$ in. ℔,

$M_0^B = +\, 3.604\, H_0 + 175{,}650 = +\, 193{,}800$ in. ℔,

$C_1^A = +\, 0.276\, H_0 + 10{,}040 = +\, 11{,}425,$

$C_1^B = -\, 0.276\, H_0 - 10{,}040 = -\, 11{,}425,$

$\Delta_0^a = 0\, H_0 + 0 = 0,$

$S_0^A = -\, S_0^B = -\, 5{,}020$ ℔,

$X_0^A = X_0^B = \dfrac{193{,}800}{5{,}020} = +\, 38.62$ in.,

$\delta_0^A = +\, 10.0\, M_0^A - 333.33\, S_0^A = -\, 264{,}700$ in.,

$\delta_0^B = +\, 10.0\, M_0^B + 333.33\, S_0^A = +\, 264{,}700$ in.,

$\lambda_0^A = -\, 8.0\, S_0^A = +\, 40{,}160$ in.,

$$
\begin{aligned}
V_0^A &= +\,0.005\ M_0^A + 0.005\ M_0^B + 30{,}000 \\
&\qquad\qquad = +\,30{,}000 \text{ lb.} = +\,V_0^B, \\
\alpha_0^A &= +\,0.0417\ M_0^A + 0.0417\ M_0^B + 250{,}000 \\
&\qquad\qquad = +\,250{,}000 \text{ in.} = +\,\alpha_0^B.
\end{aligned}
$$

Girder Stresses.

Top flange:

$$
\text{Max. comp.} = \frac{100}{50} S_0^A - \frac{1}{50} M_0^A + \frac{100}{50} V_0^A - \frac{\overline{w_0 100^2}}{2 \times 50}
$$

$$
= +\,23{,}836 \text{ lb} \quad (x = 100 \text{ in.}),
$$

$$
\text{Max. tens.} = -\tfrac{100}{50} S_0^A + \tfrac{1}{50} M_0^A = +\,6{,}164 \text{ lb}
$$

$$
(x = 0 \text{ in.}, \ \epsilon x = 200 \text{ in.}).
$$

Bottom flange:

$$
\text{Max. tens.} = \frac{150}{50} S_0^A - \frac{1}{50} M_0^A + \frac{100}{50} V_0^A - \frac{\overline{w_0 100^2}}{2 \times 50}
$$

$$
= +\,18{,}816 \text{ lb} \quad (x = 100 \text{ in.}),
$$

$$
\text{Max. comp.} = -\tfrac{100}{50} S_0^A + \tfrac{1}{50} M_0^B = +\,11{,}814 \text{ lb}
$$

$$
(x = 0 \text{ in.}, \ \epsilon x = 200 \text{ in.}).
$$

Max. Compressive Column Fibre Stress.

$$
\text{For Col. } A = \frac{(100\ S_0^A - M_0^A) \times 7.00}{500} + \frac{30{,}000}{15}
$$

$$
= 4{,}315 + 2{,}000 = +\,6{,}315 \text{ lb per sq. in.}
$$

This occurs at the inside top fibre.

For Col. *B*.

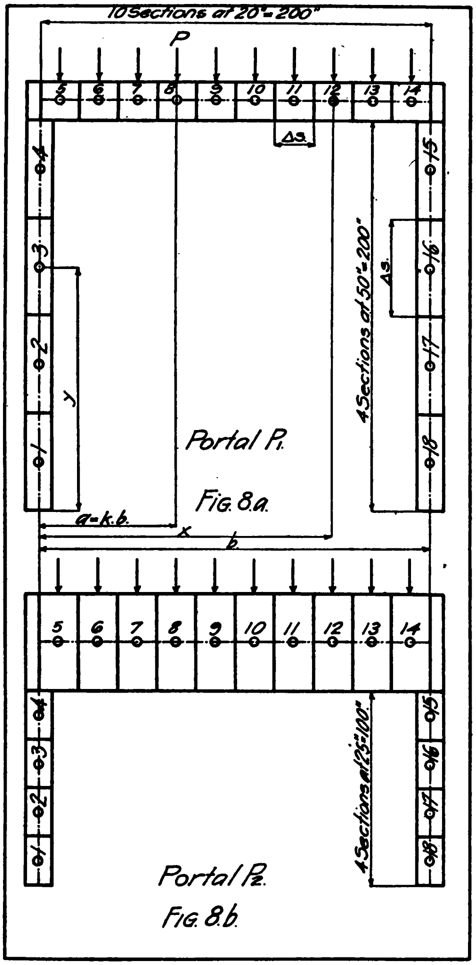
Same as for Column *A*.

Checks.

Using the geometrical check of Equation *A*,

$$
\left.
\begin{aligned}
\delta_0^A + \frac{d_0}{2} C_1^A - \lambda_0^a &= -\,19{,}235 \text{ in.} \\
\delta_1^B + \frac{d_0}{2} C_1^B \qquad &= -\,20{,}925 \text{ in.}
\end{aligned}
\right\}
\text{Considering the large quantities involved this checks.}
$$

The Arch Theory Applied to Portals P_1 *and* P_2.—As has just been shown, the effect of the *floor loads* in producing *bending moments* in the columns is very large. These results were so large and considered so important that it was decided to test the theory of



Portal P1.
Fig. 8a.

Portal P2.
Fig. 8b.

Chapter III by computing the quantities S_0^B, the "thrust of the arch," by the theory so well developed by Professor Howe, for the floor load w_0 only. Adapting his formulae for a fixed arch, to the present case, the value of S_0^B is given thus:

$$S_0^B = \frac{(1-k)\Sigma_0^a \dfrac{xy\Delta s}{I} + k\Sigma_a^b \dfrac{b(b-x)y\Delta s}{I} - \dfrac{(1-k)\Sigma_0^a \dfrac{x\Delta s}{I} + k\Sigma_a^b \dfrac{(b-x)\Delta s}{I}}{\Sigma_0^b \dfrac{\Delta s}{I}} \times \Sigma_0^b \dfrac{y\Delta s}{I}}{\Sigma_0^b \dfrac{y^2\Delta s}{I} - \dfrac{\left(\Sigma_0^b \dfrac{y\Delta s}{I}\right)^2}{\Sigma_0^b \dfrac{\Delta s}{I}}}$$

In which $a = kb$, where k is a decimal fraction, a being the value of x, measured from the left support, corresponding to the single concentrated load producing S_0^B.

In making the calculations the uniform floor load is divided into ten equal parts, each part then being considered as a concentrated load applied at its center of gravity. Figs. 8, *a* and *b*, show the subdivisions of the portals for the calculations which are given in the following tables.

Portal P 1.—For Total Uniform Floor Load of 60000 lbs.

Substituting in the equation for S_0^B the values in the tables given on pages 103 and 104 we find:

For load P at 5,

$$S_0^B = P\left\{\frac{0.95\times 35.0+0.05\times 2{,}835.0 - (0.95\times 0.1667+0.05\times 13.5)\times\dfrac{75.00}{0.5667}}{12{,}599 - \dfrac{(75.00)^2}{0.5667} = 2{,}679}\right\} = 0.0242P.$$

For load P at 6,

$$S_0^B = P\left\{\frac{0.85\times 140.0+0.15\times 2{,}240.0-(0.85\times 0.6667+0.15\times 10.667)\times\dfrac{75.00}{0.5667}}{2{,}679}\right\} = 0.0628P.$$

For load P at 7,

$$S_0^B = P\left\{\frac{0.75\times 315.0+0.25\times 1{,}715.0 - (0.75\times 1.500+0.25\times 8.167)\times\dfrac{75.00}{0.5667}}{2{,}679}\right\} = 0.0920P.$$

PORTAL P_1—FOR TOTAL UNIFORM FLOOR LOAD OF 60,000 POUNDS.

Pt.	x	y	I	Δs	$\Delta s/I$	$y\Delta s/I$	$y^2\Delta s/I$	$xy\Delta s/I$	$x\Delta s/I$
1	0	25	1,000	50	0.050	1.250	31.2	0	0
2	0	75	1,000	50	0.050	3.75	281.5	0	0
3	0	125	1,000	50	0.050	6.25	781.0	0	0
4	0	175	1,000	50	0.050	8.75	1531.0	0	0
5	10	210	1,200	20	0.01667	3.50	735.0	35	0.1667
6	30	210	1,200	20	0.01667	3.50	735.0	105	0.500
7	50	210	1,200	20	0.01667	3.50	735.0	175	0.833
8	70	210	1,200	20	0.01667	3.50	735.0	245	1.167
9	90	210	1,200	20	0.01667	3.50	735.0	315	1.500
10	110	210	1,200	20	0.01667	3.50	735.0	385	1.833
11	130	210	1,200	20	0.01667	3.50	735.0	455	2.167
12	150	210	1,200	20	0.01667	3.50	735.0	525	2.500
13	170	210	1,200	20	0.01667	3.50	735.0	595	2.833
14	190	210	1,200	20	0.01667	3.50	735.0	665	3.167
15	200	175	1,000	50	0.050	8.75	1531.0	1,750	10.000
16	200	125	1,000	50	0.050	6.25	781.0	1,250	10.000
17	200	75	1,000	50	0.050	3.75	281.5	750	10.000
18	200	25	1,000	50	0.050	1.25	31.2	250	10.000
Σ					0.5667	75.00	12599	7,500	56.667

Pt.	For Load at 5.			For Load at 6.			For Load at 7.			For Load at 8.			For Load at 9.		
	$(b-x)_a^b$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)_a^b$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)_a^b$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)_a^b$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)_a^b$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$
1	$k = 0.05$			$k = 0.15$			$k = 0.25$			$k = 0.35$			$k = 0.45$		
2	$1 - k = 0.95$			$1 - k = 0.85$			$1 - k = 0.75$			$1 - k = 0.65$			$1 - k = 0.55$		
3															
4															
5															
6	170	2.833	595												
7	150	2.500	525	150	2.500	525									
8	130	2.167	455	130	2.167	455	130	2.167	455						
9	110	1.833	385	110	1.833	385	110	1.833	385	110	1.833	385			
10	90	1.500	315	90	1.500	315	90	1.500	315	90	1.500	315	90	1.500	315
11	70	1.167	245	70	1.167	245	70	1.167	245	70	1.167	245	70	1.167	245
12	50	0.833	175	50	0.833	175	50	0.833	175	50	0.833	175	50	0.833	175
13	30	0.500	105	30	0.500	105	30	0.500	105	30	0.500	105	30	0.500	105
14	10	0.167	35	10	0.167	35	10	0.167	35	10	0.167	35	10	0.167	35
15	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
16	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
17	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
18	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
Σ		13.500	2835		10.667	2240		8.167	1715		6.000	1260		4.167	875

For load P at 8,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.65\times 560.0+0.35\times 1{,}260.0\\ \quad -(0.65\times 2.667+0.35\times 6.000)\times \dfrac{75.00}{0.5667}\end{array}}{2{,}679}\right\} = 0.1112P.$$

For load P at 9,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.55\times 875.0+0.45\times 875.0\\ \quad -(0.55\times 4.167+0.45\times 4.167)\times \dfrac{75.00}{0.5667}\end{array}}{2{,}679}\right\} = 0.1208P.$$

For all loads on,

$$S_0^B = 2(0.0242 + 0.0628 + 0.0920 + 0.1112 + 0.1208)P$$
$$= 0.8220P = 0.8220 \times \frac{60{,}000}{10} = 4{,}930 \text{ ℔.}$$

The value of S_0^B, by the method of Chapter III was found to be 4,680 ℔.

The arch formula used does not consider the effect of the axial thrust.

Portal P 2.—For Total Uniform Floor Load of 60000 lbs.

Substituting in the equation for S_0^B the values of the tables on pages 106 and 107 give:

For load P at 5,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.95\times 5.0+0.05\times 405\\ \quad -(0.95\times 0.040+0.05\times 3.240)\times \dfrac{23.750}{0.440}\end{array}}{1{,}859.6 - \dfrac{(23.750)^2}{0.440} = 580}\right\} = 0.0245P.$$

For load P at 6,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.85\times 20.0+0.15\times 320\\ \quad -(0.85\times 0.160+0.15\times 2.560)\times \dfrac{23.750}{0.440}\end{array}}{580}\right\} = 0.0637P.$$

PORTAL P_2 FOR TOTAL UNIFORM FLOOR LOAD OF 60,000 POUNDS.

Pt.	x	y	I	$\Delta s.$	$\Delta s/I.$	$y\Delta s/I.$	$y^2\Delta s/I.$	$x\Delta s/I.$	$xy\Delta s/I.$
1	0	12.5	500	25	0.050	0.625	7.8	0.000	0.0
2	0	37.5	500	25	0.050	1.875	70.3	0.000	0.0
3	0	62.5	500	25	0.050	2.500	156.2	0.000	0.0
4	0	87.5	500	25	0.050	4.375	383.0	0.000	0.0
5	10	125.0	5,000	20	0.004	0.500	62.5	0.040	5.0
6	30	125.0	5,000	20	0.004	0.500	62.5	0.120	15.0
7	50	125.0	5,000	20	0.004	0.500	62.5	0.200	25.0
8	70	125.0	5,000	20	0.004	0.500	62.5	0.280	35.0
9	90	125.0	5,000	20	0.004	0.500	62.5	0.360	45.0
10	110	125.0	5,000	20	0.004	0.500	62.5	0.440	55.0
11	130	125.0	5,000	20	0.004	0.500	62.5	0.520	65.0
12	150	125.0	5,000	20	0.004	0.500	62.5	0.600	75.0
13	170	125.0	5,000	20	0.004	0.500	62.5	0.680	85.0
14	190	125.0	5,000	20	0.004	0.500	62.5	0.760	95.0
15	200	87.5	500	25	0.050	4.375	383.0	10.000	875.0
16	200	62.5	500	25	0.050	2.500	156.2	10.000	500.0
17	200	37.5	500	25	0.050	1.875	70.3	10.000	375.0
18	200	12.5	500	25	0.050	0.625	7.8	10.000	125.0
Σ					0.440	23.750	1,859.6	44.000	2,375.0

Pt.	For Load at 5.			For Load at 6.			For Load at 7.			For Load at 8.			For Load at 9.		
	$(b-x)\frac{b}{a}$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)\frac{b}{a}$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)\frac{b}{a}$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)\frac{b}{a}$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$	$(b-x)\frac{b}{a}$	$\frac{(b-x)\Delta s}{I}$	$\frac{(b-x)y\Delta s}{I}$
1	$k = 0.05$			$k = 0.15$			$k = 0.25$			$k = 0.35$			$k = 0.45$		
2	$1 - k = 0.95$			$1 - k = 0.85$			$1 - k = 0.75$			$1 - k = 0.65$			$1 - k = 0.55$		
3															
4															
5															
6	170	0.680	85												
7	150	0.600	75	150	0.600	75									
8	130	0.520	65	130	0.520	65	130	0.520	65						
9	110	0.440	55	110	0.440	55	110	0.440	55	110	0.440	55			
10	90	0.360	45	90	0.360	45	90	0.360	45	90	0.360	45	90	0.360	45
11	70	0.280	35	70	0.280	35	70	0.280	35	70	0.280	35	70	0.280	35
12	50	0.200	25	50	0.200	25	50	0.200	25	50	0.200	25	50	0.200	25
13	30	0.120	15	30	0.120	15	30	0.120	15	30	0.120	15	30	0.120	15
14	10	0.040	5	10	0.040	5	10	0.040	5	10	0.040	5	10	0.040	5
15	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
16	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
17	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
18	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0	0	0.000	0
Σ		3.240	405		2.560	320		1.960	245		1.440	180		1.000	125

For load P at 7,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.75\times45.0+0.25\times245\\ \quad-(0.75\times0.360+0.25\times1.960)\times\dfrac{23.750}{0.440}\end{array}}{580}\right\}=0.09315P.$$

For load P at 8,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.65\times80.0+0.35\times180\\ \quad-(0.65\times0.640+0.35\times1.440)\times\dfrac{23.750}{0.440}\end{array}}{580}\right\}= 0.11275P.$$

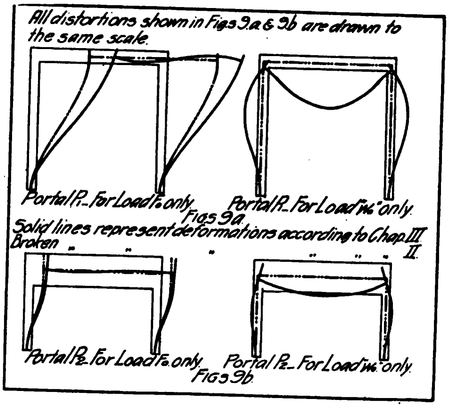
For load P at 9,

$$S_0^B = P\left\{\frac{\begin{array}{l}0.55\times125.0+0.45\times125.0\\ \quad-(0.55\times1.000+0.45\times1.000)\times\dfrac{23.750}{0.440}\end{array}}{580}\right\}= 0.1224P.$$

For all loads on,

$$S_0^B = 2(0.0245 + 0.0637 + 0.09315 + 0.11275 + 0.1224)P = 0.8330P = 0.8330 \times \frac{60,000}{10} = 4,998 \text{ lb}.$$

The value of S_0^B, by the method of Chapter III, was found to be 5,020 lb.


All distortions shown in Figs 9a & 9b are drawn to the same scale.
Portal P₁ – For Load F₀ only
Portal P₁ – For Load "w" only
Figs 9a.
Solid lines represent deformations according to Chap. III
Broken " " " " " " II.
Portal P₂ – For Load F₀ only
Portal P₂ – For Load "w" only
Figs 9b.

	For Load F_0.				For Load W_0.			
	Portal P_1.		Portal P_2.		Portal P_1.		Portal P_2.	
	Rigid Gir.	Elastic.	Rigid Gir.	Elastic.	Rigid Gir.	Elastic.	Rigid Gir.	Elastic.
S_0^B	+ 5,000	+ 4,965	+ 5,000	+ 4,915	0	+ 4,680	0	+ 5,020
S_0^A	+ 5,000	+ 5,035	+ 5,000	+ 5,085	0	− 4,680	0	− 5,020
M_0^A	+ 500,000	+ 572,800	+250,000	+268,780	0	−*325,050	0	−†193,800
M_0^B	+ 500,000	+ 566,700	+250,000	+259,220	0	+*325,050	0	+†193,800
X_0^A	100	113.8	50	52.85	0	69.4	0	38.62
X_0^B	100	114.1	50	52.75	0	69.4	0	38.62
H_0	+ 5,000	+ 4,965	+ 5,000	+ 4,915	0	+ 4,680	0	+ 5,020
V_0^A	− 5,500	− 4,802	− 3,750	− 3,610	+ 30,000	+ 30,000	+ 30,000	+ 30,000
V_0^B	+ 5,500	+ 4,802	+ 3,750	+ 3,610	+ 30,000	+ 30,000	+ 30,000	+ 30,000
C_1^A	0	+ 13,807	0	+ 2,860	0	+ 28,554	0	+ 11,425
C_1^B	0	+ 14,090	0	+ 2,740	0	− 28,554	0	− 11,425
δ_0^A	+3,333,333	+4,742,700	+833,333	+992,800	0	− 261,000	0	− 264,700
δ_0^B	+3,333,333	+4,714,000	+833,333	+953,900	0	+ 261,000	0	+ 264,700
Δ_0^a	+ 115,500?	+ 100,900	+ 62,500?	+ 60,167	0	0	0	0
λ_0^a	+ 50,000?	+ 49,650	+ 40,000?	+ 39,320	0	+ 46,800	0	+ 40,160
α_0^A	− 57,750?	− 50,427	− 31,250?	− 30,065	+315,000	+ 315,000	+250,000	+ 250,000
α_0^B	+ 57,750?	+ 50,427	+ 31,250?	+ 30,065	+315,000	+ 315,000	+250,000	+ 250,000
Gir. Str. Top. Flg. +	+ 30,000	+ 26,710	+ 10,000	+ 9,794	75,000	+ 44,450	+ 30,000	+ 23,836
−	+ 25,000	+ 21,315	+ 5,000	+ 4,646	0	+ 30,550	0	+ 6,164
Bott. Flg. +	+ 30,000	+ 26,280	+ 10,000	+ 9,561	0	+ 35,230	0	+ 11,814
−	+ 25,000	+ 21,745	+ 5,000	+ 4,879	75,000	+ 39,770	+ 30,000	+ 18,816
Fib. Str. Col. A	+ 3,225	+ 3,770	+ 3,250	+ 3,522	+ 1,500	+ 5,777	+ 2,000	+ 6,315
Col. B	+ 3,775	+ 4,207	+ 3,750	+ 3,870	+ 1,500	+ 5,777	+ 2,000	+ 6,315

? These quantities given for comparison only. They can not exist under the assumptions of Chapter II.

* Maximum occurs at top and equals 610,800 inch pounds.

† Maximum occurs at top and equals 308,200 inch pounds.

Thus the arch theory checks the methods developed in Chapter III very nicely. The table on page 109 shows a comparison of the stresses and deformations for the cases worked out in this chapter. In addition to this table, Figs. 9, *a* and *b* (page 108), are drawn to scale to compare the deformations of the structure.

CHAPTER VI.

A STUDY OF THE EFFECT OF WIND PRESSURE ON AN ELASTIC PORTAL BRACED BUILDING — WITH COMPARISONS.

The five story, two column, structure designed in Chapter II is now to be investigated in this and the three succeeding chapters by the methods of Chapter III. As was mentioned in that chapter and demonstrated in Chapter V, for any given structure the symbolic coefficients, with the exception of "z," and the solution of Plate 17, with the exception of the fourth column, are entirely independent of the loading. Therefore, by providing only a few extra columns in the tables of this chapter the values of the symbolic coefficients and the solution of Plate 17 may be most conveniently given here for all the loadings. The calculations for eccentric column loads alone (Chapter IX) will be made by combining the calculations of Chapter VII and VIII. These tables in this chapter then will provide for the following loadings:

(*a*) Wind load only on face *A*.

(*b*) Floor and eccentric column loads combined.

(*c*) Floor loads only.

Wind loads will be taken as used in the design, Chapter II.

Floor loads will here be taken as uniformly distributed over the girders instead of being concentrated at the beam connections. They will be considered as extending only to the centres of the columns and the unit floor loads of Chapter II will be used.

Eccentric column loads and their eccentricities will be taken as given in Chapter II, exclusive of the floor loads and weight of columns and fireproofing.

The structure will be computed first by the methods of Chapter III; then a comparison will be made with the results by the methods of Chapter II for which, of course, the loadings as modified above are to be used.

The properties of the structure and the values of the loads are as given in the following table.

STORIES.

		0	1	2	3	4
	c	125.0	125.0	125.0	125.0	130.0
	d	25.0	25.0	25.0	25.0	20.0
	b	200.0	200.0	200.0	200.0	200.0
	I^A	1,550.9	1,550.9	878.6	878.6	461.0
	I^B	1,550.9	1,122.6	1,122.6	660.8	660.8
	I^a	2,740.0	2,378.0	1,972.0	1,850.0	1,041.0
	A^A	34.73	34.73	19.73	19.73	11.52
	A^B	34.73	24.65	24.65	15.23	15.23
	A^a	23.67	20.33	17.77	17.00	14.47
	P^A	23,500	23,500	23,500	23,500	12,500
	P^B	31,225	31,225	31,225	31,225	0
	e^A	10.75	10.75	10.75	10.75	10.75
	e^B	4.50	4.50	4.50	4.50	4.50
	w	*353.35	*233.75	*233.75	*233.75	200.00
	F	3,750	7,500	7,500	7,500	6,750
	ΣF	33,000	29,250	21,750	14,250	6,750
Wind load	X	+15,354,375	+10,451,250	+ 6,157,500	+2,988,750	+ 945,000
	Y	0	0	0	0	0
Floor and column loads	X	−46,974,835	−35,095,720	−25,608,605	−16,121,490	−6,634,375
	Y	+ 482,320	+ 356,925	+ 255,450	+ 153,975	+ 52,500
Floor loads	X	−25,092,000	−18,025,000	−13,350,000	− 8,675,000	−4,000,000
	Y	+ 250,920	+ 180,250	+ 133,500	+ 86,750	+ 40,000

* These quantities should have been 350.00 and 233.33 respectively.

Using the quantities in the above table the values of the symbolic coefficients as given in Plate 15 are as follows:

VALUES OF THE SYMBOLIC COEFFICIENTS.

Exp'n.	a_0	a_1	a_2	a_3	a_4	b_0	b_1	b_2	b_3	b_4
$C-A$	+0.080598	+0.080598	+0.14227	+0.14227	+0.28200	0	0	0	0	0
$C-B$	0	0	0	0	0	+0.080598	+0.11135	+0.11135	+0.18917	+0.19673
$\delta-A$	+5.0374	+5.0374	+8.8920	+8.8920	+18.330	0	0	0	0	0
$\delta-B$	0	0	0	0	0	+5.0374	+6.9593	+6.9593	+11.823	+12.788
$H-a$	0	0	0	0	0	0	0	0	0	0
$\lambda-a$	0	0	0	0	0	0	0	0	0	0
$V-A$	+0.005	+0.005	+0.005	+0.005	+0.005	+0.005	+0.005	+0.005	+0.005	+0.005
$V-B$	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005	−0.005
$\alpha-A$	+0.019795	+0.02160	+0.03800	+0.03800	+0.06614	+0.019795	+0.02160	+0.03800	+0.03800	+0.06614
$\alpha-B$	−0.019795	−0.03042	−0.03042	−0.04926	−0.05010	−0.019795	−0.03042	−0.03042	−0.04926	−0.05010
$\Delta-a$	−0.03959	−0.05202	−0.06842	−0.08726	−0.11624	−0.03959	−0.05202	−0.06842	−0.08726	−0.11624
$Q-a$	−0.0003650	−0.0004205	−0.0005072	−0.0005405	−0.0009600	0	0	0	0	0
$R-a$	+0.000001825	+0.000002102	+0.000002536	+0.000002702	+0.000004800	+0.000001825	+0.000002102	+0.000002536	+0.000002702	+0.000004800
$A-a$	+6.044	+6.044	+10.669	+10.669	+21.150	−6.044	−8.350	−8.350	−14.186	−14.755
$B-a$	−0.11710	−0.12265	−0.19299	−0.19632	−0.37800	+0.11710	+0.15340	+0.16207	+0.24322	+0.29273
$C-a$	−2.393	−2.751	−3.313	−3.516	−6.284	+21.027	+27.927	+29.101	+45.126	+52.262

VALUES IN THE TRANSFORMED EQUATIONS.

$C-A$	+0.080598	+0.080598	+0.14227	+0.14227	+0.28200	0	0	0	0	0
$C-B$	0	0	0	0	0	+0.080598	+0.11135	+0.11135	+0.18917	+0.19673
$\Delta-a$	−0.03959	−0.05202	−0.06842	−0.08726	−0.11624	−0.03959	−0.05202	−0.06842	−0.08726	−0.11624
$A-a$	+6.044	+6.044	+10.669	+10.669	+21.150	−6.044	−8.350	−8.350	−14.186	−14.755
$B-a$	−0.11710	−0.12265	−0.19299	−0.19632	−0.37800	+0.11710	+0.15340	+0.16207	+0.24322	+0.29273
$C-a$	−2.393	−2.751	−3.313	−3.516	−6.284	+21.027	+27.927	+29.101	+45.126	+52.262

VALUES IN THE FINAL SIMPLIFIED EQUATIONS.

$A-a$	+0.71545	+0.61450	+0.94810	+0.90705	+1.5301	−0.71545	−0.84890	−0.74200	−1.2060	−1.0675
$B-a$	+5.4329	+4.8520	+6.6316	+6.2884	+6.8894	+2.2246	+2.6666	+2.2162	+3.5239	+2.0662
$C-a$	+2.2246	+1.9353	+2.8259	+2.6563	+2.9538	+5.4329	+6.3146	+5.4122	+8.0236	+5.1142

Exp'n.	e_0	e_1	e_2	e_3	e_4	f_0	f_1	f_2	f_3	f_4
$C-A$	0	0	0	0	0	0	0	0	0	0
$C-B$	0	0	0	0	0	0	0	0	0	0
$\delta-A$	0	0	0	0	0	0	0	0	0	0
$\delta-B$	0	0	0	0	0	0	0	0	0	0
$H-a$	0	0	0	0	0	0	0	0	0	0
$\lambda-a$	0	0	0	0	0	0	0	0	0	0
$V-A$	0	0	0	0	0	0	0	0	0	0
$V-B$	0	0	0	0	0	0	0	0	0	0
$\alpha-A$	0	0	0	0	0	0	0	0	0	0
$\alpha-B$	0	0	0	0	0	0	0	0	0	0
$\Delta-a$	0	0	0	0	0	0	0	0	0	0
$Q-a$	+0.0003650	+0.0004205	+0.0005072	+0.0005405	+0.0009600	0	0	0	0	0
$R-a$	−0.000001825	−0.000002102	−0.000002536	−0.000002702	−0.000004800	−0.000001825	−0.000002102	−0.000002536	−0.000002702	−0.000004800
$A-a$	0	0	0	0	0	0	0	0	0	0
$B-a$	+0.03650	+0.04205	+0.05072	+0.05405	+0.09600	−0.03650	−0.04205	−0.05072	−0.05405	−0.09600
$C-a$	+2.433	+2.803	+3.381	+3.603	+6.400	−4.866	−5.606	−6.762	−7.206	−12.800
VALUES IN THE TRANSFORMED EQUATIONS.										
$C-A$	0	0	0	0	0	0	0	0	0	0
$C-B$	0	0	0	0	0	0	0	0	0	0
$\Delta-a$	0	0	0	0	0	0	0	0	0	0
$A-a$	0	0	0	0	0	0	0	0	0	0
$B-a$	+0.03650	+0.04205	+0.05072	+0.05405	+0.09600	−0.03650	−0.04205	−0.05072	−0.05405	−0.09600
$C-a$	+2.433	+2.803	+3.381	+3.603	+6.400	−4.866	−5.606	−6.762	−7.206	−12.800
VALUES IN THE FINAL SIMPLIFIED EQUATIONS.										
$A-a$	0	0	0	0	0	0	0	0	0	0
$B-a$	+1.0	+1.0	+1.0	+1.0	+1.0	0	0	0	0	0
$C-a$	0	0	0	0	0	+1.0	+1.0	+1.0	+1.0	+1.0

Exp'n.	i_0	i_1	i_2	i_3	i_4	l_0	l_1	l_2	l_3	l_4
$C-A$	− 5.0374	− 5.0374	− 8.8920	− 8.8920	− 18.330	0	0	0	0	0
$C-B$	+ 5.0374	+ 6.9593	+ 6.9593	+ 11.823	+ 12.788	0	0	0	0	0
$\delta-A$	− 210.0	− 210.0	− 371.0	− 371.0	− 795.0	0	0	0	0	0
$\delta-B$	+ 210.0	+ 290.0	+ 290.0	+ 493.0	+ 554.0	0	0	0	0	0
$H-a$	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0
$\lambda-a$	− 8.450	− 9.845	− 11.255	− 11.760	− 13.825	+ 8.450	+ 9.845	+ 11.255	+ 11.760	+ 13.825
$V-A$	0	0	0	0	0	0	0	0	0	0
$V-B$	0	0	0	0	0	0	0	0	0	0
$\alpha-A$	0	0	0	0	0	0	0	0	0	0
$\alpha-B$	0	0	0	0	0	0	0	0	0	0
$\Delta-a$	0	0	0	0	0	0	0	0	0	0
$Q-a$	+ 0.05020	+ 0.05780	+ 0.06975	+ 0.07435	+0.13440	+ 0.004565	+ 0.005250	+ 0.006340	+ 0.006750	+ 0.009600
$R-a$	0	0	0	0	0	0	0	0	0	0
$A-a$	− 537.5	− 631.7	− 838.0	−1099.9	−1634.6	− 8.450	− 9.845	− 11.255	− 11.760	− 13.825
$B-a$	+ 20.114	+ 23.557	+ 29.801	+ 35.585	+ 57.998	+ 0.9130	+ 1.0500	+ 1.2680	+ 1.3500	+ 1.9200
$C-a$	+2011.48	+2547.86	+2786.86	+3851.6	+5245.6	+ 91.30	+105.00	+126.80	+135.00	+192.00

VALUES IN THE TRANSFORMED EQUATIONS.

Exp'n.	i_0	i_1	i_2	i_3	i_4	l_0	l_1	l_2	l_3	l_4
$C-A$	+ 5.0374	+ 5.0374	+ 8.8920	+ 8.8920	+ 18.330	0	0	0	0	0
$C-B$	− 5.0374	− 6.9593	− 6.9593	− 11.823	− 12.788	+ 0	0	0	0	0
$\Delta-a$	0	0	0	0	0	0	0	0	0	0
$A-a$	+ 545.94	+ 649.98	+ 859.09	+1122.93	+1660.18	− 8.450	− 9.845	− 11.255	− 11.760	− 13.825
$B-a$	− 21.027	− 24.607	− 31.069	− 36.935	− 59.918	− 0.9130	+ 1.0500	+ 1.2680	+ 1.3500	+ 1.9200
$C-a$	−2102.8	−2652.9	−2913.7	−3986.6	−5437.6	+ 91.30	+105.00	+126.80	+135.00	+192.00

VALUES IN THE FINAL SIMPLIFIED EQUATIONS.

Exp'n.	i_0	i_1	i_2	i_3	i_4	l_0	l_1	l_2	l_3	l_4
$A-a$	+ 64.585	+ 66.055	+ 76.285	+ 95.380	+ 120.070	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0
$B-a$	+ 288.03	+ 224.08	+ 363.46	+ 260.29	+ 398.51	− 12.500	− 12.500	− 12.500	− 12.500	− 10.00
$C-a$	− 288.03	− 361.21	− 249.13	− 422.98	− 225.44	+ 12.500	+ 12.500	+ 12.500	+ 12.500	+ 10.00

Exp'n.	s_0	s_1	s_2	s_3	s_4	t_0	t_1	t_2	t_3	t_4
C–A	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	0	0	0	0	0
C–B	0	0	0	0	0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0
δ–A	+ 125.0	+ 125.0	+ 125.0	+ 125.0	+ 130.0	0	0	0	0	0
δ–B	0	0	0	0	0	+125.0	+125.0	+125.0	+125.0	+130.0
H–a	0	0	0	0	0	0	0	0	0	0
λ–a	0	0	0	0	0	0	0	0	0	0
V–A	0	0	0	0	0	0	0	0	0	0
V–B	0	0	0	0	0	0	0	0	0	0
α–A	0	0	0	0	0	0	0	0	0	0
α–B	0	0	0	0	0	0	0	0	0	0
Δ–a	0	0	0	0	0	0	0	0	0	0
Q–a	0	0	0	0	0	0	0	0	0	0
R–a	0	0	0	0	0	0	0	0	0	0
A–a	+ 137.5	+ 150.0	+ 150.0	+ 150.0	+ 152.5	−137.5	−150.0	−150.0	−150.0	−152.5
B–a	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0
C–a	0	0	0	0	0	+200.0	+200.0	+200.0	+200.0	+200.0

VALUES IN THE TRANSFORMED EQUATIONS.

Exp'n.	s_0	s_1	s_2	s_3	s_4	t_0	t_1	t_2	t_3	t_4
C–A	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	0	0	0	0	0
C–B	0	0	0	0	0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0
Δ–a	0	0	0	0	0	0	0	0	0	0
A–a	+ 137.5	+ 150.0	+ 150.0	+ 150.0	+ 152.5	−137.5	−150.0	−150.0	−150.0	−152.5
B–a	− 1.0	− 1.0	− 1.0	− 1.0	− 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	+ 1.0
C–a	0	0	0	0	0	+200.0	+200.0	+200.0	+200.0	+200.0

VALUES IN THE FINAL SIMPLIFIED EQUATIONS.

Exp'n.	s_0	s_1	s_2	s_3	s_4	t_0	t_1	t_2	t_3	t_4
A–a	0	+ 15.247	+ 13.328	+ 12.750	+ 11.034	0	− 15.247	− 13.328	− 12.750	− 11.034
B–a	+ 54.800	+ 47.560	+ 39.440	+ 37.000	+ 20.820	+ 27.400	+ 23.780	+ 19.720	+ 18.500	+ 10.410
C–a	+ 27.400	+ 23.780	+ 19.720	+ 18.500	+ 10.410	+ 54.800	+ 47.560	+ 39.440	+ 37.000	+ 20.820

Exp'n.	w_0	w_1	w_2	w_3	w_4	P_0	P_1	P_2	P_3	P_4
$C-A$	0	0	0	0	0	0	0	0	0	0
$C-B$	0	0	0	0	0	0	0	0	0	0
$\delta-A$	0	0	0	0	0	0	0	0	0	0
$\delta-B$	0	0	0	0	0	0	0	0	0	0
$H-a$	0	0	0	0	0	0	0	0	0	0
$\lambda-a$	0	0	0	0	0	0	0	0	0	0
$V-A$	0	0	0	0	0	0	0	0	0	0
$V-B$	0	0	0	0	0	0	0	0	0	0
$\alpha-A$	0	0	0	0	0	0	0	0	0	0
$\alpha-B$	0	0	0	0	0	0	0	0	0	0
$\Delta-a$	0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	0	0	0	0	0
$Q-a$	0	0	0	0	0	0	0	0	0	0
$R-a$	0	0	0	0	0	0	0	0	0	0
$A-a$	0	0	0	0	0	0	− 8.450	− 9.845	− 11.255	− 11.760
$B-a$	0	0	0	0	0	0	0	0	0	0
$C-a$	0	− 1.0	− 1.0	− 1.0	− 1.0	0	0	0	0	0
VALUES IN THE TRANSFORMED EQUATIONS.										
$C-A$	0	0	0	0	0	0	0	0	0	0
$C-B$	0	0	0	0	0	0	0	0	0	0
$\Delta-a$	0	+ 1.0	+ 1.0	+ 1.0	+ 1.0	0	0	0	0	0
$A-a$	0	0	0	0	0	0	+ 8.450	+ 9.845	+ 11.255	+ 11.760
$B-a$	0	0	0	0	0	0	0	0	0	0
$C-a$	0	− 1.0	− 1.0	− 1.0	− 1.0	0	0	0	0	0
VALUES IN THE FINAL SIMPLIFIED EQUATIONS.										
$A-a$	0	0	0	0	0	0	+ 0.85890	+ 0.87405	+ 0.95665	+ 0.85120
$B-a$	− 0.41100	− 0.35670	− 0.29580	− 0.27750	− 0.15615	0	0	0	0	0
$C-a$	− 0.41100	− 0.35670	− 0.29580	− 0.27750	− 0.15615	0	0	0	0	0

	For Wind on Face *A* Only.					For Floor and Eccentric Column Loads.				
Exp'n.	z_0	z_1	z_2	z_3	z_4	z_0	z_1	z_2	z_3	z_4
C–A	0	0	0	0	0	0	0	0	0	0
C–B	− 166,300	− 203,500	− 151,300	− 168,500	− 86,350	0	0	0	0	0
δ–A	0	0	0	0	0	0	0	0	0	0
δ–B	− 6,930,000	− 8,485,000	− 6,315,000	− 7,030,000	− 3,740,000	0	0	0	0	0
H–a	+ 3,750	+ 7,500	+ 7,500	+ 7,500	+ 6,750	0	0	0	0	0
λ–a	+ 31,670	+ 73,800	+ 84,450	+ 88,250	+ 93,250	0	0	0	0	0
V–A	− 76,772	− 52.256	− 30,788	− 14,944	− 4,725	+ 234,874	+ 175,479	+ 128,043	+ 80,607	+ 33,172
V–B	+ 76,772	+ 52,256	+ 30,788	+ 14,944	+ 4,725	+ 247,446	+ 181,446	+ 127,407	+ 73,368	+ 19,328
α–A	− 303,950	− 225,680	− 234,000	− 113,600	− 62,500	+ 930,270	+ 758,000	+ 973,230	+ 613,000	+ 439,120
α–B	+ 303,900	+ 318,000	+ 187,400	+ 147,210	+ 47,350	+ 979,500	+1,104,290	+ 775,000	+ 722,700	+ 193,500
Δ–a	+ 607,850	+ 543,680	+ 421,400	+ 260,810	+ 109,850	+ 49,770	+ 346,290	− 198,230	+ 109,700	− 245,620
Q–a	0	0	0	0	0	− 92.20	− 106.3	− 128.1	− 136.6	− 129.0
R–a	− 8.945	− 9.030	− 8.030	− 5.525	− 4.535	+ 13.10	+ 10.07	+ 12.14	+ 12.94	+ 19.8
A–a	+ 8,977,080	+10,986,620	+ 8,195,600	+ 9,132,450	+ 4,598,500	0	0	0	0	0
B–a	− 345,200	− 384,100	− 311,900	− 279,000	− 177,050	+ 71,760	+ 49,040	+ 59,180	+ 62,980	+ 115,200
C–a	−57,721,100	−65,323,700	−52,094,700	−48,694,100	−29,473,100	+7,269,200	+4,716,000	+6,309,500	+6,390,000	+12,195,600
Values in the Transformed Equations.										
C–A	− 166,300	− 147,400	− 193,400	− 126,700	− 123,700	0	0	0	0	0
C–B	0	0	0	0	0	0	0	0	0	0
Δ–a	+ 607,850	+ 543,680	+ 421,400	+ 260,810	+ 109,850	+ 49,770	+ 346,290	− 198,230	+ 109,700	− 245,620
A–a	− 9,011,300	− 7,971,200	−10,483,700	− 6,863,700	− 6,596,700	0	0	0	0	0
B–a	+ 345,290	+ 327,750	+ 354,350	+ 237,200	+ 214,450	+ 71,760	+ 49,040	+ 59,180	+ 62,980	+ 115,200
C–a	+11,328,900	+11,485,300	+10,326,300	+ 7,100,900	+ 5,935,900	+7,269,200	+4,716,000	+6,309,500	+6,390,000	+12,195,600
Values in the Final Simplified Equations.										
A–a	− 1,065,700	− 811,350	− 930,800	− 583,400	− 477,450	0	0	0	0	0
B–a	−14,262,545	−11,496,090	−10,921,410	− 6,804,722	− 3,538,554	− 945,664	− 650,072	− 467,902	− 556,984	− 493,937
C–a	− 4,804,620	− 3,698,140	− 3,938,660	− 2,416,672	− 1,305,354	+1,016,865	+ 515,125	+ 697,295	+ 608,213	+ 705,022

FOR FLOOR LOADS ONLY.

	z_0	z_1	z_2	z_3	z_4
C–A	0	0	0	0	0
C–B	0	0	0	0	0
δ–A	0	0	0	0	0
δ–B	0	0	0	0	0
H–A	0	0	0	0	0
λ–a	0	0	0	0	0
V–A	+ 125,460	+ 90,125	+ 66,750	+ 43,375	+ 20,000
V–B	+ 125,460	+ 90,125	+ 66,750	+ 43,375	+ 20,000
α–A	+ 496,300	+ 389,000	+ 507,800	+ 330,000	+ 264,700
α–B	+ 496,300	+ 548,180	+ 406,500	+ 427,442	+ 200,200
Δ–a	0	+ 159,180	− 101,300	+ 97,442	− 64,500
R–a	0	0	0	0	0
Q–a	+ 12.90	+ 9.825	+ 11.85	+ 12.64	+ 19.22
A–a	0	0	0	0	0
B–a	+ 86,200	+ 65,400	+ 79,000	+ 84,300	+ 128,200
C–a	+8,630,000	+6,242,500	+8,001,300	+8,334,300	+12,887,800
C–A	0	0	0	0	0
C–B	0	0	0	0	0
Δ–a	0	+ 159,180	− 101,300	+ 97,442	− 64,500
A–a	0	0	0	0	0
B–a	+ 86,200	+ 65,400	+ 79,000	+ 84,300	+ 128,200
C–a	+8,630,000	+6,242,500	+8,001,300	+8,334,300	+12,887,800
A–a	0	0	0	0	0
B–a	−1,177,833	− 835,947	− 749,205	− 806,207	− 656,596
C–a	+1,177,833	+ 722,387	+ 809,129	+ 752,127	+ 676,738

The solution corresponding to Plate 17, for all loadings is now given in the following tables. The tables give the results of every slide rule calculation and, as will be seen, the results are quite accurate. A Thacher rule was used for all these calculations.

For the O Story.

										For Wind on Face A Only.			For Floor and Ec. Col. Loads.		For Floor Loads Only.		
A_0	$+350.85$ -206.06	$+0.99233$ $+1.4233$	$A_0-A_0'=+2.0015$	B_0			C_0	$+5.5757$ -1.9894		D_0	$+6,362,500$ $-2,120,000$		D_0		D_0	$+525,330$ 0	
den.	$+144.79$	$+2.4156$			$+0.44605$			$+3.5863$			$+4,242,500$			$+421,780$		$+525,330$	
A_0'	$+143.67$ $+206.06$	$+1.0033$ -0.58923		B_0'			C_0'	-2.3085 $+0.82362$		D_0'	$+887,250$ $+877,750$		D_0'		D_0'	$-217,490$ 0	
den.	$+349.73$	$+0.4141$			$+0.18467$			-1.4849			$+1,765,000$			$-187,760$		$-217,490$	
α_0		-1.7921 -0.74202	$\alpha_0+\alpha_0'=-0.0004$	β_0		$\beta_0+\beta_0'=+0.13056$	γ_0		$\gamma_0+\gamma_0'=+0.13062$	δ_0	$+2,130,100$ $-\ 877,800$	$\delta_0+\delta_0'=+2,490,100$	δ_0	$-226,600$ $-\ 87,265$	δ_0	$-262,490$ $-108,680$	$\delta_0+\delta_0'=-60$
		-2.5341			-0.092290			$+0.22288$			$+1,252,300$			$-313,865$		$-371,170$	
α_0'				β_0'			γ_0'			δ_0'			δ_0'		δ_0'		
		$+2.5337$			$+0.22285$			-0.092265			$+1,237,800$			$+304,530$		$+371,110$	
α_0''		$+0.0154835$ $+0.028072$ $+0.028068$		β_0''	$+0.0010224$ $+0.0024688$		γ_0''	-0.0024690 -0.0010222		δ_0''	$+16,502$ $-13,873$ $+13,713$		δ_0''	0 $+3,477.1$ $+3,373.5$	δ_0''	0.0 $+4,112.0$ $+4,111.2$	
$\alpha_0''-1=-0.928376$		$+0.071624$			$+0.0034912$			-0.0034912			$+16,342$			$+6,850.6$		$+8,223.2$	
α_0'''		-0.20424 $+0.36081$	$\alpha_0'''-\alpha_0^{iv}=+0.31318$	β_0'''	-0.0074385 $+0.017588$	$\beta_0'''-\beta_0^{iv}=+0.009776$	γ_0'''	$+0.017964$ -0.017588	$\gamma_0'''-\gamma_0^{iv}=-0.009776$	δ_0'''	$-166,234$ $+100,930$ $+\ 82,325$	$\delta_0'''-\delta_0^{iv}=-419$	δ_0'''	0 $-25,296$ $+34,509$	δ_0'''	0 $-29,916$ $+41,424$	$\delta_0'''-\delta_0^{iv}=+23,020$
		$+0.15657$			$+0.010150$			$+0.000376$			$+17,021$			$+9,213$		$+11,508$	
α_0^{iv}		$+0.20420$ -0.36081		β_0^{iv}	$+0.017962$ -0.017588		γ_0^{iv}	-0.0074363 $+0.017588$		δ_0^{iv}	$+99,765$ $-82,325$		δ_0^{iv}	$+24,544$ $-34,509$	δ_0^{iv}	0 $+29,912$ $-41,424$	
		-0.15661			$+0.000374$			$+0.010152$			$+17,440$			$-9,965$		$-11,512$	

For the O Story.—*Continued.*

						For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.	
α_0^v	+0.0000158	β_0^v	−0.0051687	γ_0^v	−0.0051710	δ_0^v	+607,850 − 98,590 +509,260	δ_0^v	+49,770 + 370 +50,140	δ_0^v	0 +2.4 +2.4
θ_0	−4.6765 +0.15657 −4.5199	ϕ_0	+0.080598 +0.017588 +0.010150 +0.108336	ψ_0	−0.017588 +0.000376 −0.017212	ω_0	−147,344 + 82,325 + 17,021 −47,998	ω_0	0 +34,509 + 9,213 +43,722	ω_0	0 +11,508 +41,424 +52,932
θ_0'	+6.4608 −0.15661 +6.3042	ϕ_0'	−0.024297 +0.000374 −0.023923	ψ_0'	+0.11135 +0.024297 +0.010152 +0.14580	ω_0'	−113,720 + 17,440 −96,280	ω_0'	−47,672 −9,965 −57,637	ω_0'	0 −11,512 −57,228 −68,740
θ_0''	+0.0000158	ϕ_0''	−0.052023 −0.0051687 −0.057192	ψ_0''	−0.052023 −0.0051710 −0.057194	ω_0''	+543,680 +509,260 +1,052,940	ω_0''	+346,290 + 50,140 +396,430	ω_0''	+159,180 + 2.4 +159,182.4

For the First Story.

c_1	−61.325 + 4.7750 + 0.8589 −55.691	a_1	+0.61450 +0.23061 +0.14905 +0.99416	b_1	−0.84890 −0.23061 −0.14905 +0.99416	d_1	− 811,350 +1,079,400 − 6,389 +261,661	d_1	0 +452,510 +292,410 +744,920	d_1	0 +543,220 +351,000 +894,220
i_1	−208.03 + 7.447 − 3.724 − 0.000 −204.31	g_1	+4.8520 +0.78233 +0.48272 +0.00889 +0.00184 +6.1278	h_1	+2.6666 −0.78233 +0.01788 +0.24141 +0.00184 +2.1454	k_1	+11,496,090 − 3,662,100 − 809,600 − 414,750 + 181,650 +6,791,290	k_1	+ 650,072 −1,535,100 − 438,160 + 236,970 + 17,885 −1,068,333	k_1	+ 835,947 −1,842,700 − 547,350 + 273,775 + 1 −1,280,327

For the First Story.—*Continued.*

							For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.	
p_1		+335.340 + 3.724 − 7.449 0.000	n_1	+1.9353 −1.2611 +0.24136 +0.01779 +0.00184	o_1	+6.3146 +1.2611 +0.00894 +0.48282 +0.00184	q_1	+3,698,140 +5,903,200 − 404,800 − 829,500 + 181,650	q_1	− 515,125 +2,474,600 − 219,080 + 473,940 + 17,885	q_1	− 722,387 +2,970,300 − 273,675 + 547,550 + 1
		+331.62		+0.9352		+8.0693		+8,548,690		+2,232,220		+2,521,789
A_1	−341.27 +203.12	+0.86495 +1.8172	B_1		C_1	+5.0393 −1.4790	D_1	+2,737,800 + 387,000	D_1	− 430,630 +1,101,600	D_1	− 516,070 +1,322,400
den.	−138.15	+2.6821		+0.40314		+3.5603		+3,124,800		+670,970		+806,330
A_1'	− 52.083 −329.69	+1.1771 −1.0673	B_1'		C_1'	−1.8235 +0.86872	D_1'	+1,247,100 − 227,300	D_1'	+325,630 −647,100	D_1'	+367,880 −776,780
den.	−381.77	+0.1098		+0.14588		−0.9548		+1,019,800		−321,470		−408,900
α_1		−0.99550 −0.15198	β_1		γ_1		δ_1	+1,063,300 − 133,390	δ_1	−335,180 − 28,641	δ_1	−426,340 − 34,419
		−1.14748		−0.017209		+0.15211		+929,910		−363,821		−460,759
α_1'		+1.7552	β_1'	+0.15671	γ_1'	−0.056710	δ_1'	+818,370	δ_1'	+385,820	δ_1'	+472,400
α_1''		−0.017955 −0.020484	β_1''	−0.0003072 −0.0034571	γ_1''	+0.0027155 +0.0012510	δ_1''	+ 4,698.3 +16,600.0	δ_1''	+13,375 − 6,494.5	δ_1''	+16,056 − 8,225.0
		−0.038722						−18,053.0		− 8,512.2		−10,422.0
$\alpha_1''-1=-0.922839$		−0.077161		−0.0037643		+0.0039665		+3,245		−1,631.7		−2,591

$A_1-A_1'=+2.5723$

For the First Story.—*Continued.*

						For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.	
α_1'''	+0.34876 −0.12434 −0.030211 +0.19421	β_1'''	+0.017014 −0.0018644 −0.0026972 +0.012452	γ_1'''	−0.017929 +0.016481 +0.0009761 −0.000472	δ_1'''	− 14,667 +100,750 − 14,085 − 47,998 +24,000	δ_1'''	+ 7,374.8 −39,418.0 − 6,641.2 +43,722.0 +5,037.6	δ_1'''	+52,932 +11,712 −49,925 − 8,132.0 +6,587
	$\alpha_1'''-\alpha_1^{iv}=+0.39727$		$\beta_1'''-\beta^{iv}=+0.012920$		$\gamma_1'''-\gamma^{iv}=-0.013571$		$\delta_1'''-\delta_1^{iv}=+2,760$				$\delta_1'''-\delta_1^{iv}=+11,756$
α_1^{iv}	−0.48643 +0.027456 +0.25591 −0.20306	β_1^{iv}	−0.023730 +0.0004117 +0.022850 −0.000468	γ_1^{iv}	+0.025006 −0.0036392 −0.0082680 +0.013099	δ_1^{iv}	+ 20,457 − 22,247 +119,310 − 96,280 +21,240	δ_1^{iv}	−10,286 + 8,703.5 +56,255 −57,637 −2,964.5	δ_1^{iv}	−68,740 −16,335 +11,023 +68,883 −5,169
α_1^{v}	−0.0000012 +0.065625 −0.10040 −0.034776	β_1^{v}	−0.000000059 +0.00098420 −0.00896350 −0.00797936	γ_1^{v}	+0.000000063 −0.0087000 +0.0032435 −0.0054566	δ_1^{v}	+ 0.1 − 53,180 − 46,802 +1,052,940 +952,958	δ_1^{v}	0 + 20,806 − 22,067 +396,430 +395,169	δ_1^{v}	+159,182 0 + 26,351 − 27,020 +158,513
α_1^{vi}	+0.071637 −0.0040065 −0.0061283 $\alpha_1^{vi}-1=-0.938498$ +0.061502	β_1^{vi}	+0.0034947 −0.00006008 −0.00054712 + 0.0028875	γ_1^{vi}	−0.0036825 +0.00053105 +0.00019798 −0.0029535	δ_1^{vi}	− 3,012.6 + 3,246.4 − 2,857.0 +16,342.0 +13,719	δ_1^{vi}	+1,514.8 −1,270.1 −1,347.0 +6,850.6 +5,748.3	δ_1^{vi}	+8,223.2 +2,405.4 −1,608.6 −1,649.4 +7,370.6
θ_1	−8.3455 +0.19421 −8.1513	ϕ_1	+0.14227 +0.025675 +0.012452 +0.18040	ψ_1	−0.026262 −0.000472 −0.026734	ω_1	−193,401 +121,990 + 24,000 −47,411	ω_1	0 +51,113 + 5,037.6 +56,151	ω_1	0 + 6,587 +65,541 +72,128
θ_1'	+6.5312 −0.20306 +6.3281	ϕ_1'	−0.020093 −0.000468 −0.020561	ψ_1'	+0.11135 +0.020553 +0.013099 +0.14500	ω_1'	−95,470 +21,240 −74,230	ω_1'	−40,003 − 2,964.5 −42,968	ω_1'	− 5,169 −52,190 −56,459
θ_1''	−0.034776	ϕ_1''	−0.068442 −0.0079794 −0.076421	ψ_1''	−0.068442 −0.0054566 −0.073899	ω_1''	+421,400 +952,958 +1,374,358	ω_1''	−198,230 +395,169 +196,939	ω_1''	−101,300 +158,513 +57,213

For the Second Story

								For Wind on Face A Only.		For Floor and Ec. Col. Loads·		For Floor Loads Only.
c_2		−71.598 + 5.2945 + 0.8740 −65.429	a_2	+0.94810 +0.22027 +0.17220 +1.34057	b_2	−0.74200 −0.22531 −0.18087 −1.14818	d_2	− 930,800 +1,046,500 + 36,784 +152,484	d_2	0 +438,500 +106,640 +545,140	d_2	0 +562,250 +156,690 +718,940
i_2		−341.11 + 7.660 − 4.005 + 0.010 −337.44	g_2	+6.6316 +1.0495 +0.49115 −0.00923 +0.00236 +8.1654	h_2	+2.2162 −1.0735 −0.01862 +0.25831 +0.00161 +1.3840	k_2	+10,921,410 − 4,986,200 − 946,570 − 418,860 + 281,880 +4,851,660	k_2	+ 467,902 −2,089,300 − 198,670 + 58,460 + 116,900 −1,644,708	k_2	+ 749,205 −2,678,900 − 259,790 + 101,930 + 46,887 −1,280,327
p_2		+233.81 + 3.830 − 8.009 + 0.010 +229.64	n_2	+2.8259 −0.71938 +0.24557 −0.018458 +0.002360 +2.3360	o_2	+5.4122 +0.73582 −0.009307 +0.51662 +0.001614 +6.6570	q_2	+3,938,660 +3,418,000 − 473,285 − 837,720 + 281,880 +6,327,535	q_2	− 697,295 +1,432,200 − 99,335 + 116,920 + 116,900 +869,390	q_2	− 809,129 +1,836,300 − 129,895 + 203,860 + 46,887 +1,148,023
A_2	−534.23 +452.39 den. − 81.84	+1.1065 +4.7345 +5.8410	B_2	+0.79950	C_2	+9.9940 −4.1232 +5.8708	D_2	+3,878,900 + 628,750 +4,507,650	D_2	−1,315,000 +2,247,700 +932,700	D_2	−1,023,600 +2,964,300 +1,940,700
A_2'	−152.85 −307.88 den. −460.73	+0.94540 −0.57230 +0.37310	B_2'	+0.14202	C_2'	−1.7753 +0.49840 −1.2769	D_2'	+898,600 − 76,000 +822,600	D_2'	+123,460 −271,720 −148,260	D_2'	+163,040 −358,330 −195,290
α_2		−1.3640 −0.40062 −1.7646	β_2	−0.054555	γ_2	+0.15172	δ_2	+878,750 −307,590 +571,160	δ_2	−158,390 − 63,645 −222,035	δ_2	−208,610 −132,430 −341,040

$A_2 - A_2' = +5.4679$

For the Second Story.—*Continued.*

			For Wind on Face A Only.	For Floor and Ec. Col. Loads.	For Floor Loads Only.
α_2' +1.3071	β_2' +0.14621	γ_2' −0.025973	δ_2' +673,950	δ_2' +197,700	δ_2' +390,650
α_2'' −0.015283 −0.036158 −0.022939 −0.074380	β_2'' −0.0011177 −0.0025659 −0.0036836	γ_2'' +0.0031091 +0.00045581 +0.0035649	δ_2'' + 2,330.4 +11,703.0 −11,826.0 +2,207	δ_2'' +8,332.0 −4,549.8 −3,469.4 +312.8	δ_2'' +10,987 − 6,988.0 − 6,855.5 −2,856.5
α_2''' +0.60630 −0.31834 −0.034946 +0.25301	β_2''' +0.030026 −0.0098420 −0.0039091 +0.016275	γ_2''' −0.029059 +0.027373 +0.00069438 −0.000992	δ_2''' − 17,990 +103,030 − 18,017 − 47,411 +19,612	δ_2''' − 2,549.6 −40,052 − 5,285.0 +56,151 +8,264	δ_2''' +72,128 +23,283 −61,528 −10,444 +23,439
$\alpha_2''' - \alpha_2^{iv} = +0.49787$	$\beta_2''' - \beta_2^{iv} = +0.017263$	$\gamma_2''' - \gamma_2^{iv} = -0.016664$	$\delta_2''' - \delta_2^{iv} = -6101$	$\delta_2''' - \delta_2^{iv} = +16,020$	$\delta_2''' - \delta_2^{iv} = +34,313$
α_2^{iv} −0.47066 +0.036284 +0.18952 −0.24486	β_2^{iv} −0.023310 +0.0011217 +0.021200 −0.000988	γ_2^{iv} +0.022558 −0.0031198 −0.0037660 +0.015672	δ_2^{iv} +13,966 −11,743 +97,720 −74,230 +25,713	δ_2^{iv} + 1,979.4 + 4,565.3 +28,667 −42,968 −7,756.3	δ_2^{iv} −56,459 −18,075 + 7,012.8 +56,647 −10,874
α_2^{v} +0.0025867 +0.13486 −0.096600 +0.04085	β_2^{v} +0.00012810 +0.0041690 −0.010805 −0.006508	γ_2^{v} −0.00012397 −0.011593 +0.0019194 −0.009798	δ_2^{v} − 76.8 − 43,647 − 49,800 +1,374,358 +1,280,834	δ_2^{v} − 10.9 + 16,969 − 14,610 +196,939 +199,287	δ_2^{v} +57,213 + 99.3 +26,064 −28,870 +54,506
α_2^{vi} +0.069807 −0.0050952 −0.0038610	β_2^{vi} +0.0034571 −0.00015753 −0.00043185	γ_2^{vi} −0.0033456 +0.00043812 +0.00007672	δ_2^{vi} − 2,071.3 + 1,649.2 − 1,990.5 +13,719.0	δ_2^{vi} − 293.6 − 641.2 − 584.0 +5,748.3	δ_2^{vi} +7,370.6 +2,680.8 − 984.8 −1,153.9
$\alpha_2^{vi} - 1 = -0.939149$ +0.060851	+0.0028677	−0.0028308	+11,306	+4229.5	+7,912.7

For the Second Story.—*Continued.*

							For Wind on Face A Only.		For Floor and Ec Col. Loads.		For Floor Loads Only.
θ_2	−8.3510 +0.25301 −8.0980	ϕ_2	+0.14227 +0.025498 +0.016275 +0.18404	ψ_2	−0.025170 −0.000992 −0.026162	ω_2	−126,711 +100,540 + 19,612 −6,559	ω_2	0 +37,609 + 8,264 +45,873	ω_2	0 +23,439 +70,363 +93,802
θ_2'	+11.103 − 0.24486 +10.858	ϕ_2'	−0.033905 −0.000988 −0.034893	ψ_2'	+0.18917 +0.033469 +0.015672 +0.23831	ω_2'	−133,680 + 25,713 −107,967	ω_2'	−50,008 − 7,756.3 −57,764	ω_2'	−10,874 −93,555 −104,429
θ_2''	+0.04085	ϕ_2''	−0.087260 −0.006508 −0.093768	ψ_2''	−0.087260 −0.009798 −0.097058	ω_2''	+ 260,810 +1,280,834 +1,541,644	ω_2''	+109,700 +199,287 +308,987	ω_2''	+97,442 +54,506 +151,948

For the Third Story.

c_3	−89.578 + 6.3480 + 0.9566 −82.273	a_3	+0.90705 +0.27353 +0.22011 +1.40069	b_3	−1.2060 −0.27000 −0.21247 −1.60885	d_3	− 583,400 +1,078,500 − 77,787 +417,313	$,d_3$	0 +403,410 +204,250 +607,660	d_3	0 +754,730 +437,500 +1,192,230
i_3	−244.44 + 9.3610 − 4.5300 − 0.0113 −239.62	g_3	+6.2884 +0.74640 +0.60220 −0.018279 +0.001806 +7.6205	h_3	+3.5239 −0.73682 −0.03670 +0.289960 +0.002719 +3.04306	k_3	+6,804,722 −2,942,900 − 725,650 − 475,700 + 355,400 +3,015,872	k_3	+ 556,984 −1,100,900 − 305,770 + 143,500 + 55,300 −650,886	k_3	+ 806,207 −2,059,700 − 867,270 + 201,185 + 15,124 −1,904,454
p_3	+397.26 + 4.6805 − 9.060 − 0.011 +392.87	n_3	+2.6563 −1.2130 +0.30110 −0.03656 +0.001806 +1.7096	o_3	+8.0236 +1.1974 −0.018351 +0.579920 +0.002719 +9.78529	g_3	+2,416,672 +4,782,700 − 362,825 − 951,400 + 355,400 +6,240,547	g_3	− 608,213 +1,789,000 − 152,885 + 287,000 + 55,300 +1,370,202	g_3	− 752,127 +3,346,900 − 433,635 + 402,370 + 15,124 +2,578,632

For the Third Story.—*Continued.*

							For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.	
A_3		+0.85940	B_3		C_3	+3.5300	D_3	+851,700	D_3	−183,800	D_3	−537,750
	−626.97	+1.38890				−0.82252		+343,240		+499,770		+980,620
	+335.63											
den.	−291.34	+2.2483		+0.28240		+2.7075		+1,194,940		+315,970		+442,870
A_3'		+1.1652	B_3'		C_3'	−1.4885	D_3'	+743,150	D_3'	+163,180	D_3'	+307,070
	−140.65	−0.96020				+0.56862		−237,290		−345,550		−677,980
	−550.27											
den.	−690.92	+0.2050		+0.11908		−0.9199		+505,860		−182,370		−370,910
α_3		−1.0122	β_3		γ_3		δ_3	+556,600	δ_3	−200,670	δ_3	−408,130
		−0.27163						−119,880		− 31,701		− 44,432
		−1.2838		−0.028333		+0.13103		+436,720		−232,371		−452,562
α_3'			β_3'		γ_3'		δ_3'		δ_3'		δ_3'	
		+1.7752		+0.13820		−0.05828		+337,230		+243,900		+398,250
α_3''		−0.012154	β_3''	−0.00048237	γ_3''	+0.0022309	δ_3''	+5,072.5	δ_3''	+7,385.8	δ_3''	+14,491
		−0.021857		−0.0028362		+0.0011960		+7,435.5		−3,956.2		− 7,705.5
		−0.036432						−6,921.0		−5,005.7		− 8,173.8
		−0.070443		−0.0033186		+0.0034269		+5,587.0		−1,576.1		−1,388
α_3'''		+0.57045	β_3'''	+0.026876	γ_3'''	−0.027752	δ_3'''	−45,245	δ_3'''	+12,764	δ_3'''	+93,802
		−0.23629		−0.0052145		+0.024118		+80,382		−42,768		+11,239
		−0.046443		−0.0036157		+0.0015248		− 8,823		− 6,381.5		−83,295
								− 6,559		+45,873		−10,419
		+0.28772		+0.018046		−0.002109		+19,755		+9,488		+11,327
α_3^{iv}		−0.76497	β_3^{iv}	−0.036036	γ_3^{iv}	+0.037210	δ_3^{iv}	+ 60,670	δ_3^{iv}	−17,115	δ_3^{iv}	−104,429
		+0.044798		+0.0009887		−0.004572		− 15,239		+ 8,108.3		− 15,071
		+0.42310		+0.032937		−0.013890		+ 80,353		+58,125		+ 15,792
								−107,967		−57,764		+ 94,910
		−0.29707		−0.002110		+0.018748		+17,817		−8,646		−8,798
		$\alpha_3'''-\alpha_3^{\text{iv}}=+0.58479$		$\beta_3'''-\beta_3^{\text{iv}}=+0.020156$		$\gamma_3'''-\gamma_3^{\text{iv}}=-0.020857$		$\delta_3'''-\delta_3^{\text{iv}}=+1938$		$\delta_3'''-\delta_3^{\text{iv}}=+18134$		$\delta_3'''-\delta_3^{\text{iv}}=+20125$

For the Third Story.—*Continued.*

							For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.
α_3^{v}	−0.0028778 +0.12038 −0.17231	β_3^{v}	−0.00013557 +0.0026568 −0.013413	γ_3^{v}	+0.00014000 −0.012288 +0.0056562	δ_3^{v}	+ 228.2 − 40,950 − 32,730 +1,541,644	δ_3^{v}	− 64.4 + 21,790 − 23,671 +308,987	δ_3^{v}	+151,948 − 56 + 42,438 − 38,653
	−0.05481		−0.010892		−0.006492		+1,468,192		+307,042		+155,677
α_3^{vi}	+0.066160 −0.0036817 −0.0050253	β_3^{vi}	+0.0031168 −0.00008125 −0.00039121	γ_3^{vi}	−0.0032184 +0.00037575 +0.00016498	δ_3^{vi}	− 5,247.0 + 1,252.4 − 954.6 +11,306	δ_3^{vi}	+1,480.1 − 666.4 − 690.4 +4,229.5	δ_3^{vi}	+7,912.7 +1,303.6 −1,297.9 −1,127.3
$\alpha_3^{vi}-1=-0.942547$	+0.057453		+0.0026443		−0.0026777		+6,356.8		+4,352.8		+6,791.1
θ_3	−17.278 + 0.28772	ϕ_3	+0.28200 +0.048473 +0.018046	ψ_3	−0.049083 −0.002109	ω_3	−123,726 +116,520 + 19,755	ω_3	0 +79,788 + 9,488	ω_3	0 + 11,327 +124,490
	−16.990		+0.34852		−0.051192		+12,549		+89,276		+135,817
θ_3'	+12.053 − 0.29707	ϕ_3'	−0.033816 −0.002110	ψ_3'	+0.19673 +0.034242 +0.018748	ω_3'	−81,295 +17,817	ω_3'	−55,662 − 8,646	ω_3'	− 8,798 −86,855
	+11.756		−0.035926		+0.24972		−63,478		−64,308		−95,653
θ_3''		ϕ_3''	−0.116240 −0.010892	ψ_3''	−0.116240 −0.006492	ω_3''	+ 109,850 +1,468,192	ω_3''	−245,620 +307,042	ω_3''	− 64,500 +155,677
	−0.05481		−0.127132		−0.122732		+1,578,042		+61,422		+91,177

For the Fourth Story.

c_4	−113.170 + 6.4527 + 0.85120	a_4	+1.5301 +0.31752 +0.22241	b_4	−1.0675 −0.32152 −0.23015	d_4	−477,450 +763,300 + 21,385	d_4	0 +522,630 +200,100	d_4	0 +815,500 +222,090
	−105.866		+2.0700		−1.6192		+307,235		+722,730		+1,037,590

For the Fourth Story.—*Continued.*

							For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.	
i_4		−375.61 + 5.9907 − 3.0926 + 0.0085	g_4	+6.8894 +1.0537 +0.37572 −0.02196 +0.00170	h_4	+2.0662 −1.0670 −0.04391 +0.19516 +0.001014	k_4	+3,538,554 −2,533,300 − 411,300 − 185,470 + 229,250	k_4	+ 493,937 −1,734,800 − 197,530 + 90,005 + 47,945	k_4	+ 656,596 −2,706,600 − 235,820 = 91,585 + 24,310
		−372.70		+8.2986		+1.1515		+637,734		−1,300,443		−2,169,929
p_4		+212.50 + 2.9954 − 6.1852 + 0.0086	n_4	+2.9538 −0.59618 +0.18786 −0.04393 +0.00170	o_4	+5.1142 +0.60370 −0.021955 +0.39032 +0.001014	g_4	+1,305,354 +1,433,300 − 205,650 − 370,940 + 229,250	g_4	−705,022 +981,400 − 98,765 +180,010 + 47,945	g_4	− 676,738 +1,531,200 − 117,910 + 183,170 + 24,310
		+209.32		+2.5032		+6.0873		+2,391,314		+405,568		+944,032
A_4	−878.65 +771.50	+1.1378 +5.6322	B_4		C_4	+9.8812 −3.4783	D_4	+ 630,150 +1,068,600	D_4	−1,285,100 +2,513,600	D_4	−2,144,000 +3,609,400
den.	−107.15	+6.7700		+0.98812		+6.4029		+1,698,750		+1,228,500		+1,465,400
A_4'	−265.03 −433.29	+0.92295 −0.48545 $A_4 - A_4' =$ +6.3325	B_4'		C_4'	−1.5162 +0.29980	D_4'	+362,580 − 92,105	D_4'	+ 61,497 −216,640	D_4'	+143,140 −311,090
den.	−698.32	+0.43750		+0.15162		−1.2164		+270,475		−155,143		−167,950
α_4		−1.3005 −0.44236	β_4		γ_4		δ_4	+289,150 −117,370	δ_4	−165,870 − 84,885	δ_4	−179,550 −101,240
		−1.7429		0		0		+171,780		−250,755		−280,790
α_4'		+1.2032	β_4'	0	γ_4'	0	δ_4'	+257,940	δ_4'	+218,500	δ_4'	+225,550

For the Fourth Story.—*Continued.*

						For Wind on Face A Only.		For Floor and Ec. Col. Loads.		For Floor Loads Only.	
α_4''	−0.0094450	β_4''		γ_4''		δ_4''	+2,901.9	δ_4''	+6,826.0	δ_4''	+9,800.5
	−0.034078						+3,358.5		−4,902.6		−5,489.8
	−0.018401						−3,449.5		−3,341.5		−3,944.9
	−0.061924		0		0		+2,810.9		−1,418.1		+365.8
α_4^{vi}	+0.058368	β_4^{vi}		γ_4^{vi}		δ_4^{vi}	−2,649.5	δ_4^{vi}	+1,336.6	δ_4^{vi}	+6,791.1
	−0.0046088						+ 454.2		− 663.0		− 344.8
	−0.0032220						− 604.0		− 585.1		− 742.5
							+6,356.8		+4,352.8		− 690.7
	+0.050537		0		0		+3,557.5		+4,441.3		+5,013.1

Referring now to the previous tables and Plates 14 and 16 all unknowns are found as follows (*for wind on face A, only*).

$H_4 = \delta_4^{vi} \div (1 - \alpha_4^{vi}) = +\ 3{,}557.5 \div 0.949463 = +\ 3{,}746.9$ ℔.

$H_3 = -\ 0.061924\ H_4 + 2{,}810.9 = +\ 2{,}578.9$ ℔.

$M_4^B = +\ 1.2032\ H_4 + 225{,}550 = +\ 230{,}058$ in. ℔.

$M_4^A = -\ 1.7429\ H_4 + 171{,}780 = +\ 165{,}249$ in. ℔.

$S_4^B = H_4 = +\ 3{,}746.9$ ℔, $S_4^A = F_4^A - H_4 = +\ 3{,}003.1$ ℔.

$X_4^B = 230{,}058 \div 3{,}746.9 = +\ 61.403$ in.

$X_4^A = 165{,}249 \div 3{,}003.1 = +\ 55.025$ in.

$V_4^B = -\ 0.005\ M_4^A - 0.005\ M_4^B + 4{,}725 = +\ 2{,}748$ ℔ $= -\ V_4^A$.

$\lambda_4 = -\ 13.825\ S_4^A + 93{,}250 = +\ 51{,}790$ in.

$\Delta_4 = -\ 0.05481\ H_3 - 0.127132\ M_4^A - 0.122732\ M_4^B + 1{,}578{,}042$
$= +\ 1{,}528{,}655$ in.

$C_5^B = +\ 11.756\ H_3 - 0.035926\ M_4^A + 0.24972\ M_4^B - 63{,}478$
$= +\ 18{,}364.$

$C_5^A = -\ 16.990\ H_3 + 0.34852\ M_4^A - 0.051192\ M_4^B + 12{,}549$
$= +\ 14{,}548.$

$\Delta_3 = -\ 0.05481\ H_3 - 0.010892\ M_4^A - 0.006492\ M_4^B + 1{,}468{,}192$
$= +\ 1{,}464{,}757$ in.

$C_4^B = -\ 0.29707\ H_3 - 00.02110\ M_4^A + 0.018748\ M_4^B + 17{,}817$
$= +\ 21{,}015.5.$

$C_4^A = +\ 0.28772\ H_3 + 0.018046\ M_4^A - 0.002109\ M_4^B + 19{,}755$
$= +\ 22{,}994.$

$H_2 = -\ 0.070443\ H_3 - 0.0033186\ M_4^A + 0.0034269\ M_4^B + 5{,}587.0$
$= +\ 5{,}645.3$ ℔.

$M_3^B = +\ 1.7752\ H_3 + 0.13820\ M_4^A - 0.05828\ M_4^B + 337{,}230$
$= +\ 351{,}238$ in. ℔.

$M_3^A = -\ 1.2838\ H_3 - 0.028333\ M_4^A + 0.13103\ M_4^B + 436{,}720$
$= +\ 458{,}873$ in. ℔.

$S_3^B = S_4^B + H_3 = +\ 6{,}325.8$ ℔. $S_3^A = 14{,}250 - S_3^B = +\ 7{,}924.2$ ℔.

$X_3^B = 351{,}238 \div 6{,}325.8 = +\ 55.527$ in.

$X_3^A = 458{,}873 \div 7{,}924.2 = +\ 57.905$ in.

$V_3^B = -\ 0.005\ M_3^A - 0.005\ M_3^B + 14{,}944 = +\ 10{,}893$ ℔ $= -\ V_3^A$.

$\lambda_3 = +11.760\, S_4^A - 11.760\, S_3^A + 88{,}250 = +30{,}340$ in.

$\delta_4^B = +12.788\, M_4^B + 5.540\, S_4^A + 130.0\, C_4^B - 3{,}740{,}000$
$= +3{,}597{,}815$ in.

$\delta_4^A = +18.330\, M_4^A - 795.0\, S_4^A + 130.0\, C_4^A = +3{,}632{,}850$ in.

$\Delta_2 = +0.04085\, H_2 - 0.006508\, M_3^A - 0.009798\, M_3^B + 1{,}280{,}834$
$= +1{,}274{,}638$ in.

$C_3^B = -0.24486\, H_2 - 0.000988\, M_3^A + 0.015672\, M_3^B + 25{,}713$
$= +293{,}819.$

$C_3^A = +0.25301\, H_2 + 0.016275\, M_3^A - 0.000992\, M_3^B + 19{,}612$
$= +281{,}604.$

$H_1 = -0.074380\, H_2 - 0.0036836\, M_3^A + 0.0035649\, M_3^B + 2{,}207$
$= +13{,}489$ ℔.

$M_2^B = +1.3071\, H_2 + 0.14621\, M_3^A - 0.025973\, M_3^B + 673{,}950$
$= +739{,}305$ in. ℔.

$M_2^A = -1.7646\, H_2 - 0.054555\, M_3^A + 0.15172\, M_3^B + 571{,}160$
$= +589{,}454$ in. ℔.

$S_2^B = S_3^B + H_2 = +11971.1$ ℔.

$S_2^A = 21{,}750 - S_2^B = +9{,}778.9$ ℔.

$X_2^B = 739{,}305 \div 11971.1 = +61{,}759$ in.

$X_2^A = 589{,}454 \div 9{,}778.9 = +60.283$ in.

$V_2^B = -0.005\, M_2^A - 0.005\, M_2^B + 30{,}788 = +24{,}144$ ℔ $= -V_2^A.$

$\lambda_2 = +11.255\, S_3^A - 11.255\, S_2^A + 84{,}450 = +63{,}542$ in.

$\delta_3^B = +11.823\, M_3^B + 493.0\, S_3^A + 125.0\, C_3^B - 7{,}030{,}000$
$= +4{,}709{,}240$ in.

$\delta_3^A = +8.8920\, M_3^A - 371.0\, S_3^A + 125.0\, C_3^A = +4{,}664{,}350$ in.

$\Delta_1 = -0.034776\, H_1 - 0.0079794\, M_2^A - 0.0054566\, M_2^B$
$+ 952{,}958 = +944{,}174$ in.

$C_2^B = -0.20306\, H_1 - 0.000468\, M_2^A + 0.013099\, M_2^B + 21{,}240$
$= +30{,}374.4.$

$C_2^A = +0.19421\, H_1 + 0.012452\, M_2^A - 0.000472\, M_2^B + 24{,}000$
$= +31{,}253.5.$

$H_0 = -0.077161\, H_1 - 0.0037643\, M_2^A + 0.0039665\, M_2^B + 3{,}245$
$= +3{,}854.5$ ℔.

$M_1^B = +1.7552\, H_1 + 0.15671\, M_2^A - 0.056710\, M_2^B + 818{,}370$
$= +871{,}192$ in. ℔

$M_1^A = -1.14748\,H_1 - 0.017209\,M_2^A + 0.15211\,M_2^B + 929{,}910$
$= +1{,}030{,}674$ in. ℔.

$S_1^B = S_2^B + H_1 = +13{,}320.0$ ℔.

$S_1^A = 29{,}250 - S_1^B = +15{,}930.0$ ℔.

$X_1^B = 871{,}192 \div 13{,}320.0 = +65.403$ in.

$X_1^A = 1{,}030{,}674 \div 15{,}930.0 = +64.705$ in.

$V_1^B = -0.005\,M_1^A - 0.005\,M_1^B + 52{,}256 = +42{,}747$ ℔ $= -V_1^A$.

$\lambda_1 = +9.845\,S_2^A - 9.845\,S_1^A + 73{,}800 = +13{,}269$ in.

$\delta_2^B = +6.9593\,M_2^B + 290.0\,S_2^A + 125.0\,C_2^B - 6{,}315{,}000$
$= +5{,}470{,}700$ in.

$\delta_2^A = +8.8920\,M_2^A - 371.0\,S_2^A + 125.0\,C_2^A = +5{,}525{,}490$ in.

$\Delta_0 = +0.0000158\,H_0 - 0.0051687\,M_1^A - 0.0051710\,M_1^B$
$+ 509{,}260 = +499{,}428$ in.

$C_1^B = -0.15661\,H_0 + 0.000374\,M_1^A + 0.010152\,M_1^B + 17{,}440$
$= +26{,}066.3$.

$C_1^A = +0.15657\,H_0 + 0.010150\,M_1^A + 0.000376\,M_1^B + 17{,}021$
$= +28{,}414.1$.

$S_0^B = +0.071624\,H_0 + 0.0034912\,M_1^A - 0.0034912\,M_1^B$
$+ 16{,}342.0 = +17{,}175.5$ ℔.

$M_0^B = +2.5337\,H_0 + 0.22285\,M_1^A - 0.092265\,M_1^B + 1{,}237{,}800$
$= +1{,}396{,}896$ in. ℔.

$M_0^A = -2.5341\,H_0 - 0.092290\,M_1^A + 0.22288\,M_1^B + 1{,}252{,}300$
$= +1{,}341{,}547$ in. ℔.

$S_0^A = 33{,}000 - S_0^B = +15{,}825.5$ ℔.

$X_0^B = 1{,}396{,}896 \div 17{,}175.5 = +81.333$ in.

$X_0^A = 1{,}341{,}547 \div 15{,}825.5 = +84.773$ in.

$V_0^B = -0.005\,M_0^A - 0.005\,M_0^B + 76{,}772 = +63{,}080$ ℔. $= -V_0^A$.

$\lambda_0 = +8.450\,S_1^A - 8.450\,S_0^A + 31{,}670 = +32{,}567$ in.

$\delta_1^B = +6.9593\,M_1^B + 290.0\,S_1^A + 125.0\,C_1^B - 8{,}485{,}000$
$= +5{,}458{,}490$ in.

$\delta_1^A = +5.0374\,M_1^A - 210.0\,S_1^A + 125.0\,C_1^A = +5{,}400{,}460$ in.

$\delta_0^B = +5.0374\,M_0^B + 210.0\,S_0^A - 6{,}930{,}000 = +3{,}432{,}000$ in.

$\delta_0^A = +5.0374\,M_0^A - 210.0\,S_0^A = +3{,}436{,}700$.

Section.	*x*	By the Method of Chapter III.						By the Method of Chapter II.					
		V	*S*	*M*	*V+A*	*Mv+I*	Max. + Fibre.	*V*	*S*	*M*	*V+A*	*Mv+I*	Max + Fibre
A–0	125	−63,080	+15,825.5	− 636,641	−1,815	+3,337	+1,522	−66,475	+16,500	−1,031,000	−1,915	+5,400	+3,485
	0			+1,341,547	−1,815	+7,035	+5,220			+1,031,000	−1,915	+5,400	+3,485
B–0	125	+63,080	+17,175.5	− 749,916	+1,815	+3,930	+5,745	+66,475	+16,500	−1,031,000	+1,915	+5,400	+7,315
	0			+1,396,896	+1,815	+7,325	+9,140			+1,031,000	+1,915	+5,400	+7,315
A–1	125	−42,747	+15,930	− 960,576	−1,230	+5,035	+3,805	−43,112	+16,970	−1,120,000	−1,242	+5,870	+4,628
	0			+1,030,674	−1,230	+5,405	+4,175			+1,120,000	−1,242	+5,870	+4,628
B–1	125	+42,747	+13,320	− 793,808	+1,734	+5,740	+7,474	+43,112	+12,280	− 767,500	+1,748	+5,555	+7,303
	0			+ 871,192	+1,734	+6,300	+8,034			+ 767,500	+1,748	+5,555	+7,303
A–2	125	−24,144	+ 9,778.9	− 632,908	−1,225	+5,850	+4,625	−23,990	+ 9,553	− 597,500	−1,216	+5,525	+4,309
	0			+ 589,454	−1,225	+5,445	+4,220			+ 597,500	−1,216	+5,525	+4,309
B–2	125	+24,144	+11,971.1	− 757,083	+ 980	+5,475	+6,455	+23,990	+12,197	− 762,500	+ 974	+5,520	+6,494
	0			+ 739,305	+ 980	+5,345	+6,325			+ 762,500	+ 974	+5,520	+6,494
A–3	125	−10,893	+ 7,924.2	− 531,652	− 552	+4,915	+4,363	−10,480	+ 8,135	− 508,000	− 532	+4,700	+4,168
	0			+ 458,873	− 552	+4,225	+3,693			+ 508,000	− 532	+4,700	+4,168
B–3	125	+10,893	+ 6,325.8	− 439,487	+ 715	+5,405	+6,120	+10,480	+ 6,115	− 382,000	+ 688	+4,700	+5,388
	0			+ 351,238	+ 715	+4,320	+5,035			+ 382,000	+ 688	+4,700	+5,388
A–4	130	− 2,748	+ 3,003.1	− 225,154	− 239	+3,900	+3,661	− 2,532	+ 2,773	− 180,200	− 220	+3,125	+2,905
	0			+ 165,249	− 239	+2,865	+2,626			+ 180,200	− 220	+3,125	+2,905
B–4	130	+ 2,748	+ 3,746.9	− 257,039	+ 181	+3,160	+3,341	+ 2,532	+ 3,977	− 258,300	+ 166	+3,175	+3,341
	0			+ 230,058	+ 181	+2,830	+3,011			+ 258,300	+ 166	+3,175	+3,341

Checks.—

$H_4 + H_3 + H_2 + H_1 + H_0 = +\ 17{,}174.5$ ℔.

S_0^B, above, equals $+\ 17{,}175.5$ ℔; hence checks.

Fourth Story.

$$\left.\begin{array}{l} 12.5\ C_4^A + \delta_4^A + 10.0\ C_5^A - \lambda_4 = +\ 4{,}013{,}965 \text{ in.} \\ 12.5\ C_4^B + \delta_4^B + 10.0\ C_5^B - \lambda_3 = +\ 4{,}012{,}804 \text{ in.} \end{array}\right\} \text{Checks.}$$

Third Story.

$$\left.\begin{array}{l} 12.5\ C_3^A + \delta_3^A + 12.5\ C_4^A - \lambda_3 = +\ 5{,}273{,}440 \text{ in.} \\ 12.5\ C_3^B + \delta_3^B + 12.5\ C_4^B - \lambda_2 = +\ 5{,}275{,}661 \text{ in.} \end{array}\right\} \text{Checks.}$$

Second Story.

$$\left.\begin{array}{l} 12.5\ C_2^A + \delta_2^A + 12.5\ C_3^A - \lambda_2 = +\ 6{,}204{,}622 \text{ in.} \\ 12.5\ C_2^B + \delta_2^B + 12.5\ C_3^B - \lambda_1 = +\ 6{,}204{,}385 \text{ in.} \end{array}\right\} \text{Checks.}$$

First Story.

$$\left.\begin{array}{l} 12.5\ C_1^A + \delta_1^A + 12.5\ C_2^A - \lambda_1 = +\ 6{,}133{,}036 \text{ in.} \\ 12.5\ C_1^B + \delta_1^B + 12.5\ C_2^B - \lambda_0 = +\ 6{,}131{,}432 \text{ in.} \end{array}\right\} \text{Checks.}$$

O Story.

$$\left.\begin{array}{l} \delta_0^A + 12.5\ C_1^A - \lambda_0 = +\ 3{,}759{,}309 \text{ in.} \\ \delta_0^B + 12.5\ C_1^B = +\ 3{,}757{,}829 \text{ in.} \end{array}\right\} \text{Checks.}$$

The accompanying table gives a comparison of Column Stresses by the methods of Chapters II and III.

The following table gives a comparison of the stresses in the top flanges of the girders at the left end ($x = 200$ in.).

Girder.	Chapter III.	Chapter II.
a–0	+84,497	+104,885
a–1	+75,530	+ 82,003
a–2	+55,345	+ 56,105
a–3	+34,629	+ 34,051
a–4	+14,633	+ 12,385

As will be noticed the comparisons for both column and girder stresses are very close, excepting in the basement or o story. A difference would naturally be expected here since, when the column is not vertical at the first floor the point of contraflexure in the o story column will be raised above mid height.

CHAPTER VII.

A STUDY OF THE EFFECT OF FLOOR AND ECCENTRIC COLUMN LOADS ON AN ELASTIC PORTAL BRACED BUILDING — WITH COMPARISONS.

Referring to the previous chapter, the stresses and deformations due to the combined floor and eccentric column loads will now be obtained and comparisons similar to those of that chapter will also be given here. Using the tables calculated in Chapter VI the following values are derived.

$H_4 = \delta_4^{vi} \div (1 - \alpha_4^{vi}) = +4441.3 \div 0.949463 = +4{,}677.8$ ℔.

$H_3 = -0.061924\, H_4 - 1{,}418.1 = -1{,}707.8$ ℔.

$M_4^B = +1.2032\, H_4 + 218{,}500 = +224{,}128$ in. ℔.

$M_4^A = -1.7429\, H_4 - 250{,}755 = -258{,}909$ in. ℔.

$S_4^B = H_4 = +4{,}677.8$ ℔ $= -S_4^A$.

$X_4^B = 224{,}128 \div 4{,}677.8 = +47.913$ in.,

$X_4^A = -258{,}909 \div -4{,}677.8 = +55.345$ in.

$V_4^B = -0.005\, M_4^A - 0.005\, M_4^B + 19{,}328 = +19{,}502$ ℔.

$V_4^A = 52{,}500 - V_4^B = +32{,}998$ ℔.

$\lambda_4 = -13.825\, S_4^A = +64{,}660$ in.

$\Delta_4 = -0.05481\, H_3 - 0.127132\, M_4^A - 0.122{,}732\, M_4^B + 61{,}422$
$= +66{,}922$ in.

$C_5^B = +11.756\, H_3 - 0.035926\, M_4^A + 0.24972\, M_4^B - 64{,}308$
$= -19{,}116.$

$C_5^A = -16.990\, H_3 + 0.34852\, M_4^A - 0.051192\, M_4^B + 89{,}276$
$= +16{,}581.$

$\Delta_3 = -0.05481\, H_3 - 0.010892\, M_4^A - 0.006492\, M_4^B + 307{,}042$
$= +308{,}501$ in.

$C_4^B = -0.29707\, H_3 - 0.002110\, M_4^A + 0.018748\, M_4^B - 8{,}646$
$= -3{,}390.5.$

$C_4^A = +0.28772\, H_3 + 0.018046\, M_4^A - 0.002109\, M_4^B + 9{,}488$
$= +3{,}851.6.$

$H_2 = -0.070443\, H_3 - 0.0033186\, M_4^A + 0.0034269\, M_4^B - 1{,}576.1 = +170.8.$

$M_3^B = +1.7752\, H_3 + 0.13820\, M_4^A - 0.05828\, M_4^B + 243{,}900 = +192{,}021$ in. ℔.

$M_3^A = -1.2838\, H_3 - 0.028333\, M_4^A + 0.13103\, M_4^B - 232{,}371 = -193{,}474$ in. ℔.

$S_3^B = S_4^B + H_3 = +2{,}970.0$ ℔ $= -S_3^A.$

$X_3^B = 192{,}021 \div 2{,}970.0 = +64.655$ in.

$X_3^A = -193{,}474 \div -2{,}970.0 = +65.140$ in.

$V_3^B = -0.005\, M_3^A - 0.005\, M_3^B + 73{,}368 = +73{,}374$ ℔.

$V_3^A = 153{,}975 - V_3^B = +80{,}601$ ℔.

$\lambda_3 = +11.760\, S_4^A - 11.760\, S_3^A = -20{,}092$ in.

$\delta_4^B = +12.788\, M_4^B + 554.0\, S_4^A + 130.0\, C_4^B = -166{,}805$ in.

$\delta_4^A = +18.330\, M_4^A - 795.0\, S_4^A + 130.0\, C_4^A = -529{,}292$ in.

$\Delta_2 = +0.04085\, H_2 - 0.006508\, M_3^A - 0.009798 M_3^B + 199{,}287 = +198{,}672$ in.

$C_3^B = -0.24486\, H_2 - 0.000988\, M_3^A + 0.015672\, M_3^B - 7{,}756.3 = -4{,}597.3.$

$C_3^A = +0.25301\, H_2 + 0.016275\, M_3^A - 0.000992\, M_3^B + 8{,}264 = +4{,}967.8.$

$H_1 = -0.074380\, H_2 - 0.0036836\, M_3^A + 0.0035649\, M_3^B + 312.8 = +1{,}697.3$ ℔.

$M_2^B = +1.3071\, H_2 + 0.14621\, M_3^A - 0.025973\, M_3^B + 197{,}7700 = +164{,}647$ in. ℔.

$M_2^A = -1.7646\, H_2 - 0.054555\, M_3^A + 0.15172\, M_3^B - 222{,}035 = -182{,}648$ in. ℔.

$S_2^B = S_3^B + H_2 = +3{,}140.8$ ℔ $= -S_2^A.$

$X_2^B = 164{,}647 \div 3{,}140.8 = +52.422$ in.,

$X_2^A = -182{,}648 \div -3{,}140.8 = +58.158$ in.

$V_2^B = -0.005\, M_2^A - 0.005\, M_2^B + 127{,}407 = +127{,}496$ ℔.

$V_2^A = 255{,}450 - V_2^B = +127{,}954$ ℔.

$\lambda_2 = +11.255\, S_3^A - 11.255\, S_2^A = +1{,}922$ in.

$\delta_3^B = +11.823\, M_3^B + 493.0\, S_3^A + 125.0\, C_3^B = +232{,}570$ in.

$$\delta_3^A = +\,8.8920\,M_3^A - 371.0\,S_3^A + 125.0\,C_3^A = +\,1{,}020 \text{ in.}$$

$$\begin{aligned}\Delta_1 &= -\,0.034776\,H_1 - 0.0079794\,M_2^A - 0.0054566\,M_2^B \\ &\qquad + 395{,}169 = +\,395{,}669 \text{ in.}\end{aligned}$$

$$\begin{aligned}C_2^B &= -\,0.20306\,H_1 - 0.000468\,M_2^A + 0.013099\,M_2^B - 2{,}964.5 \\ &\qquad = -\,1{,}066.9.\end{aligned}$$

$$\begin{aligned}C_2^A &= +\,0.19421\,H_1 + 0.012452\,M_2^A - 0.000472\,M_2^B + 5{,}037.6 \\ &\qquad = +\,3{,}015.1.\end{aligned}$$

$$\begin{aligned}H_0 &= -\,0.077161\,H_1 - 0.0037643\,M_2^A + 0.0039665\,M_2^B \\ &\qquad - 1{,}631.7 = -\,422.0 \text{ lb.}\end{aligned}$$

$$\begin{aligned}M_1^B &= +\,1.7552\,H_1 + 0.15671\,M_2^A - 0.056710\,M_2^B + 385{,}820 \\ &\qquad = +\,350{,}837 \text{ in. lb.}\end{aligned}$$

$$\begin{aligned}M_1^A &= -\,1.14748\,H_1 - 0.017209\,M_2^A + 0.15211\,M_2^B - 363{,}821 \\ &\qquad = -\,337{,}581 \text{ in. lb.}\end{aligned}$$

$$S_1^B = S_2^B + H_1 = +\,4{,}838.1 \text{ lb} = -\,S_1^A.$$

$$X_1^B = 350{,}837 \div 4{,}838.1 = +\,72.517 \text{ in.,}$$

$$X_1^A = -\,337{,}581 \div -\,4{,}838.1 = +\,69.777 \text{ in.}$$

$$V_1^B = -\,0.005\,M_1^A - 0.005\,M_1^B + 181{,}446 = +\,181{,}379 \text{ lb.}$$

$$V_1^A = 356{,}925 - V_1^B = +\,175{,}546 \text{ lb.}$$

$$\lambda_1 = +\,9.845\,S_2^A - 9.845\,S_1^A = +\,16{,}696 \text{ in.}$$

$$\delta_2^B = +\,6.9593\,M_2^B + 290.0\,S_2^A + 125.0\,C_2^B = +\,101{,}720 \text{ in.}$$

$$\delta_2^A = +\,8.8920\,M_2^A - 371.0\,S_2^A + 125.0\,C_2^A = -\,83{,}700 \text{ in.}$$

$$\begin{aligned}\Delta_0 &= +\,0.0000158\,H_0 - 0.0051687\,M_1^A - 0.0051710\,M_1^B \\ &\qquad + 50{,}140 = +\,50{,}071 \text{ in.}\end{aligned}$$

$$\begin{aligned}C_1^B &= -\,0.15661\,H_0 + 0.000374\,M_1^A + 0.010152\,M_1^B - 9{,}965 \\ &\qquad = -\,6{,}463.3.\end{aligned}$$

$$\begin{aligned}C_1^A &= +\,0.15657\,H_0 + 0.010150\,M_1^A + 0.000376\,M_1^B + 9{,}213 \\ &\qquad = +\,5{,}852.3.\end{aligned}$$

$$\begin{aligned}S_0^B &= +\,0.071624\,H_0 + 0.0034912\,M_1^A - 0.0034912\,M_1^B \\ &\qquad + 6{,}850.6 = +\,44{,}169 \text{ lb} = -\,S_0^A.\end{aligned}$$

$$\begin{aligned}M_0^B &= +\,2.5337\,H_0 + 0.22285\,M_1^A - 0.092265\,M_1^B + 304{,}530 \\ &\qquad = +\,195{,}858 \text{ in. lb.}\end{aligned}$$

$$\begin{aligned}M_0^A &= -\,2.5341\,H_0 - 0.092290\,M_1^A + 0.22288\,M_1^B - 313{,}865 \\ &\qquad = -\,203{,}443 \text{ in. lb.}\end{aligned}$$

$$X_0^B = 195{,}858 \div 4{,}416.9 = +\,44.350 \text{ in.}$$

$X_0^A = -203{,}443 \div -4{,}416.9 = +46.068$ in.
$V_0^B = -0.005\ M_0^A - 0.005\ M_0^B + 247{,}446 = +247{,}482$ ℔.
$V_0^A = 482{,}320 - V_0^B = +234{,}838$ ℔.
$\lambda_0 = +8.450\ S_1^A - 8.450\ S_0^A = -3{,}565$ in.
$\delta_1^B = +6.9593\ M_1^B + 290.0\ S_1^A + 125.0\ C_1^B = +230{,}690$ in.
$\delta_1^A = +5.0374\ M_1^A - 210.0\ S_1^A + 125.0\ C_1^A = +46{,}390$ in.
$\delta_0^B = +5.0374\ M_0^B + 210.0\ S_0^A = +59{,}690$ in.
$\delta_0^A = +5.0374\ M_0^A - 210.0\ S_0^A = -97{,}910$ in.

Checks.

$H_4 + H_3 + H_2 + H_1 + H_0 = +4{,}416.1$ ℔.
S_0^B, above, $= +4{,}416.9$; hence checks.

Fourth Story

$$\left.\begin{array}{l} 12.5\ C_4^A + \delta_4^A + 10.0\ C_5^A - \lambda_4 = -379{,}997 \text{ in.} \\ 12.5\ C_4^B + \delta_4^B + 10.0\ C_5^B - \lambda_3 = -380{,}254 \text{ in.} \end{array}\right\} \text{Checks.}$$

Third Story.

$$\left.\begin{array}{l} 12.5\ C_3^A + \delta_3^A + 12.5\ C_4^A - \lambda_3 = +131{,}354 \text{ in.} \\ 12.5\ C_3^B + \delta_3^B + 12.5\ C_4^B - \lambda_2 = +130{,}741 \text{ in.} \end{array}\right\} \text{Checks.}$$

Second Story.

$$\left.\begin{array}{l} 12.5\ C_2^A + \delta_2^A + 12.5\ C_3^A - \lambda_2 = +14{,}164 \text{ in.} \\ 12.5\ C_2^B + \delta_2^B + 12.5\ C_3^B - \lambda_1 = +14\ 222 \text{ in.} \end{array}\right\} \text{Checks.}$$

First Story.

$$\left.\begin{array}{l} 12.5\ C_1^A + \delta_1^A + 12.5\ C_2^A - \lambda_1 = +140{,}537 \text{ in.} \\ 12.5\ C_1^B + \delta_1^B + 12.5\ C_2^B - \lambda_0 = +140{,}128 \text{ in.} \end{array}\right\} \text{Checks.}$$

o Story.

$$\left.\begin{array}{l} \delta_0^A + 12.5\ C_1^A - \lambda_0 = -21{,}191 \text{ in.} \\ \delta_0^B + 12.5\ C_1^B \qquad = -21{,}101 \text{ in.} \end{array}\right\} \text{Checks.}$$

The following table gives a comparison of column stresses by the methods of Chapters II and III.

Section.	X	By the Method of Chapter III.						By the Method of Chapter II.					
		V	S	M	V + A	Mv + I	Max. + Fibre.	V	S	M	V + A	Mv + I	Max. + Fibre.
A–0	125	+234,838	−4,416.9	+348,669	+6,770	+1,828	+8,598	+234,874	0	0	+6,770	0	+6,770
	0			−203,443	+6,770	+1,066	+7,836					0	+6,770
B–0	125	+247,482	+4,416.9	−356,254	+7,130	+1,868	+8,998	+247,446	0	0	+7,130	0	+7,130
	0			+195,858	+7,130	+1,026	+8,156					0	+7,130
A–1	125	+175,546	−4,838.1	+267,181	+5,057	+1,401	+6,458	+175,479	0	0	+5,055	0	+5,055
	0			−337,581	+5,057	+1,769	+6,826					0	+5,055
B–1	125	+181,379	+4,838.1	−253,925	+7,365	+1,837	+9,202	+181,446	0	0	+7,368	0	+7,368
	0			+350,837	+7,365	+2,540	+9,905					0	+7,368
A–2	125	+127,954	−3,140.8	+209,952	+6,485	+1,942	+8,427	+128,043	0	0	+6,489	0	+6,489
	0			−182,648	+6,485	+1,690	+8,175					0	+6,489
B–2	125	+127,496	+3,140.8	−227,953	+5,175	+1,650	+6,825	+127,407	0	0	+5,171	0	+5,171
	0			+164,647	+5,175	+1,191	+6,366					0	+5,171
A–3	125	+ 80,601	−2,970.0	+177,776	+4,080	+1,644	+5,724	+ 80,607	0	0	+4,080	0	+4,080
	0			−193,474	+4,080	+1,789	+5,869					0	+4,080
B–3	125	+ 73,374	+2,970.0	−179,229	+4,820	+2,203	+7,023	+ 73,368	0	0	+4,820	0	+4,820
	0			+192,021	+4,820	+2,360	+7,180					0	+4,820
A–4	130	+ 32,998	−4,677.8	+349,205	+2,864	+6,055	+8,919	+ 33,172	0	0	+2,878	0	+2,878
	0			−258,909	+2,864	+4,485	+7,349					0	+2,878
B–4	130	+ 19,502	+4,677.8	−383,986	+1,281	+4,720	+6,001	+ 19,328	0	0	+1,270	0	+1,270
	0			+224,128	+1,281	+2,759	+4,040					0	+1,270

The following table gives a comparison of the stresses in the top flanges of the girders at mid span ($x = 100$ in.) and at the left end ($x = 200$ in.).

Girder.	Chapter III.		Chapter II.	
	$X = 200$	$X = 100$	$X = 200$	$X = 100$
a–0	−42,387	+30,111	−10,100	+62,750
a–1	−31,234	+18,384	−10,100	+38,830
a–2	−29,207	+19,455	−10,100	+38,830
a–3	−32,245	+17,417	−10,100	+38,830
a–4	−23,000	+29,490	− 6,710	+46,610

Attention is called to the large bending moments in the columns, causing stresses not taken care of in the design.

CHAPTER VIII.

A STUDY OF THE EFFECT OF UNIFORM FLOOR LOADS ALONE ON AN ELASTIC PORTAL BRACED BUILDING — WITH COMPARISONS.

As in the previous chapter these calculations refer back to the tables of Chapter VI. The values for this loading become:

$H_4 = \delta_4^{\text{vi}} \div (1 - \alpha_4^{\text{vi}}) = +\,5{,}013.1 \div 0.949463 = +\,5{,}280.0$ ℔.

$H_3 = -\,0.061924\,H_4 + 365.8 = +\,38.8$ ℔.

$M_4^B = +\,1.2032\,H_4 + 257{,}940 = +\,264{,}294$ in. ℔.

$M_4^A = -\,1.7429\,H_4 - 280{,}790 = -\,289{,}993$ in. ℔.

$S_4^B = H_4 = +\,5{,}280.0$ ℔ $= -\,S_4^A$.

$X_4^B = 264{,}294 \div 5{,}280.0 = +\,50.052$ in.,

$X_4^A = -\,289{,}993 \div -\,5{,}280.0 = +\,54.920$ in.

$V_4^B = -\,0.005\,M_4^A - 0.005\,M_4^B + 20{,}000 = +\,20{,}128$ ℔.

$V_4^A = 40{,}000 - V_4^B = +\,19{,}872$ ℔.

$\lambda_4 = -\,13.825\,S_4^A = +\,72{,}980$ in.

$\Delta_4 = -\,0.05481\,H_3 - 0.127132\,M_4^A - 0.122732\,M_4^B + 91{,}177 = +\,95{,}607$ in.

$C_5^B = +\,11.756\,H_3 - 0.035926\,M_4^A + 0.24972\,M_4^B - 95{,}653 = -\,18{,}780.$

$C_5^A = -\,16.990\,H_3 + 0.34852\,M_4^A - 0.051192\,M_4^B + 135{,}817 = +\,20{,}564.$

$\Delta_3 = -\,0.05481\,H_3 - 0.010892\,M_4^A - 0.006492\,M_4^B + 155{,}677 = +\,157{,}118$ in.

$C_4^B = -\,0.29707\,H_3 - 0.002110\,M_4^A + 0.018748\,M_4^B - 8{,}798 = -\,3{,}242.8.$

$C_4^A = +\,0.28772\,H_3 + 0.018046\,M_4^A - 0.002109\,M_4^B + 11{,}327 = +\,5{,}548.$

$H_2 = -\,0.070443\,H_3 - 0.0033186\,M_4^A + 0.0034269\,M_4^B - 1{,}388 = +\,477.3$ ℔.

$M_3^B = +1.7752\,H_3 + 0.13820\,M_4^A - 0.05828\,M_4^B + 398{,}250$
$= +342{,}840$ in. ℔.

$M_3^A = -1.2838\,H_3 - 0.028333\,M_4^A + 0.13103\,M_4^B - 452{,}562$
$= -409{,}765$ in. ℔.

$S_3^B = S_4^B + H_3 = +5{,}318.8$ ℔ $= -S_3^A$.

$X_3^B = 342{,}840 \div 5{,}318.8 = +64.458$ in.

$X_3^A = -409{,}765 \div -5{,}318.8 = +77.038$ in.

$V_3^B = -0.005\,M_3^A - 0.005\,M_3^B + 43{,}375 = +43{,}710$ ℔.

$V_3^A = 86{,}750 - V_3^B = +43{,}040$ ℔.

$\lambda_3 = +11.760\,S_4^A - 11.760\,S_3^A = +457$ in.

$\delta_4^B = +12.788\,M_4^B + 554.0\,S_4^A + 130.0\,C_4^B = +32{,}086$ in.

$\delta_4^A = +18.330\,M_4^A - 795.0\,S_4^A + 130.0\,C_4^A = -400{,}260$ in.

$\Delta_2 = +0.04085\,H_2 - 0.006508\,M_3^A - 0.009798\,M_3^B + 54{,}506$
$= +53{,}834$ in.

$C_3^B = -0.24486\,H_2 - 0.000988\,M_3^A + 0.015672\,M_3^B - 10{,}874$
$= -5{,}212.5.$

$C_3^A = +0.25301\,H_2 + 0.016275\,M_3^A - 0.000992\,M_3^B + 23{,}439$
$= +16{,}550.2.$

$H_1 = -0.074380\,H_2 - 0.0036836\,M_3^A + 0.0035649\,M_3^B - 2{,}856.5$
$= -160.3$ ℔.

$M_2^B = +1.3071\,H_2 + 0.14621\,M_3^A - 0.025973\,M_3^B + 390{,}650$
$= +322{,}451$ in. ℔.

$M_2^A = -1.7646\,H_2 - 0.054555\,M_3^A + 0.15172\,M_3^B - 341{,}040$
$= -267{,}511$ in. ℔.

$S_2^B = S_3^B + H_2 = +5{,}796.1 = -S_2^A$.

$X_2^B = 322{,}451 \div 57{,}96.1 = +55.635$ in.

$X_2^A = -267{,}511 \div -5{,}796.1 = +46.159$ in.

$V_2^B = -0.005\,M_2^A - 0.005\,M_2^B + 66{,}750 = +66{,}475$ ℔.

$V_2^A = 133{,}500 - V_2^B = +67{,}025$ ℔.

$\lambda_2 = +11.255\,S_3^A - 11.255\,S_2^A = +5{,}372$ in.

$\delta_3^B = +11.823\,M_3^B + 493.0\,S_3^A + 125.0\,C_3^B = +781{,}740$ in.

$\delta_3^A = +8.8920\,M_3^A - 371.0\,S_3^A + 125.0\,C_3^A = +395{,}770$ in.

$\Delta_1 = -0.034776\,H_1 - 0.0079794\,M_2^A - 0.0054566\,M_2^B + 158{,}513$
$= +158{,}894$ in.

$C_2^B = -\,0.20306\,H_1 - 0.000468\,M_2^A + 0.013099\,M_2^B - 5{,}169$
$= -\,787.3.$

$C_2^A = +\,0.19421\,H_1 + 0.012452\,M_2^A - 0.000472\,M_2^B + 6{,}587$
$= +\,3{,}072.4.$

$H_0 = -\,0.077161\,H_1 - 0.0037643\,M_2^A + 0.0039665\,M_2^B - 2{,}591$
$= -\,292.5$ ℔.

$M_1^B = +\,1.7552\,H_1 + 0.15671\,M_2^A - 0.056710\,M_2^B + 472{,}400$
$= +\,411{,}907$ in. ℔.

$M_1^A = -\,1.14748\,H_1 - 0.017209\,M_2^A + 0.15211\,M_2^B - 460{,}759$
$= -\,406{,}918$ in. ℔.

$S_1^B = S_2^B + H_1 = +\,5{,}635.8$ ℔ $= -\,S_1^A$.

$X_1^B = 411{,}907 \div 5{,}635.8 = +\,73.088$ in.

$X_1^A = -\,406{,}918 \div -\,5{,}635.8 = +\,72.204$ in.

$V_1^B = -\,0.005\,M_1^A - 0.005\,M_1^B + 90{,}125 = +\,90{,}100$ ℔.

$V_1^A = 180{,}250 - V_1^B = +\,90{,}150$ ℔.

$\lambda_1 = +\,9.845\,S_2^A - 9.845\,S_1^A = -\,1{,}577$ in.

$\delta_2^B = +\,6.9593\,M_2^B + 290.0\,S_2^A + 125.0\,C_2^B = +\,464{,}990$ in.

$\delta_2^A = +\,8.8920\,M_2^A - 371.0\,S_2^A + 125.0\,C_2^A = +\,152{,}550$ in.

$\Delta_0 = +\,0.0000158\,H_0 - 0.0051687\,M_1^A - 0.0051710\,M_1^B + 2$
$= -\,5$ in.

$C_1^B = -\,0.15661\,H_0 + 0.000374\,M_1^A + 0.010152\,M_1^B - 11{,}512$
$= -\,7{,}436.4$

$C_1^A = +\,0.15657\,H_0 + 0.010150\,M_1^A + 0.000376\,M_1^B + 11{,}508$
$= +\,7{,}486.9.$

$S_0^B = +\,0.071624\,H_0 + 0.0034912\,M_1^A - 0.0034912$
$M_1^B + 8{,}223.2 = +\,5{,}343.5$ ℔ $= -\,S_0^A$.

$M_0^B = +\,2.5337\,H_0 + 0.22285\,M_1^A - 0.092265\,M_1^B + 371{,}110$
$= +\,241{,}671$ in. ℔.

$M_0^A = -\,2.5341\,H_0 - 0.092290\,M_1^A + 0.22288\,M_1^B - 371{,}170$
$= -\,241{,}063$ in. ℔.

$X_0^B = 241{,}671 \div 5{,}343.5 = +\,45.228$ in.

$X_0^A = -\,241{,}063 \div -\,5{,}343.5 = +\,45.115$ in.

$V_0^B = -\,0.005\,M_0^A - 0.005\,M_0^B + 125{,}460 = +\,125{,}457$ ℔.

$V_0^A = 250{,}920 - V_0^B = +\,125{,}463$ ℔.

$$\begin{aligned}
\lambda_0 &= +\,8.450\ S_1^A - 8.450\ S_0^A = -\,2{,}471. \\
\delta_1^B &= +\,6.9593\ M_1^B + 290.0\ S_1^A + 125.0\ C_1^B = +\,30{,}2850 \text{ in.} \\
\delta_1^A &= +\,5.0374\ M_1^A - 210.0\ S_1^A + 125.0\ C_1^A = +\,68{,}860 \text{ in.} \\
\delta_0^B &= +\,5.0374\ M_0^B + 210.0\ S_0^A = +\,95{,}863 \text{ in.} \\
\delta_0^A &= +\,5.0374\ M_0^A - 210.0\ S_0^A = -\,92{,}810 \text{ in.}
\end{aligned}$$

Checks.

$$H_4 + H_3 + H_2 + H_1 + H_0 = +\,5{,}343.3 \text{ lb.}$$

S_0^B, above, equals + 5,343.5 lb; hence checks.

Fourth Story.

$$\left.\begin{aligned}
12.5\ C_4^A + \delta_4^A + 10.0\ C_5^A - \lambda_4 &= -\,197{,}336 \text{ in.} \\
12.5\ C_4^B + \delta_4^B + 10.0\ C_5^B - \lambda_3 &= -\,196{,}706 \text{ in.}
\end{aligned}\right\} \text{Checks.}$$

Third Story

$$\left.\begin{aligned}
12.5\ C_3^A + \delta_3^A + 12.5\ C_4^A - \lambda_3 &= +\,671{,}540 \text{ in.} \\
12.5\ C_3^B + \delta_3^B + 12.5\ C_4^B - \lambda_2 &= +\,670{,}677 \text{ in.}
\end{aligned}\right\} \text{Checks.}$$

Second Story

$$\left.\begin{aligned}
12.5\ C_2^A + \delta_2^A + 12.5\ C_3^A - \lambda_2 &= +\,392{,}460 \text{ in.} \\
12.5\ C_2^B + \delta_2^B + 12.5\ C_3^B - \lambda_1 &= +\,391{,}570 \text{ in.}
\end{aligned}\right\} \text{Checks.}$$

First Story.

$$\left.\begin{aligned}
12.5\ C_1^A + \delta_1^A + 12.5\ C_2^A - \lambda_1 &= +\,202{,}428 \text{ in.} \\
12.5\ C_1^B + \delta_1^B + 12.5\ C_2^B + \lambda_0 &= +\,202{,}525 \text{ in.}
\end{aligned}\right\} \text{Checks.}$$

o Story.

$$\left.\begin{aligned}
\delta_0^A + 12.5\ C_1^A - \lambda_0 &= +\,3{,}247 \text{ in.} \\
\delta_0^B + 12.5\ C_1^B \qquad &= +\,2{,}908 \text{ in.}
\end{aligned}\right\} \text{Checks.}$$

The following table gives a comparison of Column Stresses by the methods of Chapters II and III.

Section.	*X*	By the Method of Chapter III.						By the Method of Chapter II.					
		V	*S*	*M*	*V*+*A*	*Mv*+*I*	Max. + Fibre.	*V*	*S*	*M*	*V*+*A*	*Mv*+*I*	Max. + Fibre.
A–0	125	+125,463	−5,343.5	+426,875	+3,612	+2,237	+5,849	+125,460	0	0	+3,612	0	+3,612
	0			−241,063	+3,612	+1,263	+4,875				+3,612	0	+3,612
B–0	125	+125,457	+5,343.5	−426,267	+3,612	+2,235	+5,847	+125,460	0	0	+3,612	0	+3,612
	0			+241,671	+3,612	+1,267	+4,879				+3,612	0	+3,612
A–1	125	+ 90,150	−5,635.8	+297,557	+2,596	+1,559	+4,155	+ 90,125	0	0	+2,596	0	+2,596
	0			−406,918	+2,596	+2,130	+4,726				+2,596	0	+2,596
B–1	125	+ 90,100	+5,635.8	−292,568	+3,657	+2,117	+5,774	+ 90,125	0	0	+3,658	0	+3,658
	0			+411,907	+3,657	+2,980	+6,637				+3,658	0	+3,658
A–2	125	+ 67,025	−5,796.1	+457,001	+3,398	+4,225	+7,623	+ 66,750	0	0	+3,384	0	+3,384
	0			−267,511	+3,398	+2,474	+5,872				+3,384	0	+3,384
B–2	125	+ 66,475	+5,796.1	−402,061	+2,697	+2,908	+5,605	+ 66,750	0	0	+2,708	0	+2,708
	0			+322,451	+2,697	+2,333	+5,030				+2,708	0	+2,708
A–3	125	+ 43,040	−5,318.8	+255,085	+2,182	+2,357	+4,539	+ 43,375	0	0	+2,198	0	+2,198
	0			−409,765	+2,182	+3,790	+5,972				+2,198	0	+2,198
B–3	125	+ 43,710	+5,318.8	−322,010	+2,868	+3,960	+6,828	+ 43,375	0	0	+2,850	0	+2,850
	0			+342,840	+2,868	+4,220	+7,088				+2,850	0	+2,850
A–4	130	+ 19,872	−5,280.0	+370,007	+1,725	+6,415	+8,140	+ 20,000	0	0	+1,736	0	+1,736
	0			−289,993	+1,725	+5,025	+6,750				+1,736	0	+1,736
B–4	130	+ 20,128	+5,280.0	−395,706	+1,322	+4,865	+6,187	+ 20,000	0	0	+1,314	0	+1,314
	0			+264,294	+1,322	+3,253	+4,575				+1,314	0	+1,314

The following table gives a comparison of the stresses in the top flanges of the girders at mid span ($x = 100$ in.) and at the left end ($x = 200$ in.). Attention is called to the large bending moments in the columns causing stresses not taken care of in the design.

Girder.	Chapter III.		Chapter II.	
	$X = 200$.	$X = 100$.	$X = 200$.	$X = 100$.
a-0	−38,987	+31,595	0	+70,670
a-1	−28,399	+17,351	0	+46,750
a-2	−39,990	+ 9,200	0	+46,750
a-3	−27,083	+18,839	0	+46,750
a-4	−18,500	+30,860	0	+50,000

CHAPTER IX.

A STUDY OF THE EFFECT OF ECCENTRIC COLUMN LOADING ON AN ELASTIC PORTAL BRACED BUILDING — WITH COMPARISONS.

Since Chapter VII gives all values for these eccentric column loads and the floor loads combined, and Chapter VIII gives all values for the floor loads alone, it is evident that most values for this chapter may be found by subtracting the values of Chapter VIII from those of Chapter VII. The only exception will be the values of the points of contraflexure coordinates. These values are then as follows:

$H_4 = +4{,}677.8 - 5{,}280.0 = -602.2$ ℔.

$H_3 = -1{,}707.8 - 38.8 = -1{,}746.6$ ℔.

$M_4^B = +224{,}128. - 264{,}294 = -40{,}166$ in. ℔.

$M_4^A = -258{,}909 + 289{,}993 = +31{,}084$ in. ℔.

$S_4^B = H_4 = -602.2$ ℔ $= -S_4^A$.

$X_4^B = -40{,}166 \div -602.2 = +66.68$ in.

$X_4^A = 31{,}084 \div 602.2 = +51.60$ in.

$V_4^B = +19{,}502 — 20{,}128 = -626$ ℔.

$V_4^A = +32{,}998 - 19{,}872 = +13{,}126$ ℔.

$\lambda_4 = +64{,}660 - 72{,}980 = -8{,}320$ in.

$\Delta_4 = +66{,}922 - 95{,}607 = -28{,}685$ in.

$C_5^B = -19{,}116 + 18{,}780 = -336.$

$C_5^A = +16{,}581 - 20{,}564 = -3{,}983.$

$\Delta_3 = +308{,}501 - 157{,}118 = +151{,}383$ in.

$C_4^B = -3{,}390.5 + 3{,}242.8 = -147.7.$

$C_4^A = +3{,}851.6 - 5{,}548 = -1{,}696.4.$

$H_2 = +170.8 - 477.3 = -306.5$ ℔.

$M_3^B = +192{,}021 - 342{,}840 = -150{,}819$ in. ℔.

$M_3^A = -193{,}474 + 409{,}765 = +216{,}291$ in. ℔.

$S_3^B = S_4^B + H_3 = -2{,}348.8$ ℔ $= -S_3^A$.

$X_3^B = -150{,}819 \div -2{,}348.8 = +64.20$ in.

$X_3^A = 216{,}291 \div 2{,}348.8 = +92.10$ in.

$V_3^B = +73{,}374 - 43{,}710 = +29{,}664$ ℔.

$V_3^A = +80{,}601 - 43{,}040 = +37{,}561$ ℔.

$\lambda_3 = -20{,}092 - 457 = -20{,}549$ in.

$\delta_4^B = -166{,}805 - 32{,}086 = -198{,}891$ in.

$\delta_4^A = -529{,}292 + 400{,}260 = -129{,}032$ in.

$\Delta_2 = +198{,}672 - 53{,}834 = +144{,}838$ in.

$C_3^B = -4{,}597.3 + 5{,}212.5 = +615.2.$

$C_3^A = +4{,}967.8 - 16{,}550.2 = -11{,}582.4.$

$H_1 = +1{,}697.3 + 160.3 = +1{,}857.6$ ℔.

$M_2^B = +164{,}647 - 322{,}451 = -157{,}804$ in. ℔.

$M_2^A = -182{,}648 + 267{,}511 = +84{,}863$ in. ℔.

$S_2^B = S_3^B + H_2 = -2{,}655.3$ ℔ $= -S_2^A.$

$X_2^B = -157{,}804 \div -2{,}655.3 = +59.43$ in.

$X_2^A = 84{,}863 \div 2{,}655.3 = +31.98$ in.

$V_2^B = +127{,}496 - 66{,}475 = +61{,}021$ ℔.

$V_2^A = +127{,}954 - 67{,}025 = +60{,}929$ ℔.

$\lambda_2 = +1{,}922 - 5{,}372 = -3{,}450$ in.

$\delta_3^B = +232{,}570 - 781{,}740 = -549{,}170$ in.

$\delta_3^A = +1{,}020 - 395{,}770 = -394{,}750$ in.

$\Delta_1 = +395{,}669 - 158{,}894 = +236{,}775$ in.

$C_2^B = -1{,}066.9 + 787.3 = -279.6.$

$C_2^A = +3{,}015.1 - 3{,}072.4 = -57.3.$

$H_0 = -422.0 + 292.5 = -129.5$ ℔.

$M_1^B = +350{,}837 - 411{,}907 = -61{,}070$ in. ℔.

$M_1^A = -337{,}581 + 406{,}918 = +69{,}337$ in. ℔.

$S_1^B = S_2^B + H_1 = -797.7$ ℔ $= -S_1^A.$

$X_1^B = -61{,}070 \div -797.7 = +76.60$ in.

$X_1^A = 69{,}337 \div 797.7 = +86.93$ in.

$V_1^B = +181{,}379 - 90{,}100 = +91{,}279$ ℔.

$V_1^A = +175{,}546 - 90{,}150 = +85{,}396$ ℔.

$\lambda_1 = + 16{,}696 + 1{,}577 = + 18{,}273$ in.

$\delta_2^B = + 101{,}720 - 464{,}990 = - 363{,}270$ in.

$\delta_2^A = - 83{,}700 - 152{,}550 = - 236{,}250$ in.

$\Delta_0 = + 50{,}071 + 5 = + 50{,}076$ in.

$C_1^B = - 6{,}463.3 + 7{,}436.4 = + 973.1.$

$C_1^A = + 5{,}852.3 - 7{,}486.9 = - 1{,}634.6.$

$S_0^B = + 4{,}416.9 - 5{,}343.5 = - 926.6$ lb $= - S_0^A.$

$M_0^B = + 195{,}858 - 241{,}671 = - 45{,}813$ in. lb.

$M_0^A = - 203{,}443 + 241{,}063 = + 37{,}620$ in. lb.

$X_0^B = - 45{,}813 \div - 926.6 = + 49.46$ in.

$X_0^A = 37{,}620 \div 926.6 = + 40.60$ in.

$V_0^B = + 247{,}482 - 125{,}457 = + 122{,}025$ lb.

$V_0^A = + 234{,}838 - 125{,}463 = + 109{,}375$ lb.

$\lambda_0 = - 3{,}565 + 2{,}471 = - 1{,}094$ in.

$\delta_1^B = + 230{,}690 - 302{,}850 = - 72{,}160$ in.

$\delta_1^A = + 46{,}390 - 68{,}860 = - 22{,}470$ in.

$\delta_0^B = + 59{,}690 - 95{,}863 = - 36{,}173$ in.

$\delta_0^A = - 97{,}910 + 92{,}810 = - 5{,}100$ in.

Checks.

Since Chapter VII and VIII checked, this chapter will also check if no errors have been made in the subtractions. The checks are all satisfied but are not given here.

The following table gives a comparison of column stresses by the methods of Chapters II and III.

Section.	X	By the Method of Chapter III.						By the Method of Chapter II.					
		V	S	M	$V \div A$	$Mv \div I$	Max. + Fibre.	V	S	M	$V \div A$	$Mv \div I$	Max. + Fibre.
A–0	125	+109,375	+ 926.6	− 78,205	+3,152	+ 409	+3,561	+109,414	0	0	+3,152	0	+3,152
	0			+ 37,620	+3,152	+ 197	+3,349				+3,152	0	+3,152
B–0	125	+122,025	− 926.6	+ 70,012	+3,512	+ 367	+3,879	+121,986	0	0	+3,512	0	+3,512
	0			− 45,813	+3,512	+ 240	+3,752				+3,512	0	+3,512
A–1	125	+ 85,396	+ 797.7	− 30,375	+2,459	+ 159	+2,618	+ 85,354	0	0	+2,459	0	+2,459
	0			+ 69,337	+2,459	+ 364	+2,823				+2,459	0	+2,459
B–1	125	+ 91,279	− 797.7	+ 38,642	+3,702	+ 280	+3,982	+ 91,321	0	0	+3,704	0	+3,704
	0			− 61,070	+3,702	+ 442	+4,144				+3,704	0	+3,704
A–2	125	+ 60,929	+2,655.3	−247,049	+3,086	+2,283	+5,369	+ 61,293	0	0	+3,106	0	+3,106
	0			+ 84,863	+3,086	+ 785	+3,871				+3,106	0	+3,106
B–2	125	+ 61,021	−2,655.3	+174,108	+2,477	+1,261	+3,738	+ 60,657	0	0	+2,461	0	+2,461
	0			−157,804	+2,477	+1,142	+3,619				+2,461	0	+2,461
A–3	125	+ 37,561	+2,348.8	− 77,309	+1,902	+ 715	+2,617	+ 37,232	0	0	+1,887	0	+1,887
	0			+216,291	+1,902	+2,001	+3,903				+1,887	0	+1,887
B–3	125	+ 29,664	−2,348.8	+142,781	+1,948	+1,756	+3,704	+ 29,993	0	0	+1,971	0	+1,971
	0			−150,819	+1,948	+1,855	+3,803				+1,971	0	+1,971
A–4	130	+ 13,126	+ 602.2	− 44,191	+1,139	+ 766	+1,905	+ 13,172	0	0	+1,143	0	+1,143
	0			+ 31,084	+1,139	+ 539	+1,678				+1,143	0	+1,143
B–4	130	− 626	− 602.2	+ 35,109	− 41	+ 432	+ 391	− 672	0	0	− 44	0	− 44
	0			− 40,166	− 41	+ 494	+ 453				− 44	0	− 44

The following table gives a comparison of the stresses in the top flanges of the girders at mid span ($x = 100$ in.) and at the left end ($x = 200$ in.).

Girder.	Chapter III.		Chapter II.	
	$X = 200$ in.	$X = 100$ in.	$X = 200$ in.	$X = 100$ in.
a–0	− 3,400	− 1,484	−10,100	−7,920
a–1	− 2,835	+ 1,033	−10,100	−7,920
a–2	+10,783	+10,255	−10,100	−7,920
a–3	− 5,162	− 1,422	−10,100	−7,920
a–4	− 4,500	− 1,370	− 6,710	−3,390

Note that the bending moments from eccentric loads do not follow the rules deduced by Professor Heller.

COMPARISON SUMMARY—CHAPTERS VI, VII, VIII, AND IX.

In the following table, computed from those of Chapters VI, VIII, and IX, the simultaneous compressive unit fibre stresses are given for wind on face A, floor, and eccentric column loadings; in each case the figures are given for the fibre which receives the maximum total combined unit stress, the object of the table being to deduce and compare the maximum possible stresses in the columns. The table gives this comparison for the methods of Chapters II and III. On the right of the table is given the percentage of increase of the total maximum stresses by Chapter *III* over those by Chapter II; these range from 20 per cent. up to 96 per cent. for Column B, which receives its maximum stress under these loadings. Column A receives its maximum for wind on face A and similar percentages will be found for Column A under such loading. For example, for Column A–0, at $x = 0$ in., wind on face B, total maximum unit by Chapter II = 13,879 ℔ per sq. in., while by Chapter III the maximum unit is 16,680 ℔ per sq. in., being an increase of 20.2 per cent. (cf. 19.7 per cent. for Column B–0); also, for Column A–4, $x = 130$ in., the maximum by Chapter II = 6,224 ℔ per sq. in. while by Chapter III it is 12,652 ℔ per sq. in. or an increase of 103.3 per cent. (cf. 96.5 per cent. for Column B–4).

Section.	*X*	By Method of Chapter III.				By Method of Chapter II.				Per Cent. Increase.	All'd. Unit.
		Wind L'd.	Floor L'd.	Ecc. Col. L'd.	Total Max.	Wind L'd.	Floor L'd.	Ecc. Col. L'd.	Total Max.		
A–0	125	0	5,849	2,743	8,592	3,485	3,612	3,152	10,249		15,600
	0	5,220	2,349	3,349	10,918	3,485	3,612	3,152	10,249	6.5	15,600
B–0	125	5,745	5,847	3,145	14,737	7,315	3,612	3,512	14,439		15,600
	0	9,140	4,879	3,272	17,291	7,315	3,612	3,512	14,439	19.7	15,600
A–1	125	3,805	1,037	2,618	7,460	4,628	2,596	2,459	9,683		15,600
	0	4,175	466	2,823	7,464	4,628	2,596	2,459	9,683		15,600
B–1	125	7,474	5,774	3,422	16,670	7,303	3,658	3,704	14,665		15,620
	0	8,034	6,637	3,260	17,931	7,303	3,658	3,704	14,665	22.4	15,620
A–2	125	4,625	− 827	5,369	9,167	4,309	3,384	3,106	10,799		14,700
	0	4,220	924	3,871	9,015	4,309	3,384	3,106	10,799		14,700
B–2	125	6,455	5,605	1,216	13,276	6,494	2,708	2,461	11,663	13.8	15,620
	0	6,325	5,030	1,335	12,690	6,494	2,708	2,461	11,663		15,620
A–3	125	4,363	− 175	2,617	6,805	4,168	2,198	1,887	8,253		14,700
	0	3,693	−1,608	3,903	5,988	4,168	2,198	1,887	8,253		14,700
B–3	125	6,120	6,828	192	13,140	5,388	2,850	1,971	10,209	28.7	13,250
	0	5,035	7,088	93	12,216	5,388	2,850	1,971	10,209		13,250
A–4	130	0	8,140	373	8,513	2,905	1,736	1,143	5,784	47.2	6,890
	0	0	6,750	600	7,350	2,905	1,736	1,143	5,784		6,890
B–4	130	3,341	6,187	− 473	9,055	3,341	1,314	− 44	4,611	96.5	13,250
	0	3,011	4,575	− 535	7,051	3,341	1,314	− 44	4,611		13,250

The following table, compiled from those of Chapters VI, VIII, and IX gives a comparison of the total maximum stresses in the top flanges of the girders at the left end ($x = 200$ in.) by the methods of Chapters II and III. On the right is given the percentage of increase, as for the columns, and here also is found an ever-increasing divergence, from basement to roof, in the results by the two methods, ranging from 10 per cent. to 97 per cent. In the case of both columns and girders the main source of increase is due to the fact that the building is virtually a series of connected *arches* set vertically over each other and the vertical loads, both floor and eccentric column loads, are therefore to be regarded as producing stresses of very great importance in designing such structures. The wind produces stresses not materially different from those deduced for the rigid girder assumption. This merely confirms the demonstrations by Professor Howe, in which he shows that if a horizontal load acts in the neutral axis of the girders, as assumed in this investigation, the arch *thrust* is practically nil, *i. e.*, there is very little tendency of the feet of the supporting columns to move *with respect to each other*, but rather they tend to move the same amount, which condition would of course be in exact accord with the results on the assumption of rigid girders.

Girder.	Chapter III.	Chapter II.	Per Cent.
a–0	−126,884	−114,985	10.4
a–1	−106,764	− 92,103	16.0
a–2	− 84,552	− 66,205	27.8
a–3	− 66,874	− 44,151	51.5
a–4	− 37,633	− 19,095	97.0

CHAPTER X.

A STUDY OF COLUMN DEFLECTIONS AND DEFORMATIONS IN AN ELASTIC PORTAL BRACED BUILDING.

A. Distortion Due to Horizontal Wind Pressure.

In considering the effect of wind in distorting a portal braced structure the five story building already used will be investigated as follows:

First, as a *cantilever beam* fixed at one end and having a variable moment of inertia corresponding to the moment of inertia of the steel columns about a horizontal axis midway between the columns, the moment of inertia being considered as an average uniformly varying one from top to bottom.

Second, as a structure with *rigid girders*, corresponding to the assumptions of Chapter II.

Third, as a perfectly *elastic structure*, corresponding to the assumptions of Chapter III.

First. *As a Cantilever Beam.*—Referring to Fig. 10,

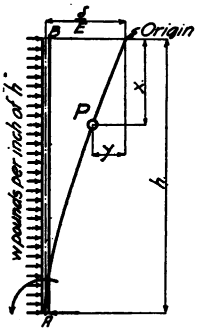


Fig 10

Let I = moment of inertia of the beam at B.
Let I_0 = moment of inertia of the beam at A.
Let I_x = moment of inertia of the beam at any point P.

Also let $I_x = I(1 + cx)$; then $I(1 + ch) = I_0$, or $c = \dfrac{I_0 - I}{hI}$.

Now, taking moments about $P \equiv (x, y)$ gives the fundamental equation

$$E\frac{d^2y}{dx^2} = \frac{wx^2}{2I_x} = \frac{w}{2I}\left(\frac{x^2}{1 + cx}\right) = \frac{w}{2Ic^2}\left[(cx - 1) + \frac{1}{cx + 1}\right];$$

integrating gives

$$E\frac{dy}{dx} = \frac{w}{2Ic^2}\left[\frac{cx^2}{2} - x + \frac{1}{c}\log_e(1 + cx) + K\right].$$

But, when $x = h, \frac{dy}{dx} = 0$; hence

$$K = -\frac{ch^2}{2} + h - \frac{1}{c}\log_e(1 + ch),$$

or

$$E\frac{dy}{dx} = \frac{w}{2Ic^2}\left[\frac{c}{2}(x^2 - h^2) - (x - h) + \frac{1}{c}\log_e\frac{1 + cx}{1 + ch}\right];$$

integrating again gives

$$Ey = \frac{w}{2Ic^2}\left\{\frac{cx}{6}(x^2 - 3h^2) - \frac{x}{2}(x - 2h) + \frac{1 + cx}{c^2}\log_e(1 + cx) - \frac{x}{c}[1 + \log_e(1 + ch)] + K'\right\}.$$

But, when $x = 0$, $y = 0$; hence $K' = 0$.

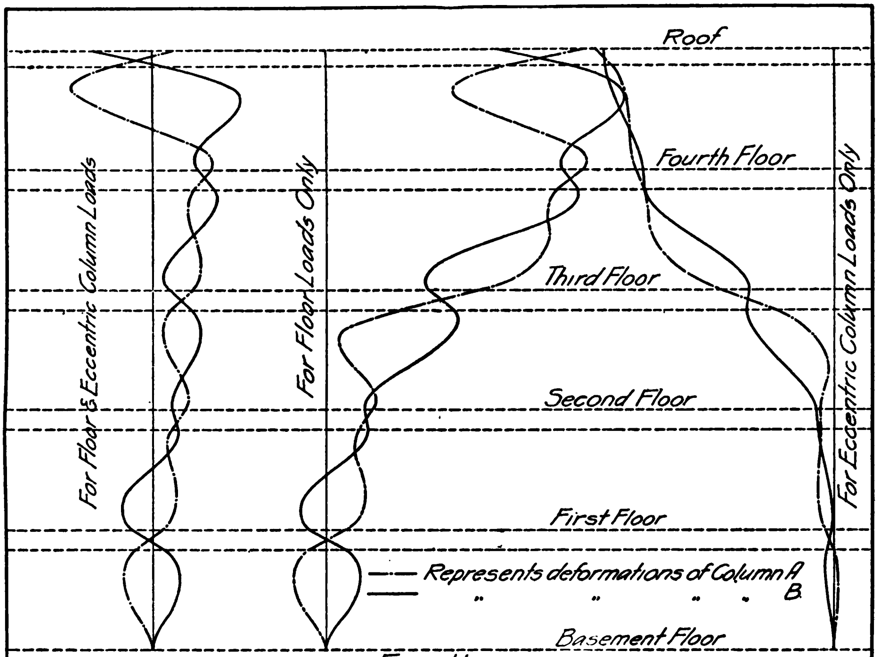
The equation for Ey or the elastic line is thus determined: Now for the five story building here considered the data required for determining this curve of distortion is as follows:

$I_0 = 34.73 \times \overline{100}^2 \times 2 = 694{,}600.$ $I = (11.52 + 15.23) \times \overline{100}^2$ $= 267{,}500$ (neglecting the comparatively small moments of inertia about the neutral axes of the columns).

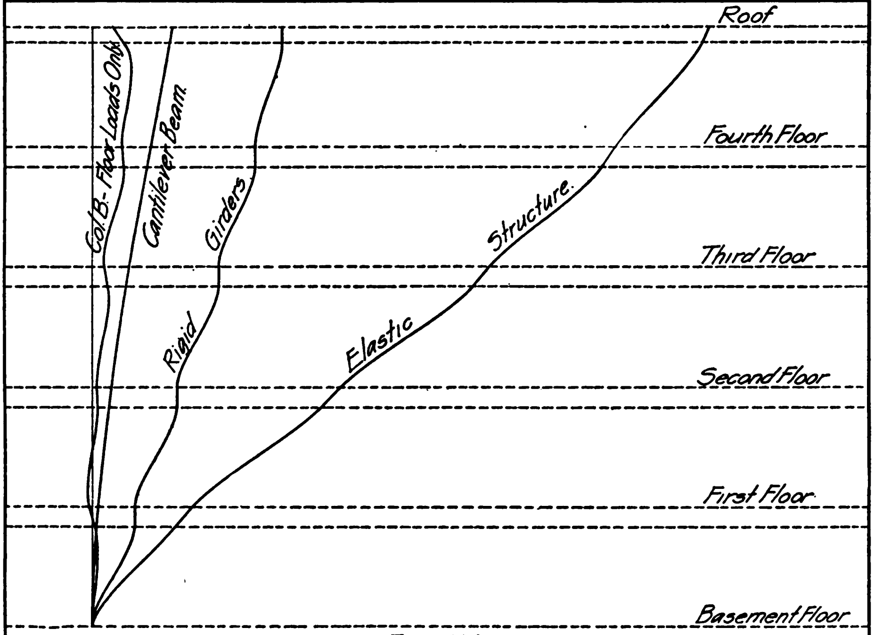
$$h = 750 \text{ in.};$$

$w = 30.0$ ℔ × 20.0 ft. ÷ 12 = 50 ℔ per. inch of height of beam.

$$c = \frac{694{,}600 - 267{,}500}{750 \times 267{,}500} = 0.002135.$$


Roof
Fourth Floor
Third Floor
Second Floor
First Floor
Basement Floor
For Floor & Eccentric Column Loads
For Floor Loads Only
For Eccentric Column Loads Only
—·— Represents deformations of Column A
—— " " " " B

FIGS 11-a



FIGS 11-b

Using these values in the value for Ey above, the deflection curve marked "cantilever beam" in Fig. 11–*b*, is obtained.

Second. As a Structure with Rigid Girders.—The values of δ for each story of the five story building may be calculated from the equation, Chapter II, $\delta = Sc^3/12I$. Since Columns *A* and *B* deflect equally, this equation may be applied to either column. Using the values of *S* given in Chapter II (or VI) the values of these deflections are;

$$\delta_0 = 1{,}730{,}000 \text{ in.}, \quad \delta_1 = 1{,}775{,}000 \text{ in.}, \quad \delta_2 = 1{,}765{,}000 \text{ in.},$$
$$\delta_3 = 1{,}505{,}000 \text{ in.}, \quad \delta_4 = 1{,}100{,}000 \text{ in.}$$

The distortion curve for this case is shown and marked "rigid girders" in Fig. 11–*b*.

Third. As an Entirely Elastic Structure.—The values of δ, C, and X coordinates of the points of contra-flexure for this case are given in Chapter VI, and the curves for Columns *A* and *B* fit so closely on each other that only that for Column *A* is shown in Fig. 11–*b*; this curve is marked "elastic structure."

B. DISTORTIONS DUE TO VERTICAL LOADING.

The values of δ, C, and X coordinates of the points of contra-flexure for uniform floor loading, eccentric column loading, and a combination of these two loadings, have been given in Chapters VIII, IX and VII respectively; by calculating, in addition, the slopes and deflections at the points of contra-flexure and a few other deflections as required, the deformation curves for these loadings have been constructed as shown in Figs. 11, *a*.

Fig. 11, *b*, also shows the curve for Column *B* for floor loads only. All curves of Figs. 11, *a*, are drawn to a horizontal scale of 1 in. = 1,000,000 in. while all those of Figs. 11, *b*, are drawn to a scale of 1 in. = 8,000,000 in. Notice in Figs. 11, *a*, the large distortions of the column *sections* due to floor loadings only.

CHAPTER XI.

A STUDY OF THE EFFECT OF HORIZONTAL AND VERTICAL LOADS ON A DOUBLE PLATE GIRDER PORTAL — WITH COMPARISONS.

This study is given to demonstrate the application of the general method of Chapter III to a structure of *three* columns per transverse section. The structure chosen for investigation is so selected as to also afford a test of the theory that the axial column unit stresses in a building subject to flexural forces, such as wind pressure, vary directly as the distance from a neutral axis. (See Chapter II.) The properties of the structure chosen are given in the following table marked "properties."

The structure will be investigated for two loadings; a horizontal wind pressure F_0^A equal to 10,000 ℔, and a uniform load of 500 ℔ per inch of girder covering the entire structure, from Column *A* to Column *C*.

PROPERTIES.

Quant.	Value.	Quant.	Value.
c_0	100 in.	A_0^B	25
d_0	20 in.	A_0^C	10
b	200 in.	I_0^a	2,000
b'	100 in.	I_0^b	1,000
I_0^A	500	A_0^a	25
I_0^B	1,000	A_0^b	20
I_0^C	200	F_0^A	10,000
A_0^A	15	w_0	500

Now, from Plates IV to VIII inclusive, of Chapter III the values of the symbolic coefficients for forming the equations for this case may be easily computed. The solution for both loadings will be carried through in one set of tables as was done in Chapter VI. The values of these symbolic coefficients then are as given in the table on page 160.

Now, using these symbolic coefficients, the solution given in Plates 11 and 12 may be calculated. The term "ω" of Plate 12 will contain only the constant terms "z," since the structure

Exp'n.	Values of Symbolic Coefficients.						x_0	
	a_0	b_0	c_0	i_0	j_0	o_0	For F_0A.	For W_0.
C–A	+ 0.200000	0	0	− 10.00000	0	0	0	0
C–B	0	+ 0.100000	0	0	− 5.00000	0	0	0
C–C	0	0	+ 0.500000	+ 25.00000	+ 25.00000	0	− 250,000	0
δ–A	+10.00000	0	0	− 333.333	0	0	0	0
δ–B	0	+ .000000	0	0	− 166.6666	0	0	0
δ–C	0	0	+25.00000	+ 833.333	+ 833.3333	0	− 8,333,333	0
H–a	0	0	0	− 1.00000	0	0	+ 10,000	0
H–b	0	0	0	− 1.00000	− 1.00000	0	+ 10,000	0
λ–a	0	0	0	− 8.00000	0	0	+ 80,000	0
a–b	0	0	0	− 5.00000	− 5.00000	0	+ 50,000	0
V–A	+ 0.0033333	+ 0.003333	+ 0.0033333	0	0	− 0.33333	− 3,667	+ 75,000
V–C	− 0.0033333	− 0.003333	− 0.0033333	0	0	− 0.66666	+ 3,667	+ 75,000
α–A	+ 0.024444	+ 0.024444	+ 0.024444	0	0	− 2.44444	− 26,900	+ 550,000
α–B	0	0	0	0	0	+ 4.40000	0	0
α–C	− 0.036666	− 0.036666	− 0.036666	0	0	− 7.33333	+ 40,333	+ 825,000
Δ–*a*	− 0.024444	− 0.024444	− 0.024444	0	0	+ 6.84444	+ 26,900	− 550,000
Δ–*b*	− 0.036666	− 0.036666	− 0.036666	0	0	− 11.73333	+ 40,333	+ 825,000
Q–a	− 0.0005000	0	0	+ 0.055000	0	0	0	0
Q–b	− 0.0003333	− 0.0003333	+ 0.0006666	+ 0.110000	+ 0.110000	− 0.066666	− 733.33	+ 5,000
R–a	+ 0.000001666	+ 0.000001666	+ 0.000001666	0	0	− 0.00016666	− 1.83333	+ 37.5
R–b	+ 0.000003333	+ 0.000003333	+ 0.000003333	0	0	+ 0.0006666	− 3.66666	− 25.0
A–a	+12.00000	− 6.00000	0	− 425.3333	+ 216.6666	0	− 80,000	0
A–b	0	+ 6.00000	−30.00000	−1,078.3333	−1,295.000	0	+10,783,333	0
B–a	− 0.266666	+ 0.133333	+ 0.033333	+ 21.00000	− 5.00000	− 3.33333	− 36,667	+ 416,667
B–b	− 0.016666	− 0.116666	+ 0.583333	+ 36.00000	+ 41.00000	− 3.33333	− 341,667	+ 291,667
C–a	− 5.531111	+24.468888	+ 4.468888	+1,100.0000	−1,000.0000	−451.2888	− 4,915,788	+50,550,000
C–b	− 0.518888	− 0.518888	+54.481111	+3,050.0000	+3,050.0000	− 99.378	−29,929,222	+ 9,591,661

Equation.	α	β	γ	θ	φ	ε	ω	
							For F_0A.	For W_0.
A-a-1	+12.000000	−6.000000	0	−425.3333	+216.6666	0	−80,000	0
A-b-1	0	+6.000000	−30.000000	−1,078.3333	−1,295.0000	0	+10,783,333	0
B-a-1	−0.266666	+0.133333	+0.033333	+21.00000	−5.000000	−3.333333	−36,667	+416,667
B-b-1	−0.016666	−0.116666	+0.583333	+36.00000	+41.000000	−3.333333	−341,667	+291,667
C-a-1	−5.531111	+24.468888	+4.468888	+1,100.0000	−1,000.0000	−451.289	−4,915,788	+50,550,000
C-b-1	−0.518888	−0.518888	+54.481111	+3,050.0000	+3,050.0000	−99.378	−29,929,222	+9,591,661
	M_0A	+0.500000	0	+35.4444	−18.05555	0	+6,666.666	0
	A-b-2	+6.000000	−30.00000	−1,078.3333	−1,295.0000	0	+10,783,333	0
	B-a-2	0	+0.033333	+11.548148	−0.18518518	−3.333333	−38,444.44	+416,667
	B-b-2	−0.1250000	+0.583333	+35.409259	+41.3009259	−3.333333	−341,777.77	+291,667
	C-a-2	+21.703333	+4.468888	+903.953	−900.133	−451.289	−4,952,663	+50,550,000
	C-b-2	−0.778333	+54.481111	+3,031.608	+3,059.3688	−99.378	−29,932,681	+9,591,661
		M_0B	+5.000000	+179.72222	+215.8333	0	−1,797,222	0
		B-a-3	+0.033333	+11.548148	−0.18518518	−3.333333	−38,444.44	+416,667
		B-b-3	−0.041666	+12.943981	+14.321759	−3.333333	−117,125.00	+291,667
		C-a-3	+112.98555	+4,804.523	+3,784.17	−451.289	−43,958,363	+50,550,000
		C-b-3	+50.589444	+2,891.724	+2,891.378	−99.378	−28,533,841	+9,591,661
			M_0C	−346.4444	+5.55555	+100.0000	+1,153,333	−12,500,000
			B-b-4	+27.379166	+14.090278	−7.500000	−165,180.5	+812,500.00
			C-a-4	−34,338.69	+4,411.87	+10,847.266	+86,351,637	−1,361,759,377
			C-b-4	−14,634.71	+3,172.43	+4,959.565	+29,812,639	−622,776,389
				S_0A	−0.514635	+0.273931	+6,033.076	−29,675.84
				C-a-5	+22,083.8	+1,440.84	−120,816,363	−342,729,400
				C-b-5	+10,703.96	+950.67	−58,479,661	−188,479,400
					S_0B	−0.0652442	+5,470.82	+15,519.5
					C-b-6	+252.30	+79,739	−22,359,400
						V_0B	−316.05	+88,622.3

consists of only one story. The calculations were carried out by Vega's logarithmic tables, because only a rough value of V_0^B can be obtained with a Thacher rule, the solution involving the differences of large quantities of very nearly the same size. The calculations are given in the table on page 161.

Now, from this table and the table of symbolic coefficients the values of all quantities may be written out as follows:

For the Wind Pressure F_0^A.—

$$V_0^B = -\ 316.05 \text{ lb.}$$

$$S_0^B = -\ 0.0652442\ V_0^B + 5{,}470.82 = +\ 5{,}491.44 \text{ lb.}$$

$$S_0^A = -\ 0.514635\ S_0^B + 0.273931\ V_0^B + 6{,}033.076 = +\ 3{,}120.41 \text{ lb.}$$

$$M_0^C = -\ 346.444\ S_0^A + 5.5555\ S_0^B + 100.0000\ V_0^B + 1{,}153{,}333$$
$$= +\ 71{,}186 \text{ in. lb.}$$

$$M_0^B = +\ 5.0000\ M_0^C + 179.7222\ S_0^A + 215.8333\ S_0^B + 0\ V_0^B$$
$$-\ 1{,}797{,}222 = +\ 304{,}755 \text{ in. lb.}$$

$$M_0^A = +\ 0.50000\ M_0^B + 0\ M_0^C + 35.4444\ S_0^A - 18.0555\ S_0^B$$
$$+\ 0\ V_0^B + 6{,}667 = +\ 170{,}495 \text{ in. lb.}$$

$$S_0^C = 10{,}000 - S_0^A - S_0^B = +\ 1{,}388.15 \text{ lb.}$$

$$X_0^A = 170{,}495 \div 3{,}120.4 = +\ 54.67 \text{ in.;}$$

$$X_0^B = 304{,}755 \div 5{,}491.44 = +\ 55.52 \text{ in.;}$$

$$X_0^C = 71{,}186 \div 1{,}388.15 = +\ 51.25 \text{ in.}$$

$$C_1^A = +\ 0.2\ M_0^A - 10.0\ S_0^A = +\ 2{,}894.9.$$

$$C_1^B = +\ 0.1\ M_0^B - 5.0\ S_0^B = +\ 3{,}018.3.$$

$$C_1^C = +\ 0.5\ M_0^C + 25.0\ (S_0^A + S_0^B) - 250{,}000 = +\ 8{,}892.$$

$$\delta_0^A = +\ 10.0\ M_0^A - 333.33\ S_0^A = +\ 664{,}813 \text{ in.}$$

$$\delta_0^B = +\ 5.0\ M_0^B - 166.66\ S_0^B = +\ 608{,}535 \text{ in.}$$

$$\delta_0^C = +\ 25.0\ M_0^C + 833.33\ (S_0^A + S_0^B) - 8{,}333{,}333$$
$$= +\ 622{,}859 \text{ in.}$$

$$H_0^a = -\ 1.0\ S_0^A + 10{,}000 = +\ 6{,}879.59 \text{ lb.}$$

$$H_0^b = -\ 1.0\ (S_0^A + S_0^B) + 10{,}000 = +\ 1{,}388.15 \text{ lb.}$$

$$\lambda_0^a = -\ 8.0\ S_0^A + 80{,}000 = +\ 55{,}037 \text{ in.}$$

$$\lambda_0^b = -\ 5.0\ (S_0^A + S_0^B) + 50{,}000 = +\ 6{,}941 \text{ in.}$$

$$V_0^A = +\ 0.003333\ (M_0^A + M_0^B + M_0^C) - 0.3333\ V_0^B - 3{,}666.7$$
$$= -\ 1{,}739.9 \text{ lb.}$$

$$V_0^C = -\,0.003333\,(M_0^A + M_0^B + M_0^C) - 0.6666\,V_0^B + 3{,}666.7 = +\,2{,}056.0 \text{ lb.}$$

$$\alpha_0^A = +\,0.02444\,(M_0^A + M_0^B + M_0^C) - 2.4444\,V_0^B - 26{,}900 = -\,12{,}770 \text{ in.}$$

$$\alpha_0^B = +\,4.400\,V_0^B = -\,1{,}390 \text{ in.}$$

$$\alpha_0^C = -\,0.03666\,(M_0^A + M_0^B + M_0^C) - 7.3333\,V_0^B + 40{,}333 = +\,22{,}600 \text{ in.}$$

$$\Delta_0^a = -\,0.02444\,(M_0^A + M_0^B + M_0^C) + 6.845\,V_0^B + 26{,}900 = +\,11{,}380 \text{ in.}$$

$$\Delta_0^b = -\,0.036667\,(M_0^A + M_0^B + M_0^C) - 11.7333\,V_0^B + 40{,}333 = +\,23{,}990 \text{ in.}$$

Checks.

$$\left.\begin{aligned} \delta_0^A + 10.0\,C_1^A - \lambda_0^a &= +\,638{,}725 \text{ in.} \\ \delta_0^B + 10.0\,C_1^B &= +\,638{,}718 \text{ in.} \end{aligned}\right\}\text{Checks.}$$

$$\left.\begin{aligned} \delta_0^B + 10.0\,C_1^B - \lambda_0^b &= +\,631{,}777 \text{ in.} \\ \delta_0^C + 10.0\,C_1^C &= +\,631\ 751 \text{ in.} \end{aligned}\right\}\text{Checks.}$$

For the Floor Load w_0.—

$$V_0^B = +\,88{,}622.3 \text{ lb.}$$

$$S_0^B = -\,0.065244\,V_0^B + 15{,}519.5 = +\,9{,}737.4 \text{ lb.}$$

$$S_0^A = -\,0.514635\,S_0^B + 0.273931\,V_0^B - 29{,}675.8 = -\,10{,}410.6 \text{ lb.}$$

$$M_0^C = -\,346.444\,S_0^A + 5.5555\,S_0^B + 100.000\,V_0^B - 12{,}500{,}000 = +\,23{,}017 \text{ in. lb.}$$

$$M_0^B = +\,5.00000\,M_0^C + 179.722\,S_0^A + 215.833\,S_0^B + 0\,V_0^B + 0 = +\,345{,}720 \text{ in. lb.}$$

$$M_0^A = +\,0.5000\,M_0^B + 0\,M_0^C + 35.444\,S_0^A - 18.0555\,S_0^B + 0\,V_0^B + 0 = -\,371{,}952 \text{ in. lb.}$$

$$S_0^C = 0 - S_0^A - S_0^B = +\,673.2 \text{ lb.}$$

$$X_0^A = -\,371{,}952 \div -\,104{,}106 = +\,35.73 \text{ in.;}$$

$$X_0^B = 345{,}720 \div 9{,}737.4 = +\,35.51 \text{ in.;}$$

$$X_0^C = +\,23{,}017 \div 673.2 = +\,34.21 \text{ in.}$$

$$C_1^A = +\,0.2\,M_0^A - 10.0\,S_0^A + \ = 29{,}715.6$$

$$C_1^B = +\,0.1\,M_0^B - 5.0\,S_0^B = -\,14{,}115.0.$$

$$C_1^C = +\,0.5\,M_0^C + 25.0\,(S_0^A + S_0^B) = -\,5{,}321.5.$$

$$\delta_0^A = +10.0\,M_0^A - 333.333\,S_0^A = -249{,}320 \text{ in.}$$

$$\delta_0^B = +5.0\,M_0^B - 166.667\,S_0^B = +105{,}700 \text{ in.}$$

$$\delta_0^C = +25.0\,M_0^C + 833.333\,(S_0^A + S_0^B) = +14{,}425 \text{ in.}$$

$$H_0^a = -1.0\,S_0^A = +10{,}410.6 \text{ lb.}$$

$$H_0^b = -1.0\,(S_0^A + S_0^B) = +673.2 \text{ lb.}$$

$$\lambda_0^a = -8.0\,S_0^A = +83{,}285 \text{ in.}$$

$$\lambda_0^b = -5.0\,(S_0^A + S_0^B) = +3{,}366 \text{ in.}$$

$$V_0^A = +0.003333\,(M_0^A + M_0^B + M_0^C) - 0.3333\,V_0^B + 75{,}000 = +45{,}448 \text{ lb.}$$

$$V_0^C = -0.003333\,(M_0^A + M_0^B + M_0^C) - 0.6666\,V_0^B + 75{,}000 = +15{,}930 \text{ lb.}$$

$$\alpha_0^A = +0.02444\,(M_0^A + M_0^B + M_0^C) - 2.4444\,V_0^B + 550{,}000 = +333{,}289 \text{ in.}$$

$$\alpha_0^B = +4.400\,V_0^B = +389{,}938 \text{ in.}$$

$$\alpha_0^C = -0.036666\,(M_0^A + M_0^B + M_0^C) - 7.3333\,V_0^B + 825{,}000 = +175{,}221 \text{ in.}$$

$$\Delta_0^a = -0.024440\,(M_0^A + M_0^B + M_0^C) + 6.845\,V_0^B - 550{,}000 = +56{,}649 \text{ in.}$$

$$\Delta_0^b = -0.036666\,(M_0^A + M_0^B + M_0^C) - 11.7333\,V_0^B + 825{,}000 = -214{,}717 \text{ in.}$$

Maximum bending in columns occurs at the top. The values are:

For Col. A — $100\,S_0^A + M_0^A = +669{,}108$ in. lb.
For Col. B — $100\,S_0^B + M_0^B = -628{,}020$ in. lb.
For Col. C — $100\,S_0^C + M_0^C = -\ 44{,}303$ in. lb.

Checks:

$$\left.\begin{aligned}\delta_0^A + 10.0\,C_1^A - \lambda_0^a &= -35{,}449 \text{ in.}\\ \delta_0^B + 10.0\,C_1^B &= -35{,}450 \text{ in.}\end{aligned}\right\}\text{Checks.}$$

$$\left.\begin{aligned}\delta_0^B + 10.0\,C_1^B - \lambda_0^b &= -38{,}816 \text{ in.}\\ \delta_0^C + 10.0\,C_1^C &= -38{,}790 \text{ in.}\end{aligned}\right\}\text{Checks.}$$

The stresses and deformations will now be computed by the methods of Chapter II.

For Wind Load F_0^A.—

$$h^A = \frac{200 \times 25 + 300 \times 10}{15 + 25 + 10} = + 160.00 \text{ in.}$$

$$X = + 10{,}000 \times 60.0 = + 600{,}000 \text{ in. ℔.}$$

$$s[15 \times \overline{160}^2 + 25 \times \overline{40}^2 + 10 \times \overline{140}^2] = + 600{,}000;$$

hence

$$s = + 0.967741 \text{ ℔.} \quad s_0^A = - 160\, s = - 154.838 \text{ ℔.};$$

$$s_0^B = + 40\, s = + 38.709 \text{ ℔.}; \quad s_0^C = + 140\, s = + 135.484 \text{ ℔.}$$

$$V_0^A = 15\, s_0^A = - 2{,}322.5 \text{ ℔.}; \quad V_0^B = 25\, s_0^B = + 967.7 \text{ ℔.};$$

$$V_0^C = 10\, s_0^C = + 1{,}354.8.$$

$$S_0^A = \tfrac{500}{1700} \times 10{,}000 = + 2{,}941.2 \text{ ℔};$$

$$S_0^B = \tfrac{1000}{1700} \times 10{,}000 = + 5{,}882.3 \text{ ℔};$$

$$S_0^C = \tfrac{200}{1700} \times 10{,}000 = + 1{,}176.5 \text{ ℔.}$$

$$C_1^A = C_1^B = C_1^C = 0; \quad X_0^A = X_0^B = X_0^C = + 50.0;$$

$$\delta_0^A = \delta_0^B = \delta_0^C = \frac{5882.3 \times \overline{100}^3}{12{,}000} = + 491{,}860 \text{ in.}$$

$$M_0^A = 50.0\, S_0^A = + 147{,}060 \text{ in. ℔.}$$

$$M_0^B = 50.0\, S_0^B = + 294{,}115 \text{ in. ℔.}$$

$$M_0^C = 50\, S_0^C = + 58{,}825 \text{ in. ℔.}$$

For Floor Load w_0.—

$h^A = 160.0$ in.; $X = - 500 \times 300 \times 10 = - 1{,}500{,}000$ in. ℔.

$s[15 \times \overline{160}^2 + 25 \times \overline{40}^2 + 10 \times \overline{140}^2] = - 1{,}500{,}000$; hence

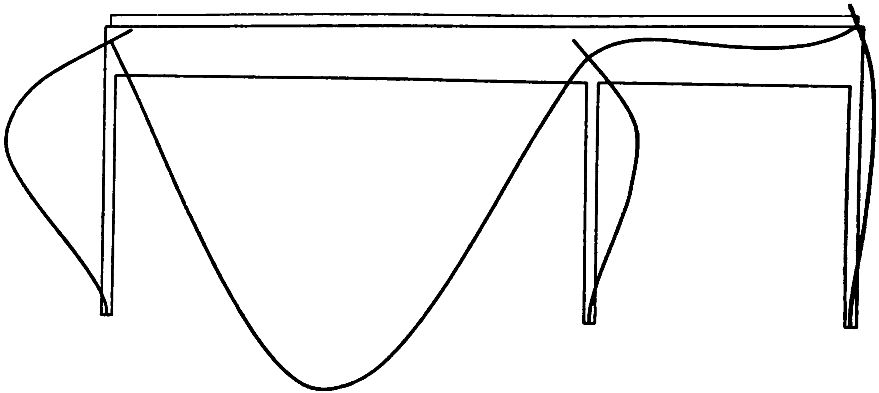
$$s = - 2.41935 \text{ ℔.}$$

$$s_0^A = - 160.0\, s = + 387.095 \text{ ℔}; \quad s_0^B = + 40\, s = - 96.773 \text{ ℔};$$

$$s_0^C = + 140\, s = - 338.71 \text{ ℔.}$$

$$p = \frac{500 \times 300}{15 + 25 + 10} = + 3{,}000 \text{ ℔}, \quad p = + 3{,}000 \text{ ℔},$$

$$p = + 3{,}000 \text{ ℔.}$$

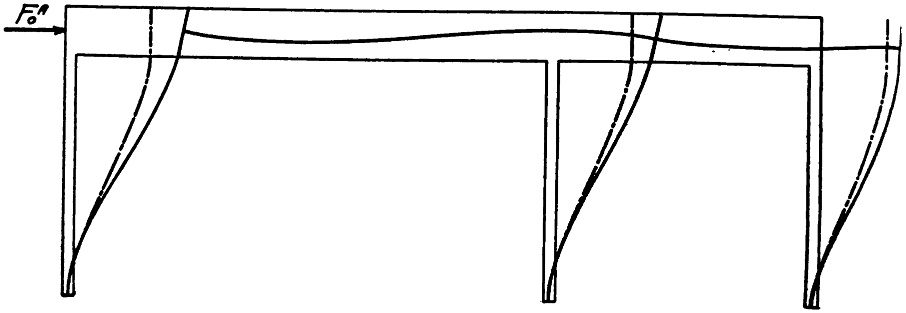


Deformation for Uniform Floor Load w.

—·— Deformation by method of Chapter II.

——— " " " " III.

All deformations are shown to the same scale.



Deformation for Wind Load F_0^a

FIGS 12.

Resultants: $s_0^A = +\ 3{,}387.095$ lb per sq. in.

$s_0^B = +\ 2{,}903.227$ lb per sq. in.,

$s_0^C = +\ 2{,}661.29$ lb per sq. in.

$V_0^A = 15\ s_0^A = +\ 50{,}806.4$ lb;

$V_0^B = 25.0\ s_0^B = +\ 72{,}580.7$ lb;

$V_0^C = 10\ s_0^C = +\ 26{,}612.9$ lb.

$$S_0^A = S_0^B = S_0^C = 0; \quad M_0^A = M_0^B = M_0^C = 0;$$

$$C_1^A = C_1^B = C_1^C = 0; \quad \delta_0^A = \delta_0^B = \delta_0^C = 0.$$

Now, for a more comprehensive comparison the results of the calculations of this chapter are arranged in tabular form as follows:

	For Wind Load F_0^A.		For Floor Load W_0.	
	Chapter III.	Chapter II.	Chapter III.	Chapter II.
S_0^A	+ 3,120.4	+ 2,941.2	− 10,410.6	0
S_0^B	+ 5,491.4	+ 5,882.3	+ 9,737.4	0
S_0^C	+ 1,388.2	+ 1,176.5	+ 673.2	0
V_0^A	− 1,739.9	− 2,322.5	+ 45,448	+50,806.4
V_0^B	− 316.1	+ 967.7	+ 88,622.3	+72,580.7
V_0^C	+ 2,056.0	+ 1,354.8	+ 15,930	+26,612.9
M_0^A	+170,495	+147,060	−371,952	0
M_0^B	+304,755	+294,115	+345,720	0
M_0^C	+ 71,186	+ 58,825	+ 23,017	0
X_0^A	+ 54.67	50.0	+ 35.73	0
X_0^B	+ 55.52	50.0	+ 35.51	0
X_0^C	+ 51.25	50.0	+ 34.21	0
δ_0^A	+664,813	+491,860	−249,320	0
δ_0^B	+608,535	+491,860	+105,700	0
δ_0^C	+622,859	+491,860	+ 14,425	0
Max. M^A	+170,495	+147,060	+669,108	0
Max. M^B	+304,755	+294,115	−628,020	0
Max. M^C	+ 71,186	+ 58,825	− 44,303	0

A graphical comparison of distortions is shown in Figs. 12.

CHAPTER XII.

THE DESIGNING OF KNEE BRACED STRUCTURES.

In this chapter the knee brace is considered as an elastic member with pivoted connections at each end.

In the segment ACB of the knee braced column shown in Fig. 13, *a*, suppose B to be the point of contraflexure, *i. e.*, that no bending moment exists at this point. The external forces acting on the segment ABC will then consist of the shear S, applied at B, the knee brace stress applied at C, and a bending moment and horizontal force at A to maintain equilibrium and keep the column vertical at A. In addition to these forces there exists the axial load on the column but, in all these discussions the effect of the axial load on the bending of the column is neglected. The effect of the shearing force S and the knee brace stress will be considered separately.

For the Shearing Force S.—Referring to Fig. 13, *a*,

$$M = Sa; \quad M_x = M - Sx = S(a - x).$$

This gives, for the elastic line

$$E\frac{d^2y}{dx^2} = \frac{S}{I}(a - x);$$

hence

$$E\frac{dy}{dx} = \frac{Sx}{I}\left(a - \frac{x}{2}\right);$$

and

$$Ey = \frac{Sx^2}{2I}\left(a - \frac{x}{3}\right);$$

both constants of integration being zero. Thus the deflection at the knee brace connection at C is given by

$$\delta_c = Ey_c = \frac{Sx_c^2}{2I}\left(a - \frac{x_c}{3}\right).$$

For the Stress in the Knee Brace.—Referring to Fig. 13, *b*, it is seen that

$$\delta'_c = Ey'_c = \frac{Px_c^3}{3I};$$

since the case is simply the deflection under a concentrated load applied at the end of a cantilever beam of length equal to X_c.

The Resultant Deflection at C.—Evidently C will deflect under the combined action of these forces to the right an amount measured by

$$\delta''_c = \delta_c - \delta'_c = Ey''_c = \frac{Sx_c^2}{2I}\left(a - \frac{x_c}{3}\right) - \frac{Px_c^3}{3I}.$$

Solution for Knee Brace Stress.—Referring to Fig. 13, *c*. Let the floor girder A–D be considered perfectly rigid. The knee brace CD then, after distortion, assumes the position $C'D$ where

$$CC' = \frac{\delta''_c}{E};$$

the shortening, CE of the knee brace will be equal to

$$\frac{\delta''_c}{E}\frac{n}{m} = \frac{\lambda}{E}.$$

The force P is the horizontal component of the knee brace stress; therefore, the stress in the brace is $P\ m/n$. Now let A_k be the cross sectional area of the knee brace. Then

$$\lambda = \frac{Pm^2}{nA_k};$$

but

$$\lambda = \delta''_c\frac{n}{m};$$

hence

$$\frac{Pm^2}{nA_k} = \delta''_c\frac{n}{m}$$

or

$$\delta''_c = P\frac{m^3}{n^2A_k} = S\frac{x_c^2}{2I}\left(a - \frac{x_c}{3}\right) - P\frac{x_c^3}{3I}$$

or

$$P\left[\frac{m^3}{n^2A_k} + \frac{x_c^3}{3I}\right] = S\frac{x_c^2}{2I}\left(a - \frac{x_c}{3}\right).$$

Designing.—Referring to the above formula, it is very evident that the larger A_k becomes, *i. e.*, the greater the cross sectional area of the knees, the larger the total stress in the knee becomes also, and δ_c'' approaches more nearly to the value zero. δ_c'' cannot, however, reach as low as zero, for this would imply an infinite value of A_k. Now, if it is desired to so determine A_k that the allowed unit stress in the brace will not be exceeded, let s be the unit stress allowed; then

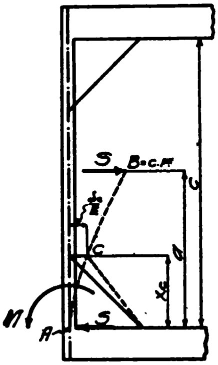


Fig 13-a

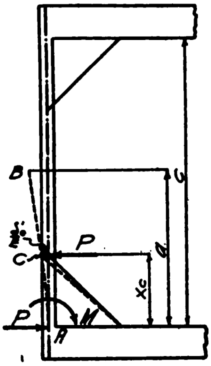


Fig. 13-b

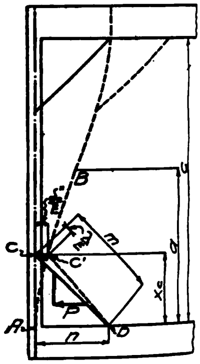


Fig. 13-c

$$s = P\frac{m}{nA_k}$$

or

$$s = \frac{S\frac{x_c^2}{2I}\left(a - \frac{x_c}{3}\right)m}{nA_k\left[\frac{m^3}{n^2A_k} + \frac{x_c^3}{3I}\right]}$$

or

$$A_k\left[\frac{m^3}{n^2A_k} + \frac{x_c^3}{3I}\right] = S\frac{mx_c^2}{2nIs}\left(a - \frac{x_c}{3}\right)$$

or

$$A_k = \frac{3I}{x_c^3}\left[S\frac{mx_c^2}{2nIs}\left(a - \frac{x_c}{3}\right) - \frac{m^3}{n^2}\right].$$

Applications.—In applying these formulae the first thing to decide on is what value shall be assigned for "a." The value of "a" will depend on what type of loading produces the bending in the column and also, to some extent, on the disposition of the bracing. It has been shown in previous chapters that, for all kinds of loading the position of the point of contraflexure does not follow any simple law, but is quite variable; however, when the bending is due to wind pressure, for all stories above the basement it is fairly close to take $a = c/2$ *when no knees are used.* For other loadings it would be much wiser to assume $a = \frac{5}{8}c$.

Now, if similar knees are used at both top and bottom of the column section, then, evidently, the values above will not be altered any. This, however, is not often a practicable way to brace a building, except in the end or wall sections of the building, because of the interference of the knee with the floor space. When knees are used only at the top (or bottom) the assumptions above will also hold quite well, as will be shown a little later.

Now, as an example, assume the following data:

$S = 16{,}000$ lb; $c = 125$ in.; $x_c = 36$ in. $= n$; $m = 1.414$;
$n = 50.85$ in. (45° bracing); $I = 1{,}550$; $s = 15{,}000$ lb per sq. in.;

assume $a = c/2 = 62.5$ in.

Solving for A_k gives

$$A_k = \frac{3 \times 1{,}550}{\overline{36}^3}\left[16{,}000 \times \frac{50.85 \times \overline{36}^2 \times 50.5}{72 \times 1{,}550 \times 15{,}000} - \frac{\overline{50.85}^3}{\overline{36}^2}\right]$$
$$= -6.94 \text{ sq. in.}$$

This is an impossible area of cross-section.

Evidently the maximum unit stress which can exist in the brace will occur when A_k is a minimum. Making $A_k = 0$, *i. e.*, no brace,

$$s = \frac{Snx_c^2\left(a - \frac{x_c}{3}\right)}{2m^2I} = \frac{16{,}000 \times 36 \times \overline{36}^2 \times 50.5}{2 \times 50.85^2 \times 1{,}550} = 4{,}720 \text{ lb per sq. in.}$$

This is then the stress that would exist if the brace were but a thin wire (tension brace) and no greater stress will exist. Certainly then, *so far as the brace is concerned,* it carries stress at a remarkably uneconomical unit.

Now, assume the brace to consist of 2 ∠'s $3\frac{1}{2}$ in. x 3 in. x $\frac{3}{8}$ in.;

then A_k = 4.60 sq. in. gross or 4.04 sq. in. net say; then

$$s = \frac{16{,}000 \times \overline{36}^2 \times \overline{50.85} \times 50.5}{72 \times 1{,}550 \times 54.04 \left[\dfrac{\overline{50.85}^3}{\overline{36}^2 \times 4.04} + \dfrac{\overline{36}^3}{3 \times 1{,}550} \right]}$$

$$= 3{,}360 \text{ ℔ per sq. in.}$$

Effect of Knee Bracing on Columns.—Using the angle bracing just mentioned gives

$$P \left[\frac{\overline{50.85}^3}{36^2 \times 4.04} + \frac{\overline{36}^3}{3 \times 1{,}550} \right] = \frac{16{,}000 \times \overline{36}^2 \times 50.5}{2 \times 1{,}550}$$

or

$$P = \frac{337{,}500}{35.15} = 9{,}610 \text{ ℔.}$$

$$\delta_c = \frac{16{,}000 \times \overline{36}^2 \times 50.5}{2 \times 1{,}550} = 337{,}500 \text{ in.,}$$

$$\delta'_c = \frac{9{,}610 \times \overline{36}^3}{3 \times 1{,}550} = 96{,}100 \text{ in.;}$$

hence

$$\delta''_c = 337{,}500 - 96{,}100 = 241{,}400 \text{ in.}$$

or

$$y''_c = \frac{241{,}400}{28{,}000{,}000} = 0.00861 \text{ in.;} \quad M' = 36 \times 9{,}610 = 346{,}000 \text{ in. ℔.;}$$

$$M = 62.5 \times 16{,}000 = 1{,}000{,}000 \text{ in. ℔.}$$

Maximum resultant bending moment in column equals, 1,000,000 − 346,000 = 654,000 in. ℔. Thus the deflection will be reduced 28.6 per cent. at the brace and the maximum bending in the column will be reduced 34.6 per cent. *if the knees* are used at both top and bottom; if used only at the top the maximum bending occurs at the bottom and equals 16,000 × 62.5 = 1,000,000 in. ℔. and no saving of material but rather a waste would seem to be indicated; however, the brace serves to stiffen the girders.

Discussion.—Professor Heller in his "Stresses in Structures" has derived very complete formulae for portals of different types. For the type used in the above calculations, viz., a rigid girder with knee bracing, it is assumed that the brace is perfectly rigid, *i. e.*, that C, Fig. 14, remains vertically under BD and the flexure curve is then as shown in the figure in solid lines. The bending moment at B is thus *reversed* from what would have existed had there been no knees. It is shown in the above calculations that the curve of flexure would be as indicated in the figure by the broken line, the effect of the knees being merely to reduce the deflection at C and the bending at B as noted above. Professor Heller shows that, for his assumptions,

$$c - a = \frac{a'(a' + c)}{3a' + c};$$

thus when $a' = 9/10\ c$, $a = 0.537\ c$ and when $a' = 0.5\ c$, $a = 0.7\ c$.

In order to determine about what the value of a should be when the knees are placed only at top or bottom, the formulae will now be derived for a knee braced plate girder portal, assuming the girder as rigid but the braces as elastic and with pivoted connections.

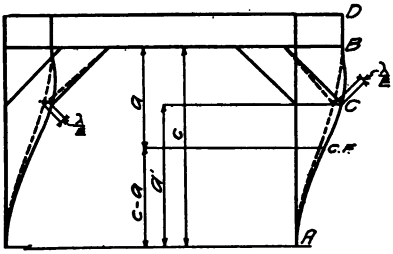


FIG. 14.

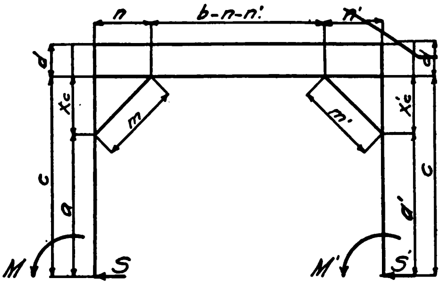


FIG. 15.

Referring now to Fig. 15,

For Column A,

$$EI\frac{d^2y}{dx^2} = M - Sx + P(x - a)_{a}^{c},$$

$$EI\frac{dy}{dx} = Mx - \tfrac{1}{2}Sx^2 + P\left(\frac{x^2}{2} - ax\right)_{a}^{c} + C,$$

$$EIy = \tfrac{1}{2}Mx^2 - \tfrac{1}{6}Sx^3 + P\left(\frac{x^3}{6} - \frac{ax^2}{2}\right)_{a}^{c} + Cx + C'.$$

From $x = 0$ to $x = a$

$$EI\frac{dy}{dx} = Mx - S\frac{x^2}{2}$$

and

$$EIy = \tfrac{1}{2}Mx^2 - \tfrac{1}{6}Sx^3.$$

From $x = a$ to $x = c$

$$C = +P\frac{a^2}{2}$$

and

$$C' = -P\frac{a^3}{6};$$

hence

$$EI\frac{dy}{dx} = Mx - \tfrac{1}{2}Sx^2 + P\left(\frac{x^2}{2} - ax\right) + P\frac{a^2}{2}$$

and

$$EIy = \tfrac{1}{2}Mx^2 - \tfrac{1}{6}Sx^3 + P\left(\frac{x^3}{6} - \frac{ax^2}{2}\right) + P\frac{a^2x}{2} - P\frac{a^3}{6}.$$

Now, when $x = C$, $dy/dx = 0$; hence, substituting and solving gives

$$M = \tfrac{1}{2}Sc - P\frac{(c-a)^2}{2c}.$$

Similarly for Column B,

$$M' = \tfrac{1}{2}S'c - P'\frac{(c-a')^2}{2c}.$$

Also, when $x = c$, δ for Column $A = \delta$ for Column B since the girder is rigid, but δ for Column A

$$= Ey = \frac{Sc^3}{12I} - \frac{P(c-a)^2(c+2a)}{12I}$$

and similarly δ for Column B

$$= \frac{S'c^3}{12I'} - \frac{P'(c-a')^2(c+2a')}{12I'};$$

equating δ for Column A to δ for Column B and solving gives

$$Sc^3\left(1 + \frac{I}{I'}\right) = P(c-a)^2(c+2a) - P'\frac{I}{I'}(c-a')^2(c+2a') + Fc^3\frac{I}{I'}$$

since $S' = F - S$.

When $x = a$,

$$Ey = \frac{Ma^2}{2I} - \frac{Sa^3}{6I}$$

and, substituting the value of M obtained above, gives

$$Ey. = \frac{Sa^2}{12I}(3c - 2a) - \frac{Pa^2}{12I}\left[\frac{3(c-a)^2}{c}\right].$$

Now the value of $\delta c''$, of the early part of this chapter, will equal the difference between the values of Ey for $x = c$ and for $x = a$; making the substitutions gives $\delta c''$, for Column A

$$= \frac{1}{12I}\left\{ Sc^3 - P(c-a)^2(c+2a) - Sa^2(3c-2a) + 3P\frac{a^2(c-a)^2}{c}\right\};$$

but

$$\delta c'' = \frac{Pm^3}{n^2A_k}$$

hence solving, gives

$$P\left\{\frac{m^3}{n^2A_k} + \frac{(c-a)^3(3a+c)}{12cI}\right\} = S\frac{(c^3 - 3ca^2 + 2a^3)}{12I},$$

and similarly may be obtained for Column B the value

$$P'\left\{\frac{m'^3}{n'^2A_k'} + \frac{(c-a')^3(3a'+c)}{12cI'}\right\} = S'\frac{(c^3 - 3ca'^2 + 2a'^3)}{12I'}.$$

Now, substituting these values of P and P' in terms of S, in the value given for S above determines S and thence P, P', S', M and M' and all other unknowns.

When the structure is symmetrical the results reduce to

$$S = S' = \tfrac{1}{2}F$$

and

$$P = P' = \frac{(c^3 - 3a^2c + 2a^3)}{12I\left[\dfrac{m^3}{n^2A_k} + \dfrac{(c-a)^3(3a+c)}{12cI}\right]}S.$$

Professor Heller shows that, for an unsymmetrical portal by his assumptions,

$$S = F\frac{I}{I + I'};$$

that is, if the portal is merely unsymmetrical with respect to the moments of inertia of the columns. It will be noticed that this simple relation does not hold in case the knees are considered elastic.

Application.s—Assume the following data, to agree with that used in previous examples.

$F = 32{,}000$ lb; $I = I' = 1{,}550$; $c = 125$ in.; $X_e = 36$ in. $= n$; $m = 50.85$ in.; $a = 89$ in. (Fig. 15); $Ak = 4.04$. Then

$$P = P' = 16{,}000 \text{ lb} \left\{ \frac{(\overline{125}^3 - 3 \times \overline{89}^2 \times 125 + 2 \times \overline{89}^3)}{\dfrac{\overline{50{,}85}^3}{\overline{36}^2 \times 4.04} + \dfrac{\overline{36}^3 \times 392}{12 \times 125 \times 1{,}550}} \right\} = 10{,}200 \text{ lb}.$$

By the approximate method already given P was found to equal 9,610 lb. The unit stress in the brace then equals

$$\frac{10{,}200 \times 50.85}{36 \times 4.04} = 3{,}600 \text{ lb per sq. in.},$$

a close agreement with the 3,360 lb per sq. in. obtained by the approximate method. Substituting in the value given for M, gives

$$M = 16{,}000 \times 6.25 - 10{,}200 \times \frac{\overline{36}^2}{250} = 947{,}000 \text{ in. lb}.$$

or a saving in the maximum column bending stress of only 5.3 per cent. by the use of knees.

The point of contraflexure will be located a distance above the foot of the column equal to

$$\frac{947{,}000}{16{,}000} = 59.1 \text{ in}.$$

or the proper value of a to use in the methods given in the earlier part of the chapter would be

$$125 - 59.1 = 65.9 \text{ in.} = 0.527c.$$

Therefore an assumption of $a = c/2$ is a fair one to use in connection with the approximate method for wind loads.

Now, supposing the columns of unequal cross sections the value of S may be written

$$S = F\left(\frac{I}{I + I'}\right) \times \alpha,$$

where α is a variable coefficient. Assuming all the data of the above problem except that the value of I' is written

$$I' = kI = k \times 1{,}550,$$

where k is a decimal and variable, the curve shown in Fig. 16 has

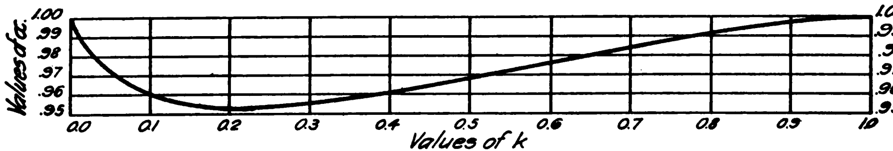


FIG. 16

been drawn to give the values of α for all values of k from zero to unity: it will be noticed that the coefficient does not vary more than 5 per cent. from unity, is always less than unity and may for all practical purposes be considered as constant and equal to 1.

It will now be plain that, for wind stresses, the structure may first be computed as if no knees existed and then the stresses in the columns modified by the use of the approximate methods of this chapter and that for the sizes of the knee braces no arbitrary limiting unit stresses can be assigned, but, rather, the cross section should be assumed in accordance with the engineer's judgment and the column designed to suit.

The economy of the brace can not be a dominant factor unless braces are used both at top and bottom, a design seriously recommended.

CHAPTER XIII.

A DISCUSSION OF WIND PRESSURES ON BUILDINGS.

Wind Velocities.

Wind velocities are measured at all U. S. Weather Bureau Stations by means of the Robinson anemometer. This instrument consists of four hollow hemispherical cups, four inches in diameter, mounted with the open face vertical on the ends of radial horizontal arms connected to a vertical shaft, whose centre is 6.72 inches distant from the centres of the cups. The wind, striking these cups, sets the apparatus revolving about the vertical axis; this motion in turn setting in motion a system of gears which automatically record the amount of the motion on a dial and usually also on an autographically recording apparatus run by clockwork. The gears are so arranged that, after a certain number of revolutions of the axis are made, the dial indicates a change of unity in its reading. Professor Robinson (about 1840) believed, from his experiments, that the wind imparted to these cups a lineal circumferential velocity of the cup centres equal to just one third of the wind velocity and the dial and mechanism was then designed so as to indicate a movement of unity, or one mile, after the cup centres have travelled a distance of one third mile; he believed this ratio was independent of the diameter of the cups, the length of the radial arms, the weight of the instrument, and, above all, the velocity of the wind. Therefore all manufacturers graduated the dials for a uniform movement and they have been made in that manner ever since. In late years it was found that the readings really were dependent on all of these quantities. However, by adopting a standard size and construction for the Weather Bureau Standard and carefully prescribing the care to be taken of all instruments, all errors are practically eliminated save that one depending on the velocity of the wind. To eliminate this latter an extensive series of anemometer tests were made at Washington by the U. S. Weather Bureau and from a study of these results a formula for correcting the anemometer readings was deduced; this formula is $V = 0.509 +$

0.9012 log v, where V is the real velocity of the wind and v is the velocity recorded by the instrument, both in miles per hour. The corrections to indicated velocities by this formula are given in the following table. For intermediate values it will be sufficiently accurate to interpolate.

Indicated velocity = v . .	0	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
Corrected velocity = V .	0	9.6	17.8	25.7	33.3	40.8	48.0	55.2	62.2	69.2

It is evident that the weight of the instrument reduces the sensitiveness to gusts. However the standard anemometer is designed to be as sensitive as is regarded practicable and, by rating the standard instrument as above, this trouble is practically eliminated.

The wear of the bearings will also cause the instrument to give too small a reading, affecting both steady and gusty wind velocities. However the U. S. Weather Bureau has one such instrument in perfect condition after thrity years service and therefore, with an instrument well taken care of, this item will be negligible.

The anemometer is ordinarily mounted on the end of an eighteen foot vertical support located near the central portion of the roof of the highest building in the vicinity.

WIND PRESSURES.

Observations have been made to determine wind pressures since about 1840 with more or less success, but probably the most trustworthy of all are those made by Professor C. F. Marvin at Mt. Washington. Using the Robinson anemometer he deduces from his experiments the formula

$$p = 0.004 \times \frac{B}{30} \times V^2,$$

where p is the pressure in pounds per square foot, B the barometric pressure in inches of mercury, and V the real or corrected velocity of the wind as given by the velocity formula above.

From this formula the following table of pressures is derived. For all practical purposes it will be sufficiently accurate to interpolate for intermediate values.

Indicated velocity = v .	0	10	20	30	40	50	60	70	80	90
Values of P	0	0.369	1.27	2.64	4.44	6.66	9.22	12.2	15.5	19.2

Professor Marvin calls attention to the fact that for velocities of over 50 or 60 miles an hour this table, while probably the best information obtainable can not be regarded as accurate, because, first, it is very difficult to make observations at high velocities, second, the experiments did not cover velocities above those mentioned (about 60 miles an hour). These experiments were made on surfaces of 4 and 9 square feet.

DISCUSSION.

Effect of Atmospheric Temperature on Wind Pressures.—Mr. A. R. Wolff, in "The Windmill as a Prime Mover," shows that, since the density of the air varies with the temperature and wind is merely air in motion, the pressure of wind varies with the temperature and he further shows this variation to amount to as much as 20 per cent in a temperature range of 100° F.

Effect of Altitude on Wind Pressures.—Since air is lighter in greater altitudes the wind pressure will also be less. Professor Marvin corrects for this in the formula already given. In addition to and quite distinct from this, it is also clearly recognized that the pressure varies as the distance above the general level of the surrounding territory, increasing towards the top of the structure. No very definite determinations of the amount and law of such variations have yet been determined. The variation is probably due to the retarding action of the earth's surface, the effect being similar to that on the flow of streams.

Effect of Size of Exposed Surface.—It is also well recognized that the wind effect on a large surface, per unit of surface, is somewhat less than on small ones such as are used in experimental determinations. Thus, Ketchum, in "Steel Mill Buildings," suggests that the unit pressure on the vertical walls of buildings be considered as 80 per cent of that for small surfaces. The pressure also varies with the shape of the surface, but in building work the surfaces may be so variable and broken that such a refinement is not practicable; most surfaces are, however, plane and rectangular.

Effect of Gusts.—Professor Marvin states that his formula for wind pressure gives the average pressure and that the effect of gusts may vary the pressure 35 per cent either way from the values of the formula (35 to 50 per cent by Langley's experiments), thus making a maximum momentary pressure of 135 per cent of that given by his formula. Professor Langley in his investigations shows that the wind velocity may have five or six maximum and minimum values during a single minute and that while the average variation is about as given by Professor Marvin, the minimum value may reach as low as zero in these variations. Professor Langley says in his conclusions: "The wind is not even an approximately uniform moving mass of air, but consists of a succession of very brief pulsations of varying amplitude, and that, relatively to the mean movement of the wind, these are of varying directions."

Now, if these pulsations should ever agree with the vibrations of the structure subject to pressure, the structure will receive a series of impacts which will, at each successive blow, increase the amount of the vibration until the structure has failed or the periods of vibration disagree. Existing experiments tend to indicate that the period of vibration of a tall building is several seconds (about two to four seconds) and thus the likelihood of such an agreement occurring does not seem impossible. It seems advisable that the increased pressure due to gusts should be considered in designing such a structure as a tall building.

Effect of Suction.—When a large mass of rapidly moving air strikes a fixed mass capable of resisting its direct movement, the central mass of the resisted atmosphere imparts a great portion of its energy of motion in doing work of a more or less evident nature on the obstructing mass and the rest of its energy is spent in deflecting the motion of the outer moving particles so as to pass around the obstruction. This central mass then produces direct pressure on the structure. The outside escaping masses, in leaving the edges and roof, produce a small suction and also on the leeward side of the obstruction, the rapidly moving air will rarefy the space close to the obstruction and create a more or less uniform suction. This has been well demonstrated by the lifting of roofs on the leeward side of houses and barns during severe wind storms.

It is still an open question whether the outward falling of the walls of buildings is due to the sudden decrease of outside atmospheric pressure over that existing inside the structure, or to the suction due to the rapid motion of the wind, as above mentioned. The confusion is due to the fact that the rapidly moving air does not occur independently of the difference in atmospheric pressure.

CONCLUSION.

It must now be evident, from the above, that the pressure of 30 pounds per square foot commonly used in designing is equivalent to an *indicated* velocity of about 130 miles an hour not allowing for gusts, or about 110 miles if gusts are assumed to increase the pressure 35 per cent; these figures are based on a coefficient of 80 per cent for the wall of the building.

It seems to the writer that the customary location of the anemometer may not be quite free from the suction effects mentioned.

It will be noticed that for the high winds that are ordinarily likely to be encountered in experiments on a tall office building (50 miles an hour indicated) the average unit pressures will not be over about 5 pounds per sq. ft. (80 per cent × 6.66). The probability of a severe storm such as a tornado, with its very narrow path, striking a particular tall building is very small indeed and especially during an investigation of only two months duration, since the Weather Bureau records show only one storm exceeding an indicated velocity of 90 indicated miles an hour in a ten year interval.

CHAPTER XIV.

DESCRIPTION OF AND DATA FOR EXPERIMENTAL BUILDING.

A study of Figs. 17 and 18 will give a comprehensive idea of the position and exposure of the seventeen story office building chosen for the experiments described in the following Chapter. It will

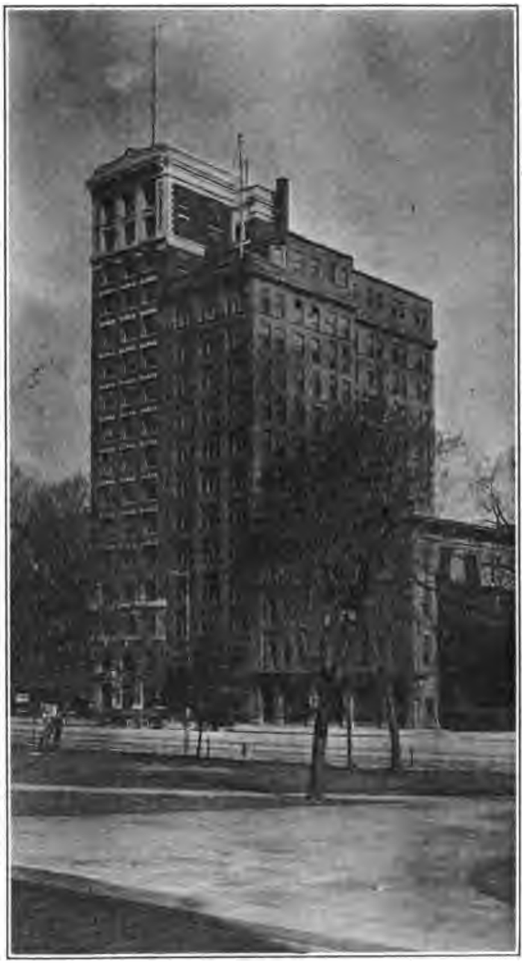


FIG. 17.

be noticed that the building faces south with a twelve story office building abutting on the east and a five story one on the west. As the surface slopes downward rapidly to the west and no obstruction of any magnitude interferes, the west winds have a most favorable opportunity for displaying the energy embodied in them.

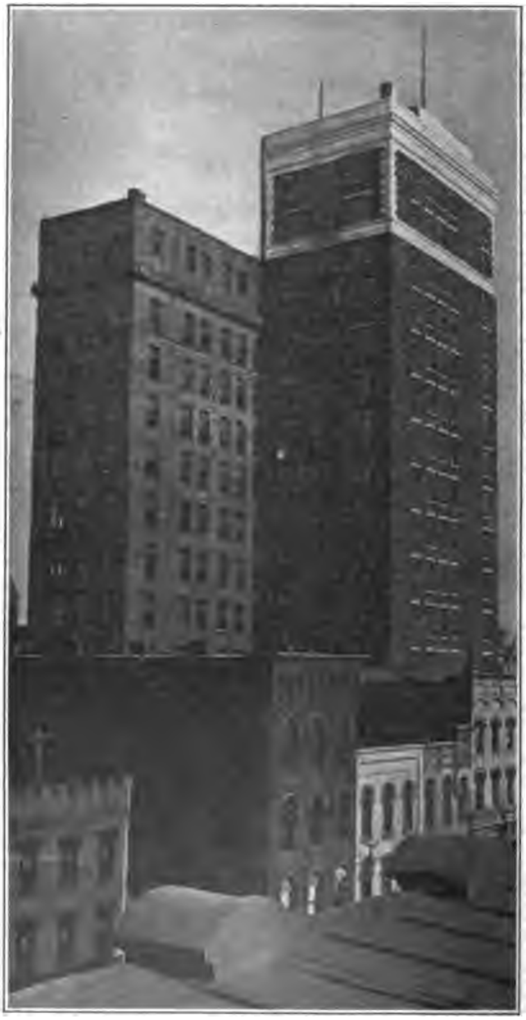


FIG. 18.

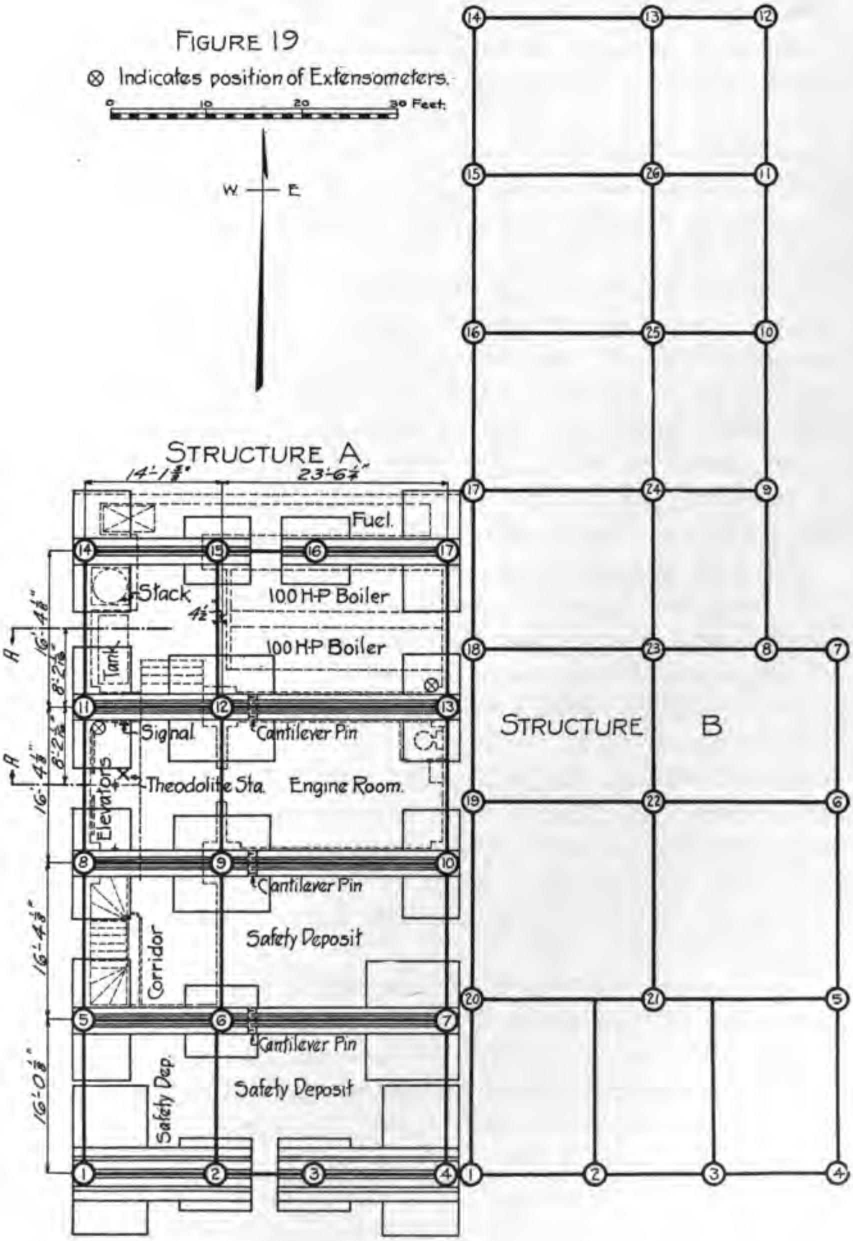


FIGURE 19

⊗ Indicates position of Extensometers.

The fact that the twelve story building abuts against the seventeen story one is, of course, an unfortunate feature, and necessitates the discussion of the characteristics of this building also. In Fig. 19 are shown the general floor plans of the two buildings in their respective positions with the column numbering given for reference throughout the discussion. The seventeen story structure will hereafter be referred to as structure *A* and the twelve story one as structure *B*.

Structure *B* is of the cage construction type with portal bracing, the latter consisting of light I beams (7 in. and 10 in.) with field connections bolted. In other words there is no structural bracing, and rigidity is furnished only by the partitions and inertia of the structure. As shown by Baier in his paper on the St. Louis tornado (*Transactions A. S. C. E.*, Vol. XXXVII, pp. 221, 307), the value of inertia in resisting high wind pressures is not a very dependable quantity, and it seems likely that the *partitions furnish whatever real rigidity this structure possesses.* The building was erected in 1901 and, during erection, was subjected to a fairly severe wind storm. This occurred when the steel frame had been erected, and partitions placed in the two top stories only. The effect of the wind was to distort the portion from the fourth to the tenth story, pushing the tenth story 7⅜ in. to the east. The two top stories were not distorted, but were kept in a vertical position by the bracing effect of the partitions. The four lower stories were protected by a four story building adjacent. When the structure *A* was built later, the wedge shaped opening caused by the leaning of structure *B* at the party wall was filled at both front and rear walls for the sake of appearance.

Fig. 20 shows the type of column used in structure *B* and the method of splicing same. The following table gives the sections of some of the columns used in this structure and the story heights. All columns are composed of 8 angles, as shown in Fig. 20, and the table gives the size of these angles. Horizontal lines indicate position of column splices. Fig. 21 shows an elevation of the frame of structure *A* on a section through columns 14, 15, 16, and 17 in the plane of the rear or north wall. Fig. 22 shows a similar section through Columns 11, 12, and 13, two of these columns, 11

Story.	Col. 9.	Col. 24.	Col. 17.	Col. 6.	Col. 22.	Col. 19.	Story Hgt.
Attic	2½ x 2½ x ¼	2½ x 2½ x ¼	2½ x 2½ x ¼	2½ x 2½ x ¼	2½ x 2½ x ¼	2½ x 2½ x ¼	Varies
12							∧ All stories 12 ft. 0 in. ∨
11	2½ x 2½ x ¼		2½ x 2½ x $\frac{5}{16}$	2½ x 2½ x $\frac{5}{16}$		2½ x 2½ x $\frac{5}{16}$	
10		2½ x 2½ x ¼			3 x 3 x $\frac{5}{16}$		
9	3 x 2½ x $\frac{5}{16}$		3 x 2½ x ⅜	3 x 2½ x ⅜		3½ x 3 x $\frac{5}{16}$	
8		2½ x 2½ x ⅜			3 x 3 x $\frac{7}{16}$		
7	3 x 3 x $\frac{7}{16}$		3½ x 3 x ⅜	3 x 3 x $\frac{7}{16}$		3½ x 3 x ⅜	
6		3 x 3 x ⅜			3½ x 3 x ½		
5	4 x 3 x $\frac{7}{16}$		4 x 3 x $\frac{7}{16}$	3½ x 3 x ½		3½ x 3 x $\frac{7}{16}$	
4		3½ x 3 x $\frac{7}{16}$			4½ x 3 x $\frac{9}{16}$		
3	4 x 3 x ½		4 x 3 x ½	4½ x 3 x ½		3½ x 3 x ½	
2		4½ x 3 x ½			5 x 3 x $\frac{11}{16}$		
1	4 x 3 x $\frac{9}{16}$		5 x 3 x $\frac{9}{16}$	4½ x 3 x $\frac{9}{16}$		4 x 3 x ½	15 ft. — 3 in.
0		5 x 3 x ½			5 x 3 x $\frac{11}{16}$		11 ft. — 4 in.

and 13, being the ones on which tests were conducted, as described in Chapter XV. Fig. 23 shows the type of column used and method of splicing same. Attention is called to the relative sizes of the columns of structures *A* and *B*. The character of the footings of structure *A* may be seen by referring to Figs. 19, 21, and 22. The following tables give the sizes and properties of columns 11, 12, and 13 and all story heights and the sections of the four bottom stories of Columns 1, 2, 3, 14, 15, 16, and 17 for comparison.

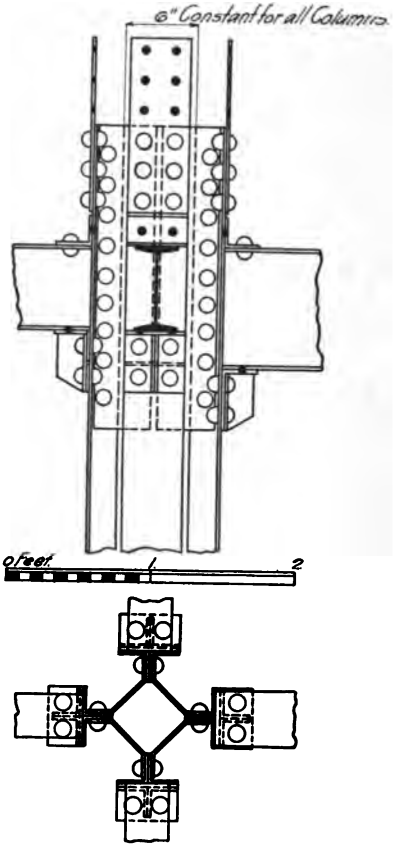


FIGURE 20

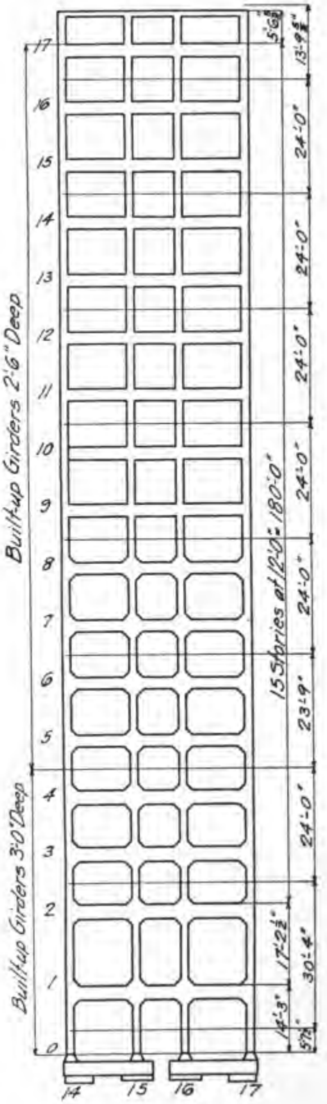


FIGURE 21

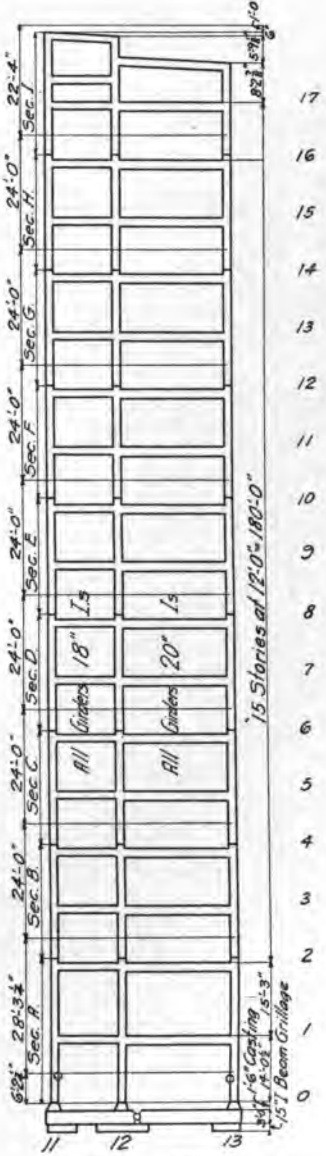


FIGURE 22

Col.	Sec.	Web. (1-Pl.)	4-∠s.	Cov. Pls.	Area.	Length.	I (Web.)	I(4 ∠s).		I(Cov. Pls.)	Total, I.
								Primary.	Secondary.		
11	*A*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{7}{16}$	54.88	32 ft. $5\frac{1}{4}$ ins.	335.8	34.8	1,740.0	1,190.0	3,300.6
11	*B*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{5}{16}$	50.88	24 0	335.8	34.8	1,740.0	837.2	2,947.8
11	*C*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$		40.88	23 7	335.8	34.8	1,740.0		2,110.6
11	*D*	$17\frac{1}{2}$ x $\frac{11}{16}$	6 x 4 x $\frac{11}{16}$		37.67	24 0	307.8	32.4	1,616.0		1,956.2
11	*E*	$17\frac{1}{2}$ x $\frac{5}{8}$	6 x 4 x $\frac{5}{8}$		34.38	24 0	279.8	30.0	1,487.5		1,797.3
11	*F*	$17\frac{1}{2}$ x $\frac{1}{2}$	6 x 4 x $\frac{1}{2}$		27.75	24 0	223.8	25.2	1,218.0		1,467.0
11	*G*	$17\frac{1}{2}$ x $\frac{3}{8}$	6 x 4 x $\frac{3}{8}$		21.00	24 0	167.9	19.6	938.0		1,125.5
11	*H*	$17\frac{1}{2}$ x $\frac{5}{16}$	5 x $3\frac{1}{2}$ x $\frac{5}{16}$		15.71	24 0	140.0	10.8	682.0		832.8
11	*I*	$17\frac{1}{2}$ x $\frac{5}{16}$	5 x $3\frac{1}{2}$ x $\frac{5}{16}$		15.71	24 0	140.0	10.8	682.0		832.8
12	*A*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{7}{16}$	54.88	32 $5\frac{1}{4}$	335.8	34.8	1,740.0	1,190.0	3,300.6
12	*B*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{5}{16}$	50.88	24 0	335.8	34.8	1,740.0	837.2	2,947.8
12	*C*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{5}{16}$	50.88	23 7	335.8	34.8	1,740.0	837.2	2,947.8
12	*D*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$		40.88	24 0	335.8	34.8	1,740.0		2,110.6
12	*E*	$17\frac{1}{2}$ x $\frac{5}{8}$	6 x 4 x $\frac{5}{8}$		34.38	24 0	279.8	30.0	1,487.5		1,797.3
12	*F*	$17\frac{1}{2}$ x $\frac{1}{2}$	6 x 4 x $\frac{1}{2}$		27.75	24 0	223.8	25.2	1,218.0		1,467.0
12	*G*	$17\frac{1}{2}$ x $\frac{7}{16}$	6 x 4 x $\frac{7}{16}$		24.42	24 0	195.9	22.4	1,083.5		1,301.8
12	*H*	$17\frac{1}{2}$ x $\frac{5}{16}$	5 x $3\frac{1}{2}$ x $\frac{3}{8}$		17.67	24 0	140.0	12.8	808.5		961.3
12	*I*	$17\frac{1}{2}$ x $\frac{5}{16}$	5 x $3\frac{1}{2}$ x $\frac{5}{16}$		15.71	23 $3\frac{1}{4}$	140.0	10.8	682.0		832.8
13	*A*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{5}{8}$	60.88	32 $5\frac{1}{4}$	335.8	34.8	1,740.0	1,733.5	3,844.1
13	*B*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{7}{16}$	54.88	24 0	335.8	34.8	1,740.0	1,190.0	3,300.6
13	*C*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{3}{8}$	52.88	23 7	335.8	34.8	1,740.0	1,013.5	3,124.1
13	*D*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$	2 Pls. 16 x $\frac{5}{16}$	50.88	24 0	335.8	34.8	1,740.0	837.2	2,947.8
13	*E*	$17\frac{1}{2}$ x $\frac{3}{4}$	6 x 4 x $\frac{3}{4}$		40.88	24 0	335.8	34.8	1,740.0		2,110.6
13	*F*	$17\frac{1}{2}$ x $\frac{5}{8}$	6 x 4 x $\frac{5}{8}$		34.38	24 0	279.8	30.0	1,487.5		1,797.3
13	*G*	$17\frac{1}{2}$ x $\frac{7}{16}$	6 x 4 x $\frac{7}{16}$		24.42	24 0	195.9	22.4	1,083.5		1,301.8
13	*H*	$17\frac{1}{2}$ x $\frac{5}{16}$	5 x $3\frac{1}{2}$ x $\frac{3}{8}$		17.67	24 0	140.0	12.8	808.5		961.3
13	*I*	$17\frac{1}{2}$ x $\frac{5}{16}$	5 x $3\frac{1}{2}$ x $\frac{5}{16}$		15.71	17 0	140.0	10.8	682.0		832.8

Note.—All Columns 18 inches b. to b. of ∠s.

Col.	Sec.	Web. (1-Pl.).	4-∠s.	Cov. Pls.	Area.
1	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x 11/16	62.88
1	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x 7/16	54.88
2	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ¾	64.88
2	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x 7/16	54.88
3	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ¾	64.88
3	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x 7/16	54.88
14	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ⅝	60.88
14	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ⅜	52.88
15	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ⅜	52.88
15	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x 5/16	50.88
16	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ⅜	52.88
16	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x 5/16	50.88
17	*A*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ½	56.88
17	*B*	17½ x ¾	6 x 4 x ¾	2 Pls. 16 x ⅜	52.88

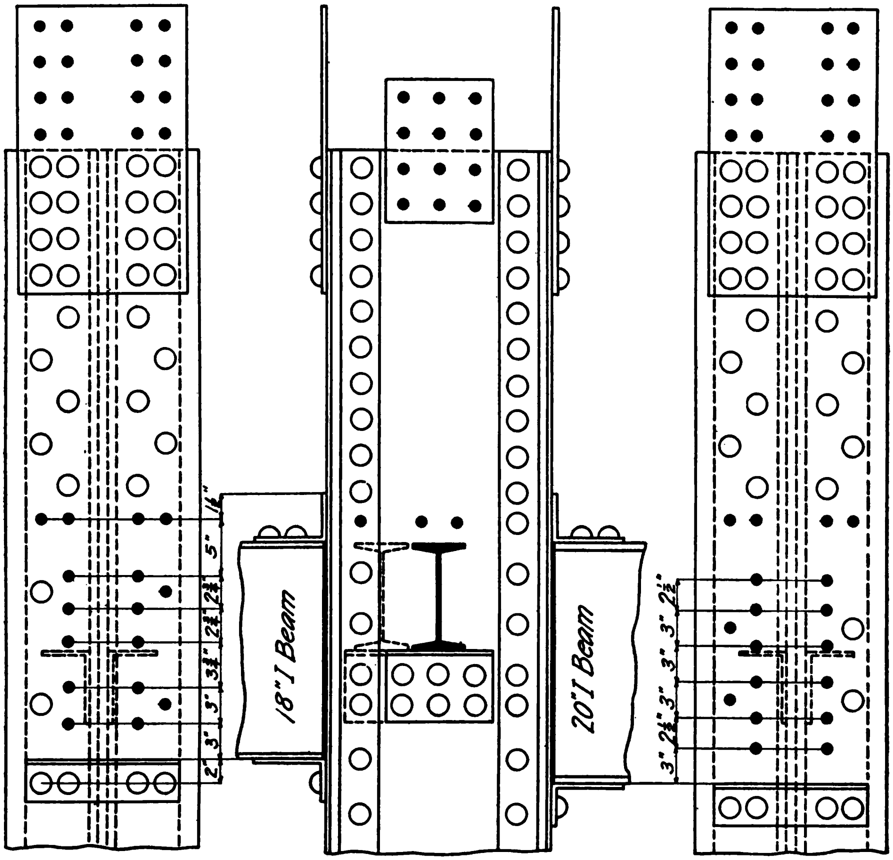
CHAPTER XV.

DESCRIPTION OF AND RESULTS FROM EXPERIMENTS.

The experiments conducted on structure *A* of the previous chapter are in some respects limited. These experiments were undertaken with the view of determining, if possible with the limited resources and working conditions available, the following features: first, the manner of variation and amount of the unit stresses in the basement columns of a high portal braced structure under the action of wind pressures; second, the period of vibration of such a structure under variable wind velocities; and third, the total deflection of the structure, at the top, under wind pressures.

The following are the features which either limit or mar the value of these experiments.

First. The only exterior columns (11 and 13) to which access could be had can not be said to be connected rigidly to the floor girders, since these girders are merely 18 in. and 20 in. I beams with the customary connections (see Fig. 22 and 23). No knees were used and the connections are capable of transferring only small bending moments. The value of these connections, in bending, will be given a little later. As will be seen from Fig. 21 the bracing in the planes of the front and rear walls is very much more rigid, although the form of connection still subjects the rivets to direct tension under the flexure due to wind pressure (as well as all other causes). These sections may however be said to be really portal braced, and, in addition, have the stiffening effect of the walls of the building. It must be realized that if the girders could not take care of *any* bending at the connections and there were no other factor to resist distortion, then the frame must simply close up on itself under lateral forces, as shown in Fig. 24. Thus, if the connections to Columns 11, 12, and 13 were purely of this type, then evidently the sections in the front and rear walls *must* take care of all lateral forces, these forces being transferred to them at each story through the floors, which act as deep, stiff girders, as was mentioned



Column 12.-Sec.C.

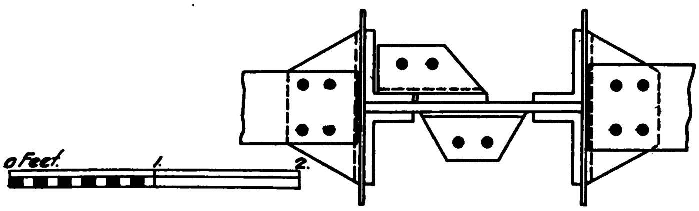


FIGURE 23.

in Chapter II. Thus it is regretted that the stresses in the columns in either front or rear could not also have been measured.

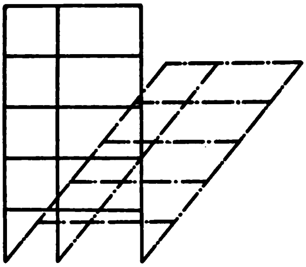


FIG. 24.

Second. Extensometers could only be attached to one corner of the columns and thus it is impossible to determine just what part of the measured stress is due to flexure and what part is axial stress.

Third. It would have been highly desirable to measure the stresses in the columns at say every third or fourth story and to obtain the measurements as nearly simultaneously as practicable.

Fourth. The element of resistance afforded by structure *B* to the west winds (the only winds whose effects are considered in this investigation) is a very uncertain one. It may be as low as nothing for ordinary wind velocities but for very violent storms it would undoubtedly amount to something.

Fifth. As will be shown later, the largest variation of unit stress to be expected under the most reasonable assumptions for the maximum winds obtained requires the most delicate measurement and careful corrections for variations of Column temperatures and instrument variations caused by variation of temperature and relative humidity.

Sixth. The observations for deflection and vibration of the building were made with a theodolite in the elevator shaft and elevator service could not be interrupted (an exception to this was made one evening).

The small space allowed for placing signals and lights and resultant poor illumination and the risk of life and instruments in the shaft, under a moving elevator run by irresponsible youths, together with the necessity of the writer to make all preparations and observations without assistance, combined to make these observations of short duration, infrequent and of a much less elaborate nature than was intended.

DESCRIPTION AND USE OF APPARATUS.

Vibration and Deflection Experiments.—A theodolite was placed in the elevator shaft at the point marked X in Fig. 19, this point being a tack set in the concrete floor of the bottom of the shaft. A signal, as shown in Fig. 25, *a*, was securely wedged in between the elevator guide and the wall at the middle of the sixteenth story so as to be just visible from the foot of the shaft when the elevator was clear at the top, *i. e.*, at the seventeenth floor. The divisions on this signal were one half inch. The signal shown in Fig. 25, *b* was nailed to the wall of the shaft at a point about two feet above the first floor level. It could not be placed any lower, as the minimum focal distance for the telescope of the theodolite was about ten feet. The graduations for this signal were one quarter inch and read from west to east as shown.

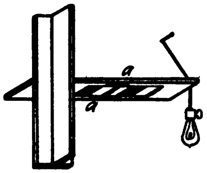


FIG. 25-a

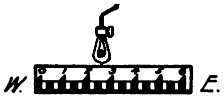


FIG. 25-b

The theodolite was equipped with a prismatic eyepiece. When set up in the shaft the plate bubbles were carefully levelled, then, turning the instrument into the plane of the signals, the striding level was placed in position, always with its adjusting end to the west, and the level carefully brought to the center with the instrument in this position. Removing the striding level and focusing on the light at the signal of Fig. 25, *a*, the cross hair was set on the line "*aa*" of the signal, an average of the vibrations of "*aa*" back and forth being judged by the equal widths of white and black covered by the cross hair as its apparent motion swung back and forth. After securing this setting the telescope was plunged to the signal of Fig. 25, *b*, and a reading recorded as "direct reading." The instrument was then reversed and, without reapplying the striding level, the above operations were repeated and the reading on the signal of Fig. 25, *b*, was recorded as "reversed reading." The striding level was then applied to test for any settlement that might affect the readings.

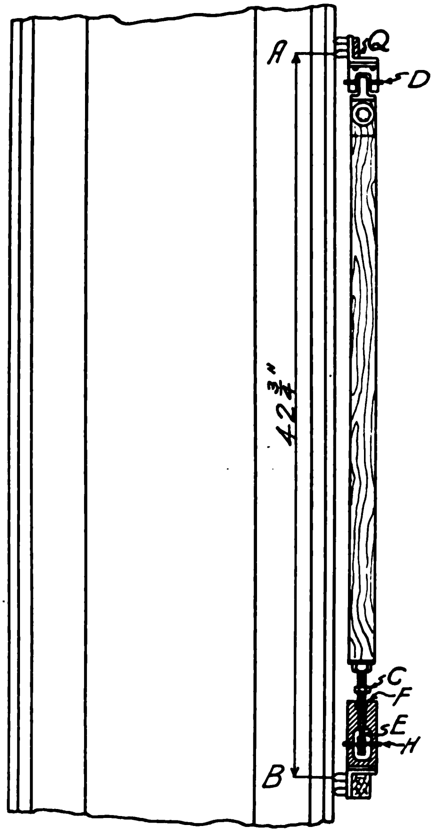
In the observations for building vibration the lower signal was not used, but vibrations were estimated by eye on the half inch scale divisions of the signal of Fig. 25, *a*.

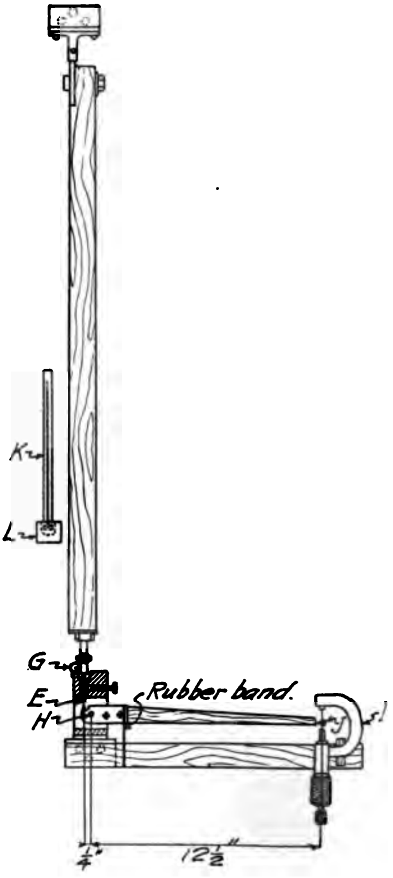
Experiments for the Determination of Column Stresses.—As has already been mentioned these experiments were made on the basement sections of columns 11 and 13. The Boyd-Morris extensometer, which has been so successfully used at the Ohio State University for some years past, was used to measure the change in length of the columns. This instrument is shown (not to scale) in Fig. 26 and in position on Column 13 in the cut, Fig. 27. Center-punch holes, about $\frac{1}{32}$ in. deep, were made in the steel columns for the reception of the hardened points shown on the brackets at *A* and *B*; these brackets were then placed in position and 8 in. malleable iron clamps used to hold them securely to the column. Center punch marks were used as an extra precaution against slipping of the brackets. The nut at *C* was then adjusted until the piston *CD* came to a bearing on the plunger *E*, did not stand out of the cup *F*, and yet left sufficient allowance for free variation in the height of the bottom of the piston. The set screws at *D* were then tightened and the spring clip *G* adjusted so as to hold the bottom of the piston against the inner wall of the cup *F*.

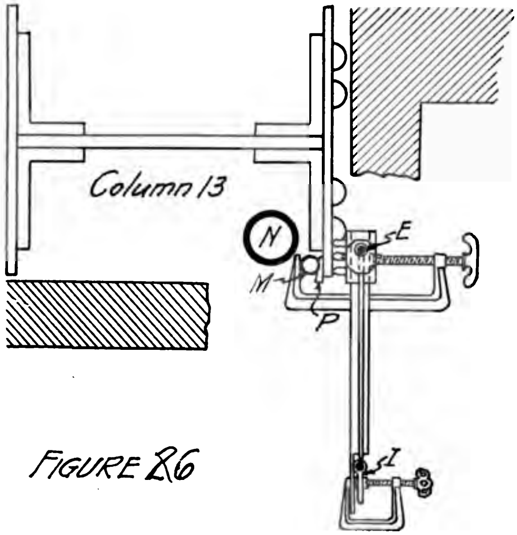
The set screws *H* were then adjusted so as to take out all play in the bearings, which are shown in Fig. 26, *a*, to a larger scale. The micrometer *I* was then adjusted so as to be in a vertical plane with the black pinhead *J* centered over the micrometer screw and proper allowance was made for the expected vertical movement of the pinhead *J*.

It will be seen then, that if there is any change in the length of the section *A–B* of the column, the bracket at *A* will move relatively to that at *B*; the amount of this movement will be the amount of the relative variation in height of the bottom of the piston *CD*, except as modified for the effects of temperature, etc. Now, if the plunger *E* is, at each observation, pushed into contact with the bottom of the piston *CD*, and a light rubber band placed around the lever as shown in the Fig. 26, to keep the knife edge of the lever *HJ* constantly in contact with the bottom of the plunger, then the vertical variation of the height of the black spherical head of the pin *J* will be 50 times the corresponding change in length of the column. Thus, by reading the micrometer *I* to the delicate tangential contact of the pinhead *J* with its reflection in the polished top of the micrometer screw, a series of readings may be obtained, indicating the range of stress in the column. A thermometer *K* was clamped to the column in the position shown and at its base a wooden cup *L*, with an open face next the column surface, was pasted and clamped to the column around the mercurial bowl of the thermometer and this cup was then filled with mercury so as to give as correct a value of column temperature variation for the height *AB* as possible. The wooden rod *CD* will be subjected to temperature correction as will also the steel column and the temperature variation for it is taken the same as indicated by the thermometer for the column. The coefficient of expansion for the wooden rod will be about one third of that for steel, according to Trautwine, and therefore, for a variation of column temperature of 1° F. the coefficient of expansion used in correcting the readings of the micrometer *I* for temperature variation will be taken as $\frac{2}{3} \times 0.0000067$.

It is realized that the varying relative humidity might affect the length of the wooden rod *CD* and the writer has been unable to find records of tests for the variation of length due to such a cause; the only related data found was a series of saturation tests (water soaked for 37 days) by De Volson Wood. These gave an elongation for saturation of 0.00065 of the length, for pine. Wood will absorb as high as 50 per cent. of its dry weight of water and from 8 to 16







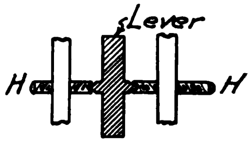


FIG. 26 a.

FIGURE 26

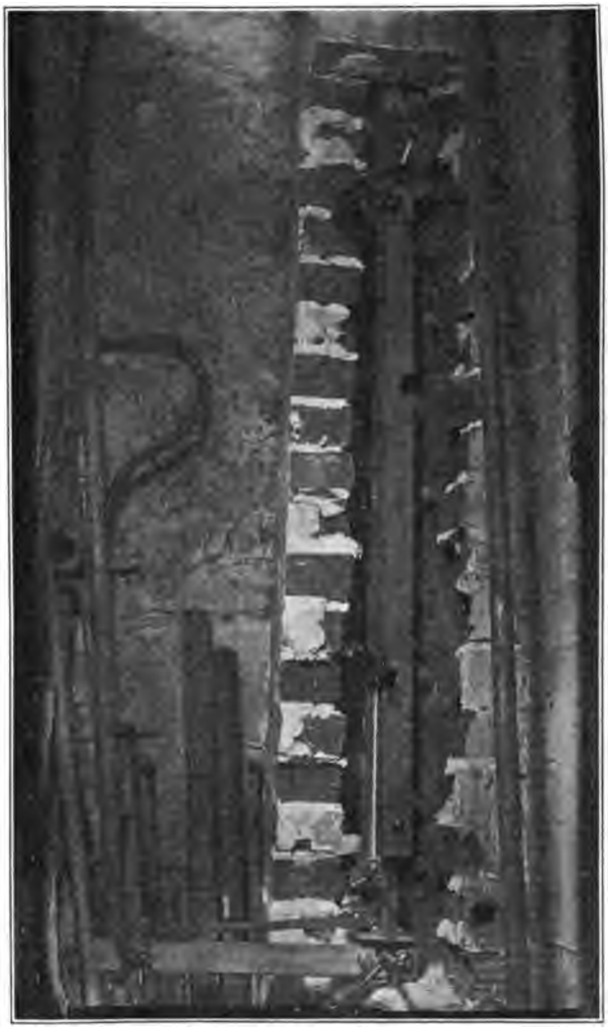


FIG. 27.

per cent. from the atmosphere. The conditions of the outside air, however, are far different from those in the basement of this structure, in which are no windows or doors communicating directly with the outside air. In addition, natural gas is burned under the boilers nearby the columns tested and a high and nearly constant temperature is maintained in this part of the building. It is considered in the absence of tests that the small changes in humidity will cause no appreciable change in length of the rod *CD*.

It has been suggested by Professor Boyd that an invar rod be used for *CD* in place of the wooden one now used; this would eliminate both the temperature correction of the extensometer and all uncertainty due to humidity.

In the experiments on Column 13 difficulty was encountered in satisfactorily clamping the brackets to the column. This was due to the presence of a one inch wrought iron water pipe *M* and a three inch cast iron water pipe *N*, as shown in Fig. 26. Wooden wedges *P* were driven in between the pipe *M* and the column and the clamps then extended from the brackets around the pipe *M*, a wooden block *Q* being placed over the bracket at *A* as a further precaution. The pipe *M* was always cold.

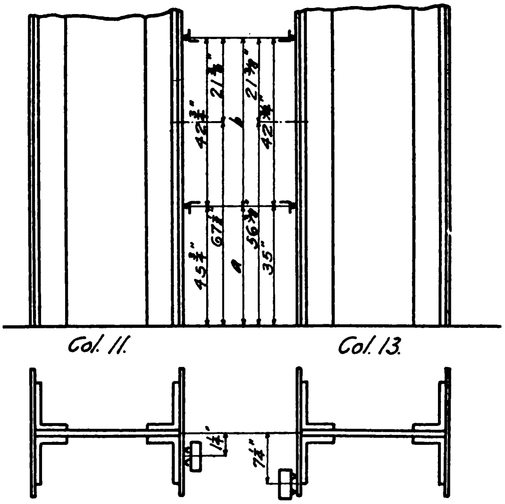


Fig. 28-a

Fig 28-b

Connections were made directly to Column 11 but no mercury contact was provided for the thermometer in that case.

The position of extensometer attachments to Columns 11 and 13 is shown in Figs. 22, 28, *a*, and 28, *b*, respectively.

Weather Bureau Records.—The station anemometer and weather vane for this vicinity are located on the rear of the top of structure *A* (see Figs. 17 and 18). On the first of the month visits were made to the Weather Bureau Office and the indicated wind velocities and wind directions were recorded for the time of each micrometer and theodolite reading taken. Also, the thermograph records were carefully traced on transparent paper, later to be reconstructed on another scale, using vertical ordinates.

ANTICIPATED AND MEASURED RESULTS.

Deflection and Vibration—Anticipated Results.—It has been shown in Chapter X that the deflections of a structure as computed by the methods of Chapter II are very much less than when computed by the methods of Chapter III.

As has also been shown, the method of Chapter III is very laborious and therefore, in calculating anticipated deflections the methods of Chapter II will be used.

Structures are usually designed for a uniform wind pressure of 30 pounds per square foot of surface and the deflection of the top of the building will be computed for this pressure. It will also be computed for the maximum wind pressure experienced during these investigations. This was for an indicated northwest wind velocity of 44 miles per hour, which interpolating in the table of Chapter 13 is equivalent to an average pressure of 5.33 pounds per square foot. Resolving into north and west components gives a west wind of $0.707 \times 5.33 = 3.77$ pounds per sq. ft. Allowing an increase of 35 per cent for gusts and using a coefficient of 80 per cent for the wall surface, there results a pressure of $3.77 \times 1.35 \times 0.80 = 4.06$ pounds per square foot. The total deflection will then be calculated for pressures of 30 and 4.06 pounds per square foot.

Referring now to Fig. 22 the heights of the stories and the depths of the girders will be found, and Fig. 19 gives the widths of the bays and the spacing of the columns.

First it will be considered that the section through columns 11, 12, and 13 is thoroughly portal braced. Then the wind on a section 16 ft. $4\frac{1}{8}$ in. wide will be assumed to be transferred to these columns. The values of the deflections for 30 pounds per sq. ft. are then as follows:

Story.	C	F	ΣF	S''	δ''
16	126	10,903	10,903	3,635	727,500
15	126	5,884	16,787	5,075	1,016,000
14	126	5,884	22,671	6,855	1,372,000
13	126	5,884	28,555	8,615	1,276,000
12	126	5,884	34,439	10,390	1,539,000
11	126	5,884	40,323	12,500	1,420,000
10	126	5,884	46,207	14,320	1,628,000
9	126	5,884	52,091	16,400	1,521,000
8	126	5,884	57,975	18,260	1,694,000
7	126	5,884	63,859	17,800	1,518,000
6	126	5,884	69,743	19,440	1,659,000
5	126	5,884	75,627	19,500	1,539,000
4	126	5,884	81,511	21,020	1,660,000
3	126	5,884	87,395	28,010	1,583,000
2	126	5,884	93,279	29,900	1,690,000
1	165	6,683	99,962	31,550	3,580,000
0	150.5	7,180	107,142	33,800	2,915,000

$\Sigma = 28{,}337{,}000$ in.

Thus the total deflection at the seventeenth floor for a pressure of 30 pounds per square foot would be only

$$\frac{28{,}337{,}000}{28{,}000{,}000} = 1.012 \text{ in.}$$

Then, for a pressure of only 4.06 pounds per sq. ft. the deflection would be

$$\frac{4.06}{30} \times 1.012 = 0.136 \text{ in.}$$

Now, if the same ratio (1 : 3.08) between the results of the methods of Chapters II and III respectively, as was found in Chapter X be considered to exist here, the value of the above deflections would be 3.12 in. and 0.420 in. respectively.

Again, suppose that the sections in the north and south walls of the structure are the only sections resisting wind pressure and that the resistance of the walls be neglected. As was noticed in Chapter XIV the section areas of the columns 14, 15, 16, and 17 are about the same as those of columns 11, 12, and 13; let them, for these calculations, be assumed all equal and the same as column 11. Remembering that S^{14} will then equal $\Sigma F/4$, the deflections will be as follows:

Story.	C	F	ΣF	S^{14}	δ^{14}
16	114	13,525	13,525	3,381	502,000
15	114	11,768	25,293	6,323	937,500
14	114	11,768	37,061	9,265	1,374,000
13	114	11,768	48,829	12,207	1,340,000
12	114	11,768	60,597	15,149	1,664,000
11	114	11,768	72,365	18,091	1,524,000
10	114	11,768	84,133	21,033	1,772,000
9	114	11,768	95,901	23,975	1,650,000
8	114	11,768	107,669	26,917	1,850,000
7	88.5	11,768	119,437	29,859	880,000
6	88.5	11,768	131,205	32,801	967,000
5	88.5	11,768	142,937	35,743	976,000
4	88.5	11,515	154,488	38,622	1,054,000
3	82.5	11,768	166,256	41,564	657,000
2	82.5	11,768	178,024	44,506	705,000
1	145	14,305	192,329	48,082	3,700,000
0	135	15,420	207,749	51,937	3,225,000

Σ = 24,777,000 in.

Thus the total deflection at the seventeenth floor for a pressure of 30 pounds per square foot would be only

$$\frac{24{,}777{,}000}{28{,}000{,}000} = 0.885 \text{ in.}$$

Then, for a pressure of only 4.06 pounds per sq. ft. the deflection would be

$$\frac{4.06}{30} \times 0.885 = 0.120 \text{ in.}$$

Now, under the assumption that the floors carry all the wind pressure to the ends of the building it will be seen from this table that the total wind load carried by any one floor is only about 24,000 pounds, and as the floors form very deep wind girders the deflection of the floors at the section through columns 11, 12, and 13 will certainly be very small. Suppose it to be negligible; then from the above figures it will be seen that, in this particular design, practically the same deflection at the section through columns 11, 12, and 13 is obtained whether this section is assumed as strictly portal braced or with no bracing whatever.

The only records of tests on tall buildings that the writer has been able to find are those on the Monadnock and Pontiac Buildings in Chicago, the first named building being a seventeen story structure which might be considered as really two structures, one of veneer cage construction and the other of skeleton construction with solid brick walls six feet thick at the sidewalk. The Pontiac building is a fourteen story one of the skeleton construction type. The maximum vibrations were observed on the Monadnock building and ranged from $\frac{1}{4}$ in. to $\frac{1}{2}$ in., the larger values being for the portion of the veneer construction type. The wind had an indicated velocity of 80 miles an hour and came from the northwest and the exposed portion of the building considered faced west. This wind is equivalent to

$$15.5 \times 0.707 \times 1.35 \times 0.80 = 11.80 \text{ pounds per square foot.}$$

It will be noticed that such a pressure on the seventeen story building considered in this Chapter would cause a maximum deflection of

$$\frac{11.80}{30} \times 1.012 = 0.394 \text{ in.}$$

under the most favorable condition for deflection, *i. e.*, that section 11–12–13 is rigidly braced.

There is no great discrepancy between this vibration of $2 \times 0.39 = 0.78$ in. and the $\frac{1}{2}$ in. observed vibration and when it is remembered that the sizes of the columns, their unsupported height and the number of columns in a cross section are the factors affecting the amount of the deflections and that when the structure is calculated as an entirely elastic one the deflections would not likely exceed $3.08 \times 0.394 = 1.21$ in., it can only be said that these actual observations merely furnish a reasonable estimate of the value of walls and partitions in resisting wind pressure and do not seem to furnish such a disagreement as Freitag in "Architectural Engineering" points out.

Although the writer has no data on the frame of the Monadnock block, he believes his estimates above to be large, as the Monadnock block is a wider structure than that considered in this chapter.

Date.	Time.	Ind. V.	Direct.	Dir. Read.	Rev. Read.	Time of Vib.	Amt. of Vib.	Act. V.	Resolved V.	Remarks.
2–28	3:10 P.M.	9	N.W.	+0.09		Not noticeable	Not noticeable	8.6	6.1	On west end of old signal.
3–2	9:35 A.M.	20	W.	−0.06		Not noticeable	Not noticeable	17.8	17.8	On west end of old signal.
3–4	8:45 A.M.	16	N.N.W.	+6.46		Not noticeable	Not noticeable	14.4	5.5	On east end of old signal.
3–4	8:50 A.M.	16	N.N.W.		+6.92	Not noticeable	Not noticeable	14.4	5.5	On east end of old signal.
3–8	3:45 P.M.	5.5	S.	+2.06		Not noticeable	Not noticeable	5.5	0	Remaining obs. on signal of Fig. 25, *a*.
3–8	3:50 P.M.	5.5	S.		+2.44	Not noticeable	Not noticeable	5.5	0	
3–15	10:15 A.M.	36	N.W.	+2.12		Ab't 8 sec.	Ab't ⅜ in.	30.2	21.3	
3–15	10:20 A.M.	38	N.W.		+2.31	Ab't 8 sec.	Ab't ⅜ in.	31.6	22.4	
3–15	10:30 A.M.	44	N.W.		+2.46	Ab't 4 sec.	Ab't ½ in.	36.3	25.6	
3–15	3:50 P.M.	41	N.W.	+1.81		From 8 to 12 sec.	From ¼ to ⅜ in.	34.0	24.0	
3–15	3:54 P.M.	41	N.W.		+2.38			34.0	24.0	
3–15	4:05 P.M.	34	N.W.		+2.12			28.7	20.3	
3–15	4:10 P.M.	27	N.W.	+1.62				23.3	16.5	
3–27	9:04 P.M.	32	W.	+1.38		About 8 sec.	Avge. ⅛ in. Max. ¼ in.	27.2	27.2	At 8–30 before ready for observing noted vibration of about ½ in. Ind. V = 24 Mi. west.
3–27	9:12 P.M.	37	N.W.		+1.88			30.7	21.7	
3–27	9:18 P.M.	35	N.W.	+1.31				29.6	20.9	
3–27	9:31 P.M.	35	W.		+1.75			29.6	29.6	
3–31	4:20 P.M.	5	W.	+1.67		Not noticeable	Not noticeable	5.0	5.0	
4–5	7:55 A.M.	22	S.W.	+1.71		Not noticeable	Not noticeable	19.4	13.7	

A tall building is such a composite structure that the time of vibration can not be closely estimated and no attempt is here made to do so. The time of vibration given for the Chicago experiments was two seconds.

Results as Observed on Structure A.—The records of observations for deflection and vibration, as made by the writer are as given in the table on page 205.

From the above table it will be seen that the largest indicated velocity during the observations was 44 miles per hour, equivalent to a pressure of 4.06 pounds per square foot normal to the west wall. The largest observed vibration was of a momentary nature and not greater than ½ in. The times of vibration seem quite variable and in fact there was evidence that any one vibration as observed was itself made up of a number of much smaller and more rapid vibrations. The times of vibration seem quite variable and do not seem closely related to the indicated velocities. It is known that the time of vibration should theoretically vary about as the square root of the pressure or almost directly as the velocity and about as the three halves power of the height of the structure for a comparison of different structures. This is on the assumption that the structure acts like a beam, an assumption which has been shown to be far from correct. As noted in the remarks column of the above table the first four readings of signal Fig. 25, *b*, were from sights on an old signal in place of that of Fig. 25, *a*, and are not related to the following readings. The readings on the signal of Fig. 25, *b*, have a variation of ¾ in., both in the direct and reversed readings. The writer can only account for this by assuming that the effects of temperature, described later, caused the building to lean; this seems especially probable since the results indicate a leaning to the west on March 27 of ¾ in. from the readings of March 15 under almost identical conditions of the wind. Striding level tests showed only very small settlements of the theodolite and could not account for more than $\frac{1}{16}$ in. variation, as tested.

Assuming a maximum vibration of ½ in., as found above, for a pressure of 4.06 pounds per square foot, this vibration will be closely twice the deflection from the normal position, since the vibrations pass back and forth through this position; therefore the observed deflection for the top of the structure under a pressure of 4.06 pounds per sq. ft. may be taken as 0.25 in. By the methods of Chapter II it was found that this deflection would be 0.14 in. and by Chapter III about 0.42 in. These figures take into account the effect of gusts and indicate that the walls and partitions reduce the deflections about 40 per cent. In other words, the walls and partitions in this structure seem to be secondary and not primary agencies in resisting distortion due to wind. This is certainly a highly desirable feature in a tall building. The dependence on the walls and partitions is probably even less than given here, for it is believed that the ratio 1 : 3.08 from Chapter X, will be found to be too small for higher structures than the five story one, of only two columns to a cross section, to which it refers.

COLUMN STRESSES—ANTICIPATED RESULTS.

The unit stresses at the position of the extensometers on columns 11 and 13 will now be computed by the method of Chapter II for pressures of 30 and 4.06 pounds per square foot.

For 30 pounds, the unit column stresses at the points of contraflexure (half the unsupported height from the casting) are as in the following table.

X	Y	h^{11}	h^{12}	h^{13}	s	s^{11}
+140,400,000	0	+215.8	+46.1	+236.1	+23.14	−4,990
X	Y	s^{12}	s^{13}	V^{11}	V^{12}	V^{13}
+140,400,000	0	−1,066	+5,465	−274,000	−58,500	+332,500

The extensometer on column 11 was placed 7.13 in. and that on column 13, 17.87 in. below the points of contraflexure and on the inner extreme fibres of the columns. If these distances are designated as "*z*," then for both of these columns the extreme fibre unit stresses will be less than those given in the above table by the amount $Sz \cdot v/I$. The total wind shear on the section is 107,142 pounds, as already given. Then

$$S^{11} = 33{,}800 \text{ ℔}, \quad S^{12} = 33{,}800 \text{ ℔}, \quad \text{and} \quad S^{13} = 39{,}540 \text{ ℔}.$$

The extreme fibre stress for column 11 at the extensometer will be

$$-4{,}990+\frac{33{,}800\times7.13\times9.44}{3{,}300.6}=-4{,}990+689=-4{,}201 \text{ lb per sq. in.}$$

For column 13 the stress would be

$$+5{,}465-\frac{39{,}540\times17.87\times9.62}{3{,}844.1}=+5{,}465-1{,}767$$
$$=+3{,}698 \text{ lb per sq. in.}$$

Now for a maximum pressure of 4.06 pounds per square foot these quantities become

$$\frac{4.06}{30}\times-4{,}201=-568 \text{ lb per sq. in.}$$

and

$$\frac{4.06}{30}\times3{,}698=+500 \text{ lb per sq. in.}$$

respectively.

It is now interesting to note a few of the relations concerning the interpretation of the micrometer readings.

The smallest reading of the micrometer is 0.001 in.

The extensometer magnifies changes in length 50 times.

One division of the micrometer (0.001 in.) is equivalent to

$$\frac{0.001\times28{,}000{,}000}{50\times42.75}=13.10 \text{ lb per sq. in.}$$

of unit stress in the column.

One degree change in temperature is equivalent to

$$\frac{0.0000067\times42.75\times50}{0.001}=14.4$$

divisions of the micrometer, or two-thirds of this, equal to 9.57 divisions when the simultaneous expansion of the wooden piston *CQ* is allowed for.

Also, one degree change of temperature is equivalent to 9.57 × 13.10 = 125.3 pounds per sq. in. of change of unit stress in the column.

The maximum number of divisions of the micrometer to be expected to be recorded under wind pressure would then be only

$$\frac{568}{13.10} = 43.4$$

(twice this, allowing for vibration back and forth).

An error of about 8° F. in column temperature or possibly a small change in relative humidity would entirely vitiate these results.

It was shown by the method of Chapter III that the points of contraflexure fall considerably above the mid height. Suppose they occur at the three quarter point; then the stresses at the extensometers on columns 11 and 13 become, for 30 pounds per sq. ft.

$$-4{,}990 + 33{,}800 \times \frac{44.25 \times 9.44}{3{,}300.6} = -700 \text{ lb per sq. in.}$$

and

$$+5{,}465 - \frac{39{,}540 \times 55.00 \times 9.62}{3{,}844.1} = +15 \text{ lb per sq. in.}$$

respectively. For a pressure of 4.06 pounds these stresses are only -95 and $+2$ lb per sq. in. respectively. In this case the stress would hardly be noticeable in column 11 and certainly not in evidence in column 13.

The rigidity of the connections of the 18 in. I beam floor girders to column 11 will now be investigated. The shear in the column in the basement story, for a 30 pound pressure, is 33,800 lb and in the first story 31,550 lb, as given in previous table. Thus the connection must be able to resist a bending moment of

$$31{,}550 \times 91.5 + 33{,}800 \times 84.25 = 5{,}735{,}000 \text{ in. lb.}$$

For a pressure of 4.06 pounds the connection must resist a moment of

$$\frac{4.06}{30} \times 5{,}735{,}000 = 775{,}000 \text{ in. lb.}$$

Referring now to Fig. 23 the rivets and spacing used for this connection will be found. The strength of the connection evidently depends on the value of a rivet in direct tension and experience has shown that no definite distinction can be made between shop and field driven rivets in this respect. Consider that the wind tends to bend the column around the toe of the upper connection angle. Then, if s represents the value of the rivet in the vertical leg of the lower connection angles and the value of the other rivets is assumed proportional to the distance from the toe of the top angle, the moment of resistance of the connection will be

$$\frac{s}{23.75}[4 \times \overline{1.5}^2 + 2 \times \overline{6.5}^2 + 2 \times \overline{9.25}^2 + 2 \times \overline{12.00}^2 + 2^2 \times \overline{15.75}^2 + 2 \times \overline{18.75}^2 + 4 \times \overline{23.75}^2] = 169s.$$

The turning moment around the toe of the top angle will be a little greater than that previously deduced for the neutral axis, but the difference is not great and it is close enough to say that

$$169s = 775{,}000$$

or

$$s = 4{,}580 \text{ lb},$$

this being then the maximum stress on a rivet. All rivets are ¾ in. It is well known that a rivet may be worth as low as zero for this type of stress or may stand a much higher stress. It is to be realized that the resultant stress on these rivets will be higher than 4,580 lb because of the vertical shearing stress which they receive. Another item to consider is that such a stress as this on the rivet will cause the joint to give sufficiently to destroy to a great extent the fixed condition assumed to start with.

These figures all indicate that the sections in the north and south walls furnish the most of the resistance to wind; especially must this be true for much higher pressures than 4.06 pounds per square foot.

The following table gives the results obtained in these experiments.

For Column 11.											
Date.	Time.	Col. T.	Microm.	Ind. V.	Direction.	Actual V.	Microm. Corr. to 66°.		Value in Lbs. per Sq. In.	W. Comp. of V.	Remarks.
3–1	8:10	64.6°	0.558	9	W.	See Col. 13 as the times of obs. are about the same.	0.571	Subtract 0.200 from all readings before converting into stress.	4,860	See Col. 13.	
1	4:40	66.2	0.546	18.5	W.S.W.		0.544		4,510		
2	8:35	64.0	0.515	22	W.		0.534		4,375		
2	4:00	64.7	0.507	25	W.		0.519		4,180		
3	3:05	66.0	0.477	9	W.		0.477		3,630		
4	8:10	67.8	0.466	13	N.W.		0.449		3,260		
6	7:50	68.0	0.496	18	N.		0.477		3,628		
6	4:10	68.7	*0.521*	14	N.E.		0.495		3,863		Reset microm. Reads 0.393 after.
7	7:20	67.9	0.458	11	S.E.		0.568		4,825		
7	4:15	69.0	0.443	15	N.E.		0.542		4,480		
8	7:35	67.7	0.426	8.5	N.N.E.		0.538		4,427		
8	4:20	69.1	0.378	5	S.S.E.		0.476		3,617		
9	8:00	69.5	0.320	10	S.S.E.		0.414		2,804		
9	3:50	70.9	0.283	12	S.		0.364		2,147		
10	7:30	71.5	0.212	5	W.		0.287		1,140		
10	3:56	71.7	0.203	5	W.		0.276		995		Tested pivots. Assume readings identical after reset.
10	4:40	72.5	0.324	5	W.		0.276		995		
11	8:19	69.4	*0.446*	20	E.N.E.		0.427		2,973		Reset microm. Reads 0.394 after.
13	7:35	68.7	0.305	9	W.		0.345		1,900		
13	4:11	71.7	0.392	16	E.N.E.		0.403		2,660		
14	7:40	70.0	0.468	4	E.		0.496		3,877		
14	3:55	69.9	0.422	5	S.W.		0.451		3,286		
15	7:36	67.8	0.371	9	W.		0.420		2,880		
15	11:00	68.0	0.351	40	N.W.		0.398		2,595		
15	3:10	67.7	0.363	40	N.W.		0.413		2,790		
16	8:00	63.3	0.413	15	W.		0.505		4,000		
16	3:30	61.6	0.430	9	W.		0.538		4,430		
17	4:09	65.8	0.440	17	S.		0.508		4,035		

For Column 13.

Date.	Time.	Col. T.	Microm.	Ind. V.	Direction.	Actual V.	Microm. Corr. to 76°.		Value in Lbs. per sq. in.	W. Comp. of V.	Remarks.
3– 1	8:00	74.0°	0.569	10	W.	9.6	0.588	Subtract 0.600 from all readings.	— 157	9.6	
1	4:45	73.8	0.598	14.5	W.S.W.	13.0	0.619		249	12.0	
2	8:25	73.0	0.608	15	W.	13.5	0.637		484	13.5	
2	3:55	73.0	0.639	27	W.	23.3	0.668		890	23.3	
3	3:00	74.8	0.664	9	W.	8.6	0.675		982	8.6	
4	7:55	75.7	0.656	12.5	N.W.	11.7	0.659		773	8.3	
6	7:40	75.8	0.670	20	N.	17.8	0.672		943	0	Reset microm. Reads 0.315 after.
6	4:05	76.2	0.328	10	N.E.	9.6	0.681		1,061	— 6.8	
7	7:15	75.2	0.352	15	S.E.	13.5	0.715		1,506	— 9.6	
7	4:05	76.0	0.362	15	N.E.	13.5	0.717		1,533	— 9.6	
8	7:30	74.2	0.335	9	E.N.E.	8.6	0.707		1,402	— 8.0	
8	4:30	76.5	0.362	5	S.S.E.	5	0.712		1,467	— 1.9	
9	7:55	76.2	0.366	10	S.S.E.	9.6	0.719		1,558	— 3.7	
9	3:45	77.4	0.361	14	S.	12.6	0.703		1,349	0	
10	7:25	77.0	0.313	5	W.	5	0.658		760	5	
10	3:50	77.2	0.302				0.645		590		
10	4:55	77.0	0.314				0.659		773		
11	8:12	76.8	0.356	16	E.N.E.	14.4	0.703		1,349	—13.4	
13	7:28	74.8	0.246	12	W.	11.5	0.613		170	11.5	
13	4:05	76.8	0.284	15	E.N.E.	13.5	0.631		406	—12.5	
14	7:30	76.3	0.309	4	E.	4	0.661		799	— 4	
14	3:50	76.7	0.322	5	S.W.	5	0.670		917	3.5	
15	7:30	76.0	0.305	9	W.	9	0.660		786	9	
15	10:40	75.6	0.318	38	N.W.	31.6	0.677		1,009	22.4	
15	3:05	75.4	0.319	40	N.W.	33.3	0.680		1,048	23.6	
16	7:53	72.4	0.374	15	W.	13.5	0.763		2,135	13.5	
16	3:22	72.0	0.397	10	W.	9.6	0.790		2,487	9.6	Changed extensometers Mar. 20. Readings *do not connect.*
17	4:03	74.0	0.470	17	S.	15.3	0.844		3,195	0	
20	4:03			23	N.W.	20.2	0.339	Add 0.100 to readings.	5,755	14.3	Rubber band placed over lever. Remains on till end.
21	7:42	76.0	0.399	2	S.E.	2	0.416		6,760	— 1.4	
21	3:42	76.0	0.399	19	W.S.W.	17.0	0.399		6,545	15.7	
22	7:41	76.0	0.361	18	W.	16.2	0.361		6,035	16.2	

For Column 13.

Date.	Time.	Col. T.	Microm.	Ind. V.	Direction.	Actual V.	Microm. Corr. to 76°.		Value in Lbs. per Sq. In.	W. Comp. of V.	Remarks.
3–22	3:32	76.6°	0.343	34	N.	28.7	0.337	Add 0.100 to all readings before converting into unit stress.	5,725	0	
23	7:44	73.7	0.336	20	N.W.	17.8	0.358		6,000	12.6	
23	3:49	73.1	0.343	20	N.	17.8	0.371		6,165	0	
24	8:04	72.3	0.376	9	N.E.	9	0.411		6,700	– 6.4	Quick drop of lever soon as plunger was pushed up.
25	7:38	72.0	0.453	7	S.E.	7	0.491		7,740	– 5.0	
27	7:48	76.8	0.343	30	S.S.W.	25.7	0.335		5,700	9.9	
27	10:22	76.7	0.338	33	W.	28.0	0.331		5,645	28.0	Rubber band on 30 min. tests. Less than 0.001 in. movement under steady observation.
27	11:00	76.7	0.343	28	S.W.	24.1	0.336		5,715	17.0	
27	4:15	75.9	0.341	15	W.	13.5	0.342		5,785	13.5	
27	9:47 P.M.	75.0	0.347	20	N.W.	17.8	0.357		5,980	12.6	
28	8:13	72.7	0.392	13	W.	12.5	0.424		6,865	12.5	
28	3:46	73.0	0.414	5	W.	5	0.443		7,120	5	
29	7:47	73.1	0.429	9	E.	9	0.457		7,300	– 9	
29	3:33	74.0	0.414	2	W.	2	0.433		6,985	2	
30	7:52	73.7	0.376	10	W.	9.6	0.398		6,530	9.6	
30	3:37	73.4	0.371	9	N.W.	9	0.396		6,500	6.4	
31	7:03	72.8	0.379	5	W.	5	0.410		6,685	5	
31	3:40	73.0	0.415	5	W.	5	0.444		7,130	5	
4– 1	11:40	73.0	0.431	7	N.W.	7	0.460		7,340	5	
3	7:30	72.8	0.534	22	E.	19.4	0.565		8,710	–19.4	
3	7:37	72.8	0.536	22	E.	19.4	0.567		8,735	–19.4	
3	7:47	72.8	0.534	22	E.	19.4	0.565		8,710	–19.4	
3	4:07	73.8	0.534	11	E.	10.5	0.555		8,580	–10.5	
4	8:00	74.7	0.507	10	S.E.	9.6	0.519		8,110	– 6.8	
4	3:35	76.0	0.468	15	S.E.	13.7	0.468		7,450	– 9.7	
5	7:25	76.8	0.314	27	S.W.	23.3	0.306		5,315	16.5	
5	8:10	76.8	0.312	20	S.W.	17.8	0.304		5,295	12.6	
5	3:28	77.9	0.313	22	W.	19.4	0.295		5,175	19.4	
6	7:22	77.5	0.314	12	S.	11.5	0.300		5,240	0	
6	3:19	77.7	0.305	22	W.	19.4	0.289		5,100	19.4	
7	7:40	75.0	0.279	5	N.	5.0	0.289		5,100	0	

For Column 13.

Date.	Times.	Col. T.	Microm.	Ind. V.	Direction.	Actual V.	Microm. Corr. to 76°.		Value in Lbs. per Sq. In.	W. Comp. of V.	Remarks.
4–7	3:21	75.5°	0.279	4	N.	4.0	0.284	Add 0.100 to all readings before converting into unit stress.	5,030	0	Top clamp worked loose. Reset 0.258
8	7:37	75.0	0.284	7	N.	7.0	0.315		5,435	0	
8	2:56	75.4	0.310	20	N.	17.8	0.337		5,725	0	
10	7:05	75.0	0.369	12	E.	11.5	0.400		6,550	−11.5	
10	3:53	76.0	0.408	12	S.E.	11.5	0.429		6,935	− 8.2	
11	7:25	76.0	0.422	10	S.E.	9.6	0.443		7,115	− 6.8	
11	3:49	76.8	0.448	11	S.E.	10.5	0.461		7,350	− 7.5	
12	8:09	75.5	0.325	20	S.E.	17.8	0.351		5,905	−12.6	
12	3:30	76.5	0.309	17	S.	15.3	0.325		5,570	0	
13	7:40	76.5	0.272	17	S.E.	15.3	0.288		5,085	−10.8	
13	3:00	77.8	0.245	20	S.	17.8	0.249		4,570	0	
14	7:27	77.5	0.179	26	N.W.	22.4	0.186		3,747	15.9	0.287 = Reading after resetting micrometer.
14	7:33	77.5	0.287	31	N.	26.5	0.186		3,747	0	
14	3:47	77.5	0.284	15	N.W.	13.5	0.183		3,707	9.6	
15	7:21	76.0	0.283	14	W.	12.5	0.196		3,880	12.5	
17	7:04	76.0	0.376	10	N.	9.6	0.289		5,100	0	
17	3:58	76.3	0.379	12	N.	11.5	0.289		5,100	0	
18	7:03	76.0	0.406	7	S.E.	7.0	0.319		5,490	− 5.0	
18	3:26	76.8	0.403	10	S.	9.6	0.308		5,350	0	
19	7:42	76.0	0.400	15	S.E.	13.5	0.313		5,420	− 9.6	
19	3:10	77.3	0.361	8	W.	8	0.262		4,740	8.0	
20	6:46	76.7	0.316	20	N.	17.8	0.222		4,220	0	
20	3:28	77.2	0.316	15	N.	13.5	0.217		4,150	0	
21	7:35	77.2	0.331	4	N.	4.0	0.232		4,350	0	
21	3:10	78.0	0.335	10	N.	9.6	0.229		4,310	0	
22	7:12	76.0	0.335	16	N.	14.4	0.248		4,560	0	
22	3:30	76.0	0.335	20	N.	17.8	0.248		4,560	0	
24	7:10	75.0	0.383	9	N.	9	0.306		5,320	0	
24	3:48	76.3	0.428	13	N.	12.5	0.338		5,745	0	
25	7:14	75.2	0.427	8	N.	8	0.348		5,875	0	
25	2:38	76.5	0.480	5	E.	5	0.388		6,400	− 5	

For Column 13.

Date.	Times.	Col. T.	Microm.	Ind. V.	Direction.	Actual F.	Microm. Corr. to 76°.		Value in Lbs. per Sq. In.	W. Comp. of V	Remarks.
4–26	3:32	76.0°	0.529	5	S.E.	5	0.442	Add 0.100 to reading.	7,100	− 3.5	
27	7:17	76.0	0.464	10	S.E.	9.6	0.377		6,250	− 6.8	
27	3:34	75.0	0.477	10	S.E.	9.6	0.400		6,550	− 6.8	
28	7:35	75.5	0.388	7	S.	7	0.306		5,315	0	
28	3:41	75.0	0.353	11	S.W.	10.4	0.276		4,930	7.4	
29	7:22	75.0	0.230	12	S.	11.5	0.153		3,315	0	
29	4:46	75.0	0.185	10	S.S.E.	9.6	0.108		2,723	− 3.7	Microm. reset. Reads 0.463 after.
5– 1	9:08	75.0	0.334	23	S.	20.2	−0.021		1,035	0	
1	4:08	74.7	0.330	52	W.	42.2	−0.023		1,008	42.2	7.17 lbs. per sq. feet press. on west wall. No variation noticeable.
2	8:08	72.5	0.324	14	N.N.W.	13.4	−0.007		1,219		
2	3:44	73.0	0.322	13	N.	12.5	−0.014		1,127	0	
3	7:40	73.2	0.324	3	N.	3	−0.014		1,127	0	
3	3:31	74.3	0.322	9	N.W.	9	−0.026		970	6.3	
4	7:19	74.0	0.322	6	N.	6	−0.024		997	0	
4	2:11	75.2	0.319	10	N.	9.6	−0.038		813	0	

N. B. An accident, May 1, prevented vibration and deflection readings during highest wind.

DISCUSSION OF RESULTS.

It is very quickly apparent that there are very large unit stresses in both these columns which can not in any way be attributed to wind pressure. In fact, during each high wind recorded above careful readings were made on the micrometers and no variations of as much as 0.001 in. could be detected over periods of thirty minutes of observation, on either Column 11 or 13. The writer noticed however that *after* a high wind the reading would usually increase with a jump; this, together with the fact that a high wind was usually followed by colder weather, suggested temperature variation as the cause of the stress variation in the column. By this is meant the variation in the temperature of the outside air and not the column temperature, which has already been corrected for, as shown in the above tables. In order to present the relation between the variation in column stress and the atmospheric temperature the curves of Fig. 29 have been constructed. The solid irregular series of straight lines represents the variation of stress in Column 13 and the solid irregular curved line represents the variation of temperature. The curve of wind velocities has not been shown since there can be no doubt but that it is not at all related to the stress curve.

An attempt will now be made to explain the curve for column 13. Column 11 is in the west wall of the building, while Column 13 is in the party wall between structures *A* and *B*. Any change of outside temperature will affect column 11 before reaching column 13, even though there is a small air space between the walls of the two structures *A* and *B*. Now, when a decrease of temperature was experienced it was noted that column 13 stretched, *i. e.*, that the micrometer readings increased, and vice versa for an increase of temperature. Another fact noted was that the change in the column stress did not take place immediately after a change of temperature, but that there appeared to be a lag in the effect on the column. In Fig. 29 the dotted line shown is an attempt to average the temperature curve: the intersections of the straight average temperature slopes are lettered and on the stress line for column 13 are marked by these letters, with a prime, the points which the writer considers to correspond with the same lettered

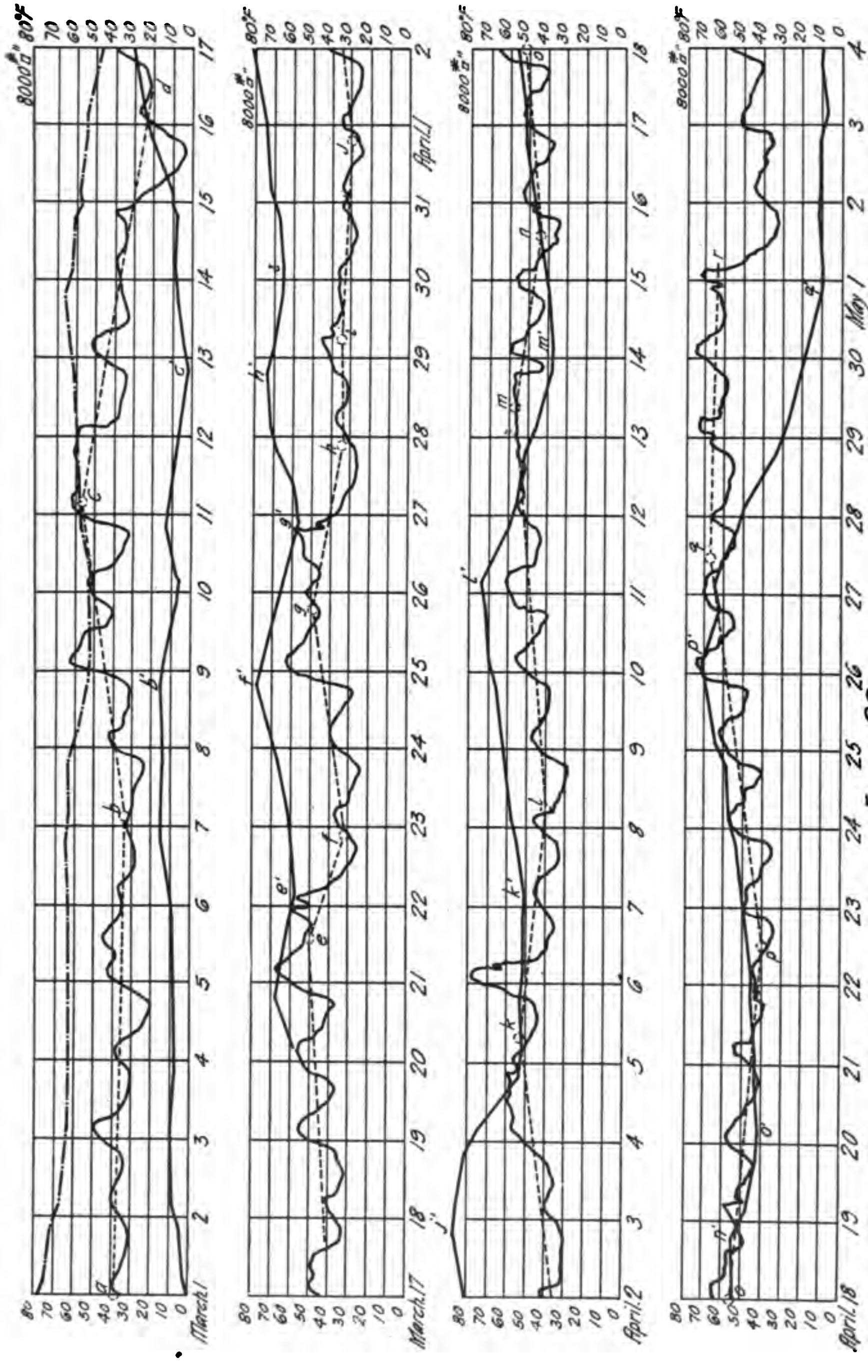

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FIGURE 29

point on the temperature line. Thus *l'* on the stress curve corresponds to *l* on the line of average temperature slopes. This means that the temperature, starting on a gradual increase at 6 P. M., April 8, does not affect column 13 until 4 P. M., April 11, or almost three days later, when the stress change becomes compressive. Stresses measured upwards in Fig. 29 are really tensile or represent a decrease in the column unit compressive stress. Thus, for every rise in the average temperature line there should be a corresponding fall in the stress curve and this seems to be proven in the figure without an exception. The amount of lag will be seen to vary from about a day to three days. Large changes of temperature of only short duration seem to have little effect on the column, but a steady change extending over about a week's time may cause a most serious range of column stress. For example note the effect of the warm wave of April 22–27 and the effect on the column 13 from April 26 to May 1, a range of over 6,000 pounds per square inch. These stresses are not only large, but they are alternating and have an effect similar to varying the load on the structure. The creaking of this building has been a very noticeable feature since its erection, and this is undoubtedly merely audible evidence of the stresses which must exist.

Now, since the instruments were only applied to the inner extreme fibre of the column, there is no way to tell whether the fibre stresses recorded above are due to bending or due to a uniform axial column stress. The writer assumes them to be uniform axial column stresses for the following reasons:

It has been shown from Fig. 29 that a drop in temperature will, after a lapse of from one to three days, produce tensile increments or a decrease in compressive stresses and likewise a rise in temperature causes compressive increments. Referring now to Fig. 30, suppose Column 11 to be shortened by the decrease of temperature which does not, however, directly affect column 13, since it is protected by structure *B*. If no resistance were offered to this shortening, the amount of the shortening would be *AA'*, say: but the point *B*, on column 12, acts as a fulcrum for the lever composed of the girders *AB* and *BC* and, while there will be sufficient freedom in the connections of the beams to allow column 11 to contract almost freely for a certain distance, when this limit is reached the lever *AC* comes into play and any further contraction will produce tensile increments, or a stretch, in Column 13, as was found in the experiments for that column. As soon as stress starts up in column 13, a similar stress (tensile), equal to b'/b times the total induced stress in column 13 will exist in column 11. This tensile stress in column 11 will offset part of the contraction of that column and an extensometer applied to the column will register the real or resultant shortening of the column. It may then be said that the free contraction that would ensue from temperature alone equals the shortening as shown by the extensometer plus the tension elongation due to b'/b times the total induced stress in column 13. There will also be some energy taken up in deforming the beam *AC*. It will be seen then that when the temperature falls Column 13 will not be affected until the freedom in the connections is first taken up and then the flexure of the beam *AC* will continue to cause a small lag in the effect on column 13. Similar results would ensue if the temperature were to rise. In the following table the stress change in Column 13 has been multiplied by b'/b times the ratio of the area of Column 13 to that of column 11 (basement story) and added to the "stresses" as already given for column 11, after first subtracting these latter figures from 5,150, as so to make the resultant reading for March 1, at 8 : 10, zero. These resultant unit stresses have been converted into degrees of temperature (F.) by dividing by

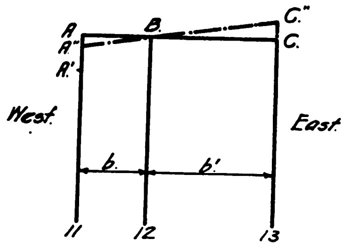

FIG 30

$$\frac{0.0000067 \times 42.75 \times 13.1 \times 50}{0.001} = 188.$$

These figures then represent the fall of temperature required to produce the effects noted. Subtracting all these figures from 80° gives a curve supposed to agree closely with the temperature curve plotted from the thermograph records. It is to be noted that the experiments on Column 11 ran only up to March 17; they were then discontinued because the extensometer was thought not to be working properly.

Col. 13 Stress.	$\frac{b'}{b}\times$ Col. 13.	5150—Col. 11.	Sum.	°*F*.	80 °*F*.
− 157	− 290	+ 290	0	0°	80.0°
249	460	640	1,100	5.8	74.2
484	895	775	1,670	8.9	71.1
890	1,645	970	2,615	13.9	66.1
982	1,817	1,520	3,337	17.8	62.2
773	1,425	1,890	3,315	17.6	62.4
943	1,740	1,522	3,262	17.3	62.7
1,061	1,960	1,287	3,247	17.2	62.8
1,506	2,780	325	3,105	16.5	63.5
1,533	2,825	670	3,495	18.6	61.4
1,402	2,590	723	3,313	17.6	62.4
1,467	2,710	1,533	4,243	22.5	57.5
1,558	2,880	2,346	5,226	27.8	52.2
1,349	2,500	3,003	5,503	29.2	50.8
760	1,405	4,010	5,415	28.8	51.2
590	1,090	4,155	5,245	27.8	52.2
773	1,427	4,155	5,582	29.7	50.3
1,349	2,500	2,177	4,677	24.8	55.2
170	314	3,250	3,554	18.8	61.2
406	750	2,490	3,240	17.2	62.8
799	1,477	1,273	2,750	14.6	65.4
917	1,693	1,864	3,557	18.9	61.1
786	1,450	2,270	3,720	19.8	60.2
1,009	1,865	2,555	4,420	23.5	56.5
1,048	1,940	2,360	4,300	22.8	57.2
2,135	3,950	1,150	5,100	27.1	52.8
2,487	4,600	720	5,320	28.2	51.8
3,195	5,900	1,115	7,015	37.3	42.7

The curve just mentioned is shown in Fig. 29 by a dash-dot symbol; it does not show the intimate relation to the thermograph record that was expected and, in the absence of further experiments, the writer can make no satisfactory explanation of the disagreement. Much confidence is, however, expressed for the results of the readings on column 13 and the deductions already given. If the principle illustrated in Fig. 30 represents existing conditions properly then the Column 12 must have a range of stress of at least 22,000 pounds per square inch. It must be explained here that the writer has been handicapped for time on this work; all of the investigation results, formulae, tables, experiments and in short this entire manuscript has been prepared since Jan. 1, 1911. (Many preliminary studies, theoretical methods and attempted solutions were undertaken during the previous year, extending intermittently from September, 1909, until the date of starting this manuscript.)

In conclusion, these experiments surely indicate that a most complete and extensive series of experiments are of the highest importance and that information on the temperature stresses in tall buildings is of much greater importance than are investigations of the effect of wind pressure on such structures. Experiments for securing this information must be on a very elaborate scale and cover a period of one year to determine the general effect of seasonal changes: it is not believed, judging from Fig. 29, that a change in the season will produce, of itself, any effect, but rather that the temporary changes of temperature of several days' duration account for most of the stress produced.

CHAPTER XVI.

CONCLUSIONS.

All references in this chapter are to be understood as applying to a portal braced structure.

A. THE PRACTICAL VALUE OF A CORRECT THEORETICAL ANALYSIS OF THE STRESSES IN TALL BUILDINGS.

Referring to the plates of Chapter III and the applications in succeeding chapters it will be seen that, for the ordinary tall building, the accurate solution of stresses involves considerable labor and the *time* required for computing stresses, to say nothing of revisions after the original computations on account of revised design, makes the accurate solution of such stresses of an *entirely impractical nature.* The writer does not believe there is any shorter cut possible for the accurate determination of stresses. It is then concluded that it is impracticable to design a high building by any correct theoretical method but that, from the application of the method of Chapter III to various types of structures under various loadings, empirical rules can be deduced, which, when applied to similar cases, will give quick and closely approximate results. It is suggested that such a large work might be systematically divided up and be undertaken as undergraduate theses in technical schools and universities.

B. A SUMMARY OF THE EFFECT OF WIND ON A STRUCTURE.

(*a*) *Wind Loads.*—It has been claimed that pressures from 60 to 85 pounds per square foot may exist during a tornado (Baier on St. Louis tornado) and, while even a 30 pound per square foot pressure seems to be equivalent to a wind higher than recorded by the U. S. Weather Bureau, yet it is probable that a unit pressure of 30 pounds will continue to be provided for in designs, at least until more accurate and direct data for tornado pressures is obtainable. It is believed that, with the common working units for a 30 pound pressure, the indefinite higher pressures possible will be taken care of.

(*b*) *The Value of the Simple Method of Chapter II for designing for Wind Pressure.*—For column shears, column bending moments, and girder stresses the method of Chapter II is all that can be desired for all stories above the basement, agreements closer than 10 per cent being the rule. In the basement story the method of Chapter II gives column bending moments too low, possibly as much as 50 per cent. For single story portals this discrepancy is not very noticeable and it is larger when the girders are lacking in rigidity, *i. e.*, of small moment of inertia.

For axial column stresses, the assumption that the building acts like a beam is undoubtedly incorrect, but the writer can suggest no better substitute. A very extensive study will be required to determine the laws of variation of the axial stresses. For a two column cross section the agreement is very close but the double portal calculations of Chapter XI show that the method fails for the general case.

For building deflections the method of Chapter II always gives too small results and as shown in Chapter X the ratio may be as small as 1 : 3.08.

(*c*) *Suggestions.*—The writer would suggest that the method of Chapter II be adopted entirely for the calculations of stresses due to wind; that the bending stresses in basement columns be then increased 25 per cent, all other stresses to remain as thus calculated.

C. A SUMMARY OF THE EFFECT OF ECCENTRIC COLUMN LOADS ON A STRUCTURE.

(*a*) *The Value of the Method of Chapter II in Providing for These Loads.*—It has been shown in Chapter IX that this method gives axial column stresses nearly correct for a two column cross section. In Chapter II it was shown that it is not reasonable that the method applies when there are more than two columns in a section. It was also shown in Chapter II that this method gives zero bending moments in the columns for such loadings. It is then concluded that the method of Chapter II should not be used in providing for eccentric column loads.

(*b*) *The Value of Professor Heller's Deductions on Eccentric Column Loadings.*—Professor Heller's methods give, as shown in Chapter IV, a maximum column moment occurring at the top of the structure equal to *Pe* and at the floors a few stories above the basement and below the roof, a nearly constant maximum of about $\frac{1}{2}$ *Pe* (see Fig. 7, *b*).

In Chapter IX it was found that the maximum moment in Column *A* (− 247,049) occurred in the *second or middle story* and was equal to 0.98 *Pe*; the maximum moment in Column *B* (+ 174,108) occurred also in the *second story* and was equal to 1.24 *Pe*. At the roof with a zero eccentric load at Column *B* there was found a bending moment of + 35,109 and, for column *A*, a moment of − 44,191 equal to 0.33 *Pe*.

It is evident then that it is not safe practice to follow the conclusions deduced in Chapter IV, which are identical with Professor Heller's. It is further shown that the value of the bending moment may materially exceed even the quantity *Pe* so commonly used by designers. Also it is shown that the bending moment is not a constant throughout the story height but reverses in sign, and at the foot of the column it may be a much larger quantity, of opposite sign, than the moment at the top of the column.

(*c*) *Suggestions for Design. Axial Column Stresses.*—In the case of several columns in a cross section it is suggested that the axial load on the column, due to the eccentric load be taken as equal to the eccentric load plus the eccentricity divided by the span of the connecting floor girder times the eccentric load. This gives but very little of the load to interior columns, as seems reasonable in contra-distinction from the method of Chapter II.

Column Bending Stresses.—It is suggested that a bending moment not less than *Pe* should be provided for, and, in combining with wind and floor load stresses, this bending should be considered as existing at both top and bottom of the story and of opposite signs at these points, realizing that the stresses thus obtained may occasionally be a little small but that, for the top and bottom portions of the structure, they are probably on the safe side.

Girder Stresses.—If the girders are proportioned by the methods of Chapter II, considering that the stresses may be either plus or minus, the results will be sufficiently close (see Chapter IX).

D. A SUMMARY OF THE EFFECT OF FLOOR LOADS ON A STRUCTURE.

(*a*) *The Value of the Method of Chapter II for Designing for Floor Loads. Axial Column Stresses.*—As shown in Chapter XI, errors in the axial stresses of the main columns, on the dangerous side, for a double portal, as high as 22 per cent may occur (cf. 88,622 and 72,581). For a two column section the agreement is very close. It is not likely that errors as high as 22 per cent exist for a several column section of more than one story.

Column Bending Stresses.—The method of Chapter II gives zero bending in the columns. Here is an enormous error in the prevailing method of designing tall buildings.

Girder Stresses.—If flange sections are made uniform for the full length of the girder, then, as shown in Chapter VIII, the method of Chapter II is on the safe side in determining flange section.

(*b*) *Results Found in This Investigation. Column Bending.*—The maximum bending in a column occurs at the top of the column in the basement; at the foot in the first, at the top in the second, etc., alternating thus to the roof line.

It was shown in Chapter VIII that the unit stress in the column due to bending from uniform floor loads may be nearly *four* times that due to the axial load (*cf.* 6,415 and 1,725; also 4,865 and 1,322). In the case of similar girders and floor loadings the maximum bending moments are about the same throughout the height of the building.

The value of the maximum bending in a column may be fairly averaged without very serious error in ordinary designs by the expression $wL^2/24$ where w is the load per lineal inch of floor girder and L is the span, in inches, of the floor girder connecting to the column. In case it is an interior column the moment will be expressed as $wL^2/24 - w'L'^2/24$ where a prime designates the quantities for the extra span.

In designing the girders the flanges should be designed to properly transmit the stresses occurring at their connections to the columns. These stresses are fairly large.

(*c*) *Suggestions for Design.*—Use the method of Chapter II for axial column stresses and for determining flange areas of girders.

Provide for a column bending moment of $wL^2/24$ as noted above, at both top and bottom of the column section in computing maximum stresses from combined loads: these moments will be of opposite sign.

Design girder connections to transmit flange stresses equal to three quarters of the maximum flange stress at the center of the girder as computed by the method of Chapter II.

E. MAXIMUM EFFECTS OF COMBINED LOADINGS.

It was shown in Chapter IX that under combined loads the maximum stresses in the extreme fibres of the columns and in the flanges of the girders will exceed those as found by the method of Chapter II by percentages increasing from the bottom to the top of the structure and it was found that these percentages reached as high as 100 per cent at the top of the building.

F. THE VALUE AND EFFICIENCY OF KNEE BRACING.

In Chapter XII it has been shown:

(*a*) That the unit stresses in knee braces can not be very large; say not over about 4,000 pounds per square inch, and that the customary design, in which a large unit stress is used, is widely in error.

(*b*) That knee braces do not appreciably diminish the maximum bending in the column unless placed at both top and bottom.

(*c*) That there can be no reversal in bending moment at the bottom of the girder, as has been assumed in mathematical discussions of plate girder portal stresses.

(*d*) That when knee braces are used, top and bottom, a considerable reduction in column bending moment can be effected.

(*e*) That the size of the knees should first be assumed and the reduction in column bending moment then computed; a few simple trials will give the most desirable brace to use.

(*f*) That the simple approximate method given in the early part of Chapter XII is a practical and closely approximate one and is recommended to the profession.

(*g*) That the assumption that the shears across the columns are proportional to the moments of inertia of the columns is close enough for all practical purposes for knee braced portals.

G. EXPERIMENTAL CONCLUSIONS (CHAPTER XV).

Effect of Wind. (*a*) *Deflection and Vibration.*—It is concluded that, when proper account is taken of the velocity and pressure of the wind, that vibrations and deflections, properly computed on the basis of the steel frame resisting all the pressure, will be only slightly in excess of those actually existing. It was shown that the walls and partitions probably reduce the deflection of the seventeen story structure tested only about 40 per cent.

(*b*) *Column Stresses.*—It was found that the wind had no noticeable effect whatever on the basement columns at the points tested. However the extensometers were necessarily located in a most unfavorable position for observations for wind stresses.

It was shown that the sections in the front and rear walls, so much more rigidly braced, probably take care of almost the entire wind pressure, because of the partial inability of the interior bracing to properly transmit the bending moments at connections.

Effect of Temperature.—It was found that in five days time there was a range of stress of over 6,000 pounds per square inch and in the course of the two months experiments a total range of probably over 8,000 pounds per square inch in the column unit stress. It was shown that in all probability these variations of stress were produced by gradual changes of temperature, but that they are probably independent of seasonal changes. Indications point to a range of 22,000 pounds per square inch in an interior column (column 12) but no tests were secured to verify it.